Student name : *Mark* : *Initials*: VF

Lancaster University Department of Physics

PHYS421 - Michaelmas Term 2010
Sheet 1 (total mark = 10)

THIS SHEET MUST BE ATTACHED TO YOUR ANSWERS — please insert your name at the top of this page and ensure your work is clearly legible. Do not submit your work in folders or plastic sleeves. Your answers should be placed in the appropriate IN-BOX in the Physics Foyer not later than 12:00 on Tuesday 26 October 2010. Work handed in after the above time and before the seminar on Thursday 12pm will be subject to a 10% reduction. Work handed in later than this will not count towards your continuous assessment.

I declare that this submission is my own work. I have not submitted it in substantially the same form towards the award of a degree or other qualification. It has not been written or composed by any other person and all sources have been appropriately referenced or acknowledged.

Signed:

1. [2] **Use of expressions involving vectors.** Consider two vectors, $\vec{R}$ and $\vec{r}$ such that $R \gg r$. (a) Using the method of Taylor expansion, show that in the first order in a small parameter r/R,

$$\frac{1}{|\vec{R}-\vec{r}|} \equiv \frac{1}{\sqrt{R^2 - 2\vec{R}\cdot\vec{r} + r^2}} \approx \frac{1}{R} + \frac{\vec{R}\cdot\vec{r}}{R^3}.$$

(b) Using the rules of the operation with vector products, show that for those two vectors and any arbitray vector $\vec{v}$ the following expression is valid $(\vec{r}\times\vec{v})\times\vec{R} = (\vec{R}\cdot\vec{r})\vec{v} - (\vec{R}\cdot\vec{v})\vec{r}$, and use that to prove that

$$(\vec{R}\cdot\vec{r})\vec{v} = \frac{1}{2}(\vec{r}\times\vec{v})\times\vec{R} + \frac{1}{2}\left\{(\vec{R}\cdot\vec{v})\vec{r} + (\vec{R}\cdot\vec{r})\vec{v}\right\}.$$

2. [2] **Vector analysis.** $\vec{R}$ is a three dimensional radius vector: $\vec{R} = (x, y, z)$. Evaluate $div\vec{R} \equiv \nabla\cdot\vec{R}$ and $curl\vec{R} \equiv \nabla\times\vec{R}$.

3. [4] **Magnetic dipole moment.** Find magnetic field generated in the vacuum by a magnetic dipole moment $\overrightarrow{m} = m\overrightarrow{l}_z$ created by a small current loop with a radius r_0 at distances R much larger than r_0. Draw a diagram of magnetic field lines illustrating the distribution of magnetic field in space.

4. [2] **Interaction of two magnetic dipole moments**. Consider two magnetic dipole moments, $\overrightarrow{m}_1$ and $\overrightarrow{m}_2$ separated by the distance R. Using the result in 3, write down the energy of their magnetic interaction and find what would be their lowest energy configuration (that is, their mutual orientation and their orientation with respect to the straight line connecting them).