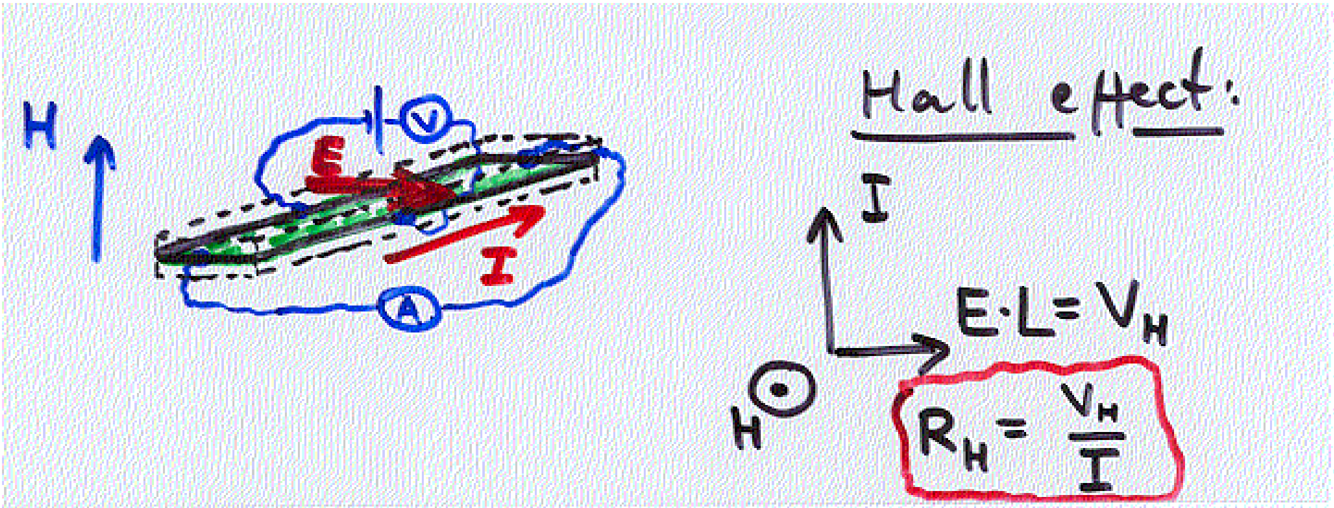
Lecture 19-20

Quantum Hall effect in 2D electron systems and its interpretation in terms of edge states of Landau levels

Two-dimensional electrons
in GaAs/AlGaAs heterostructure,
or in a Si/SiO$_2$ field-effect transistor



$$\vec{F}_{Lorentz} = e\mu_0 H\vec{v}\times\vec{l}_z$$

Quantum Hall effect - experiment

weak magnetic field regime

$$R_H = \frac{\mu_0 H}{e\rho_e}$$

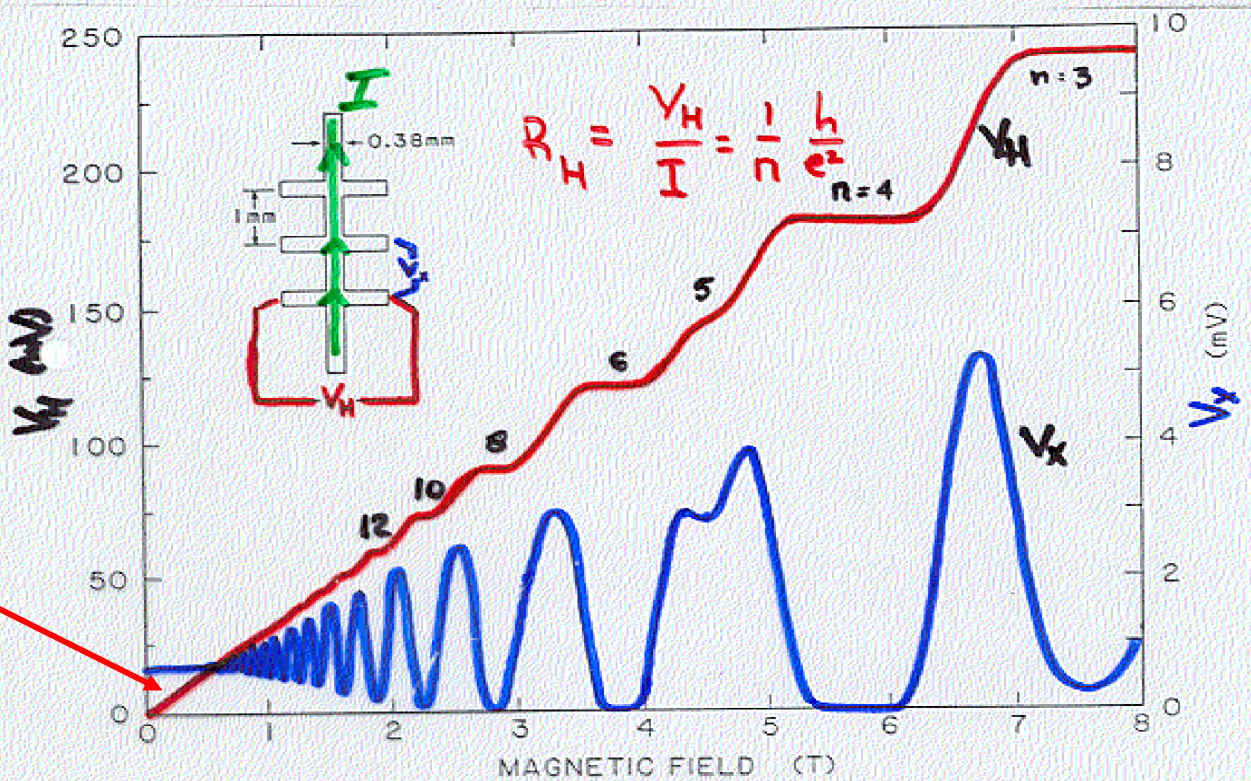
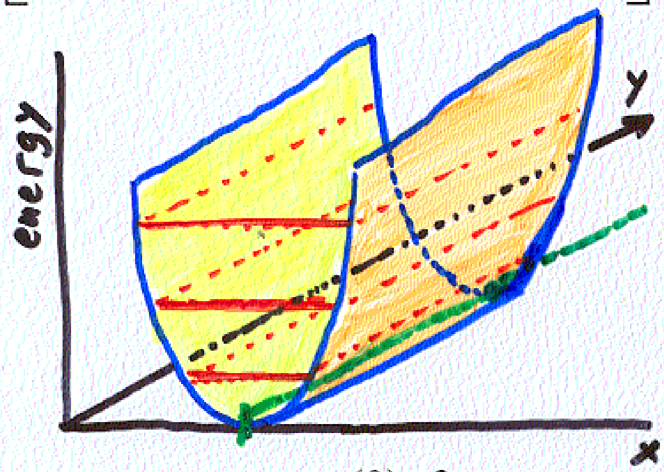


Figure 2.2. Chart recordings of V_H and V_x *vs.* B for a GaAs-AlGaAs heterostructure cooled to 1.2 K. The source-drain current is 25.5 μA and $n = 5.6 \times 10^{11}$ electrons/cm^2. Cage *et al.* (1985).

Discovered by K. von Klitzing in 1980

$$\left[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{x}^2}+\frac{m_e\omega_c^2\tilde{x}^2}{2}\right]\phi = E\phi$$

Landau levels for electrons in a magnetic field



maximal capacity of one Landau level $\rho_{LL}=\frac{1}{2\pi(r_c^{(0)})^2}=\frac{eB}{2\pi\hbar}$

$$x_{osc}(q)=(r_c^{(0)})^2 q$$

filling factor $=\frac{\rho_e}{\rho_{LL}}=\frac{2\pi\hbar\rho_e}{eB}$ is equal to the ratio of magnetic flux quantum to a magnetic flux per electron.

$$\Phi_{n,q}=\frac{e^{iqy}}{\sqrt{\pi}}h_n\exp\left\{-\frac{(x-(r_c^{(0)})^2 q)^2}{2(r_c^{(0)})^2}\right\}$$

$$n=0,1,2,\ldots.$$

$$E_n=\left[\frac{1}{2}+n\right]\hbar\omega_c$$

the same for all values of **q**, that is for all positions of a magnetic oscillator centre: infinite degeneracy.

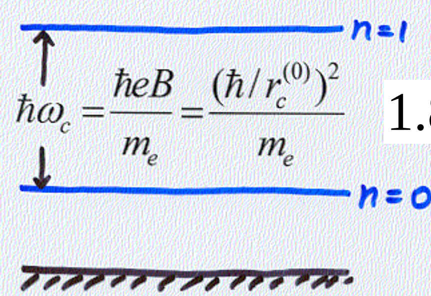
Realistic parameters for experimental conditions

B = 1 Tesla

$r_c^{(0)}=\sqrt{\hbar/eB}\approx 260\,\dot{A}$ $\quad\rho_{LL}=\frac{eB}{2\pi\hbar}\approx 2.4\cdot10^{10}cm^{-2}$

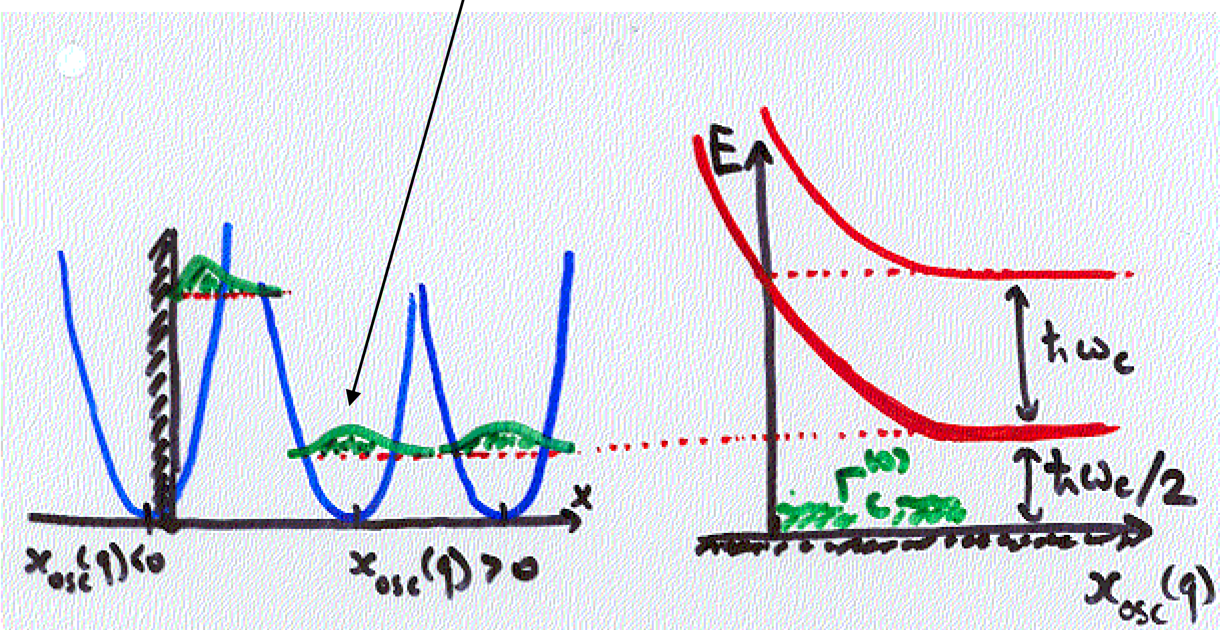
Si $(0.2m_e^{(0)})$ **GaAs** $(0.067m_e^{(0)})$

$0.6mev$ $\quad\hbar\omega_c=\frac{\hbar eB}{m_e}=\frac{(\hbar/r_c^{(0)})^2}{m_e}$ $\quad1.8mev$

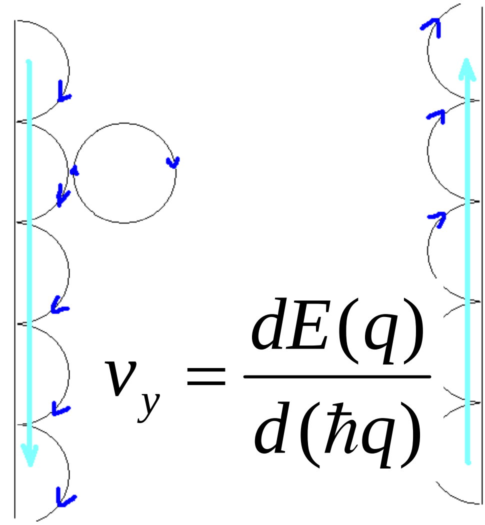


Edge states of Landau levels and edge currents

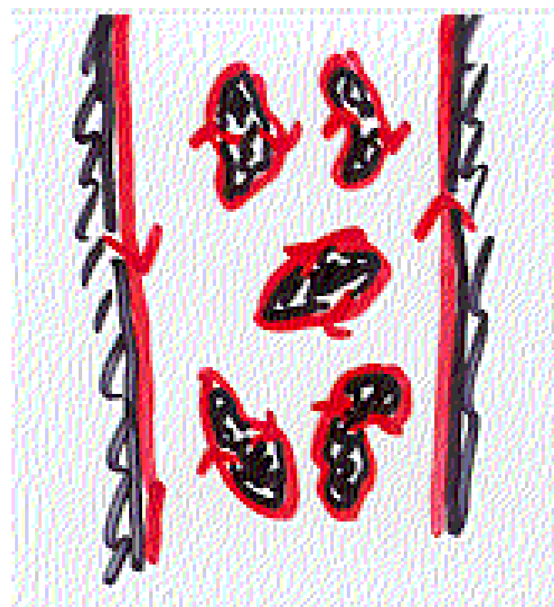
$$\Phi_{n,q} = \frac{e^{iqy}}{\sqrt{\pi}} h_n \exp\left\{ -\frac{(x-(r_c^{(0)})^2 q)^2}{2(r_c^{(0)})^2} \right\}$$



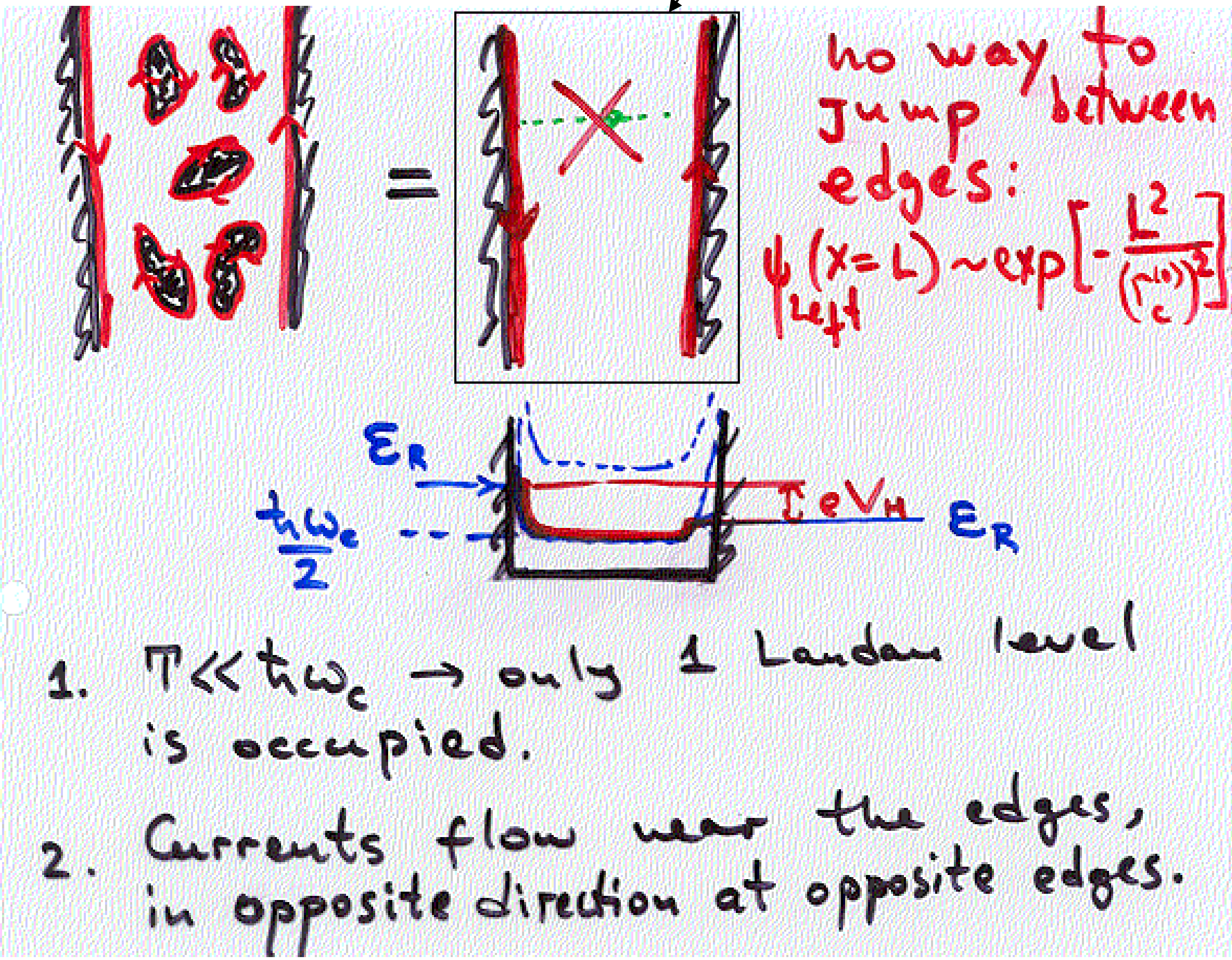
$$x_{osc} = \hbar q / eB$$



$$v_y = \frac{dE(q)}{d(\hbar q)}$$



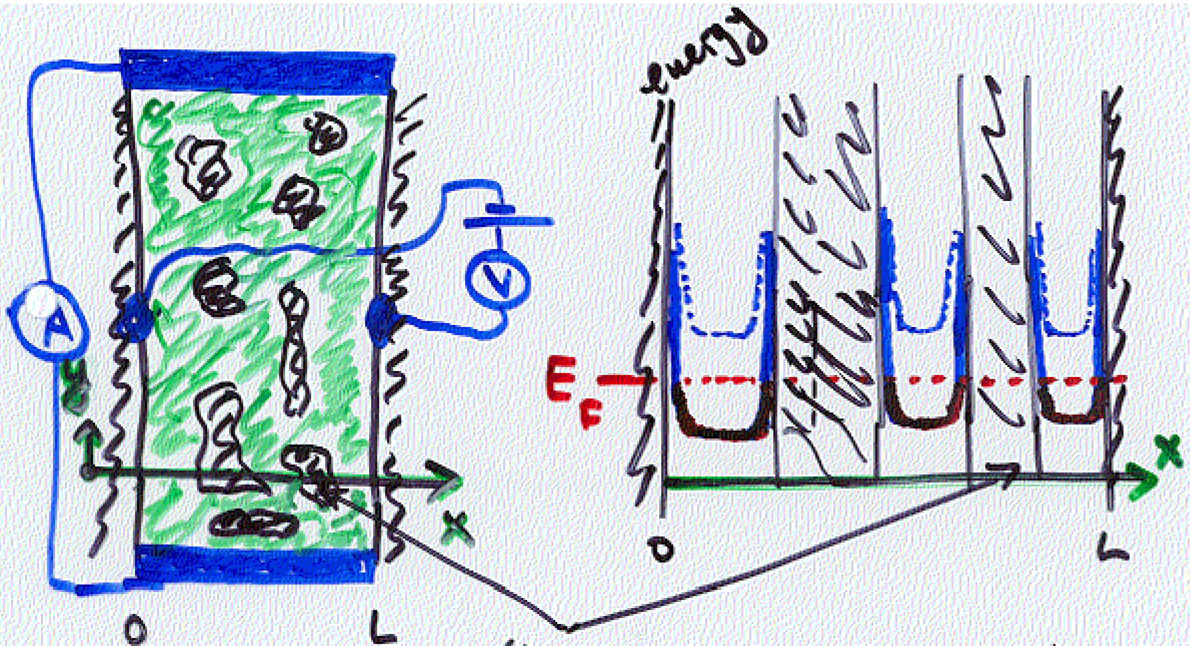
an ideal ‘ballistic quantum wire‘



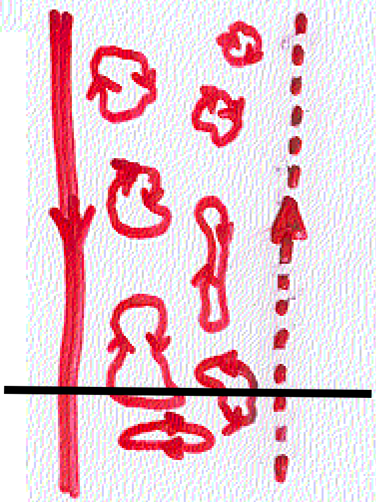
filling factor

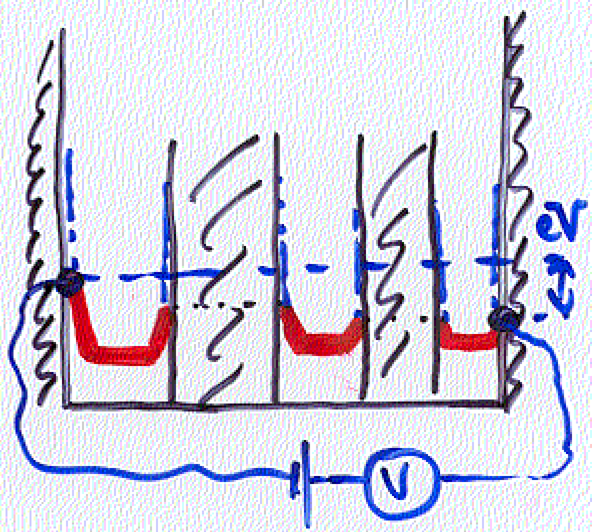
$2 \pm \delta$

(both spin states are filled int he lowest Landau level)

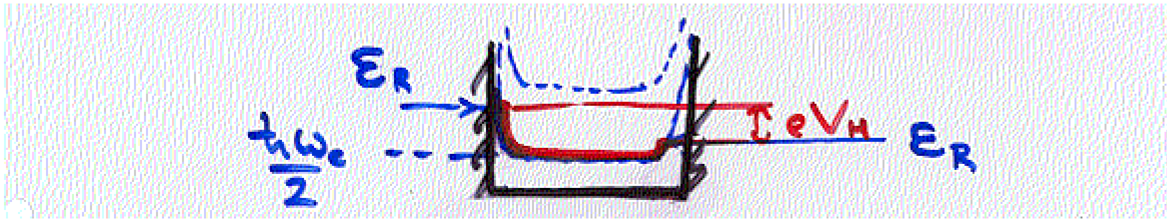


'bad regions', where doping is wrong, and there is





‘ballistic quantum wire‘ due to edge states: edge current



3. At $V = 0 \Rightarrow I_{\rightarrow} + I_{\leftarrow} = 0$

$V \neq 0 \Rightarrow I_{\rightarrow} + I_{\leftarrow} \neq 0$

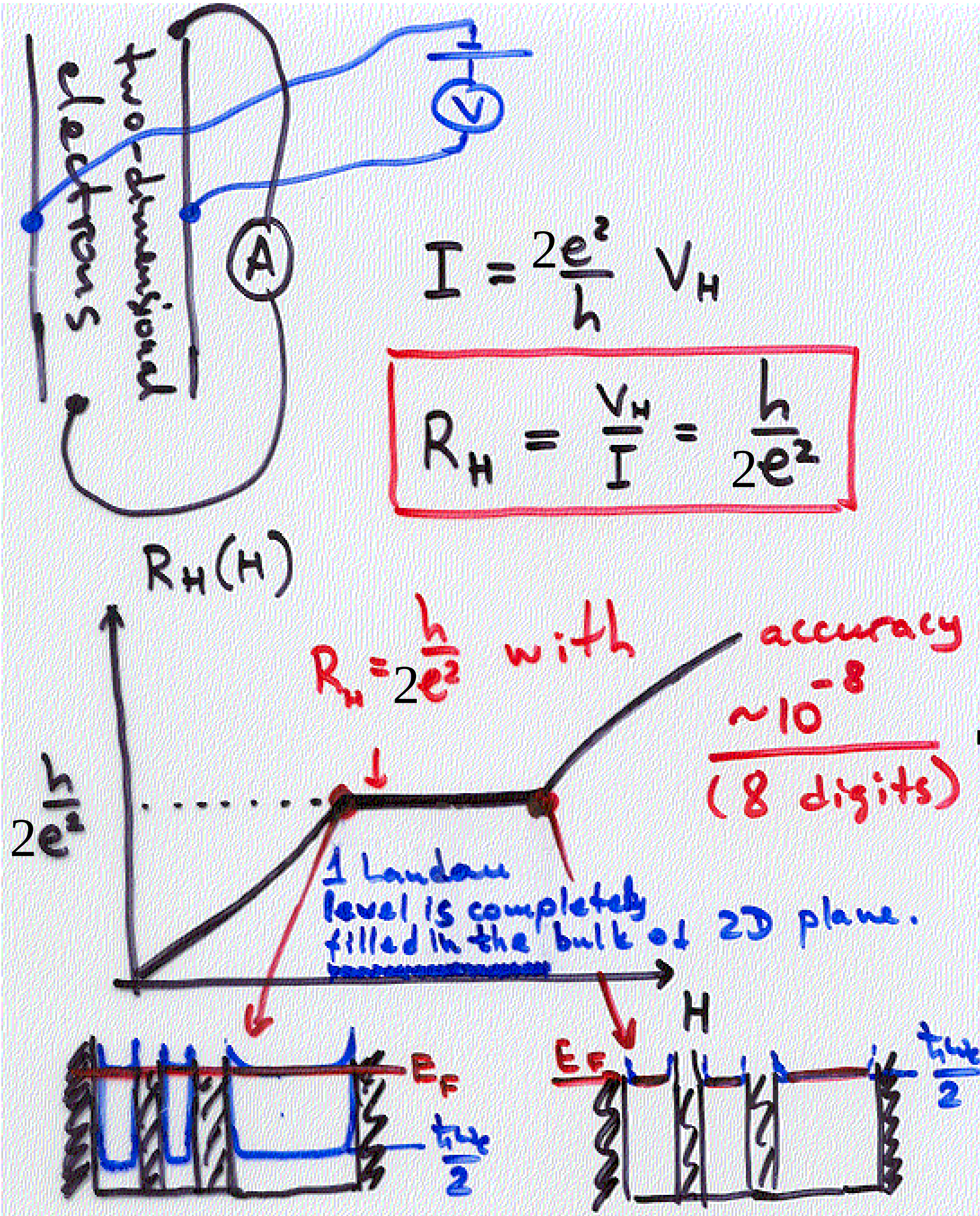
$$I_{\rightarrow} = 2\int_{q_{right}}^{\infty} e\, v_{right}(q)\, \frac{dq}{2\pi} = \frac{2e}{2\pi\hbar}\int_{\hbar\omega_c/2}^{\varepsilon_R} d\varepsilon = \frac{2e\left(\varepsilon_R - \frac{\hbar\omega_c}{2}\right)}{2\pi\hbar}$$

$$v = \frac{d\varepsilon(p)}{dp} = \frac{d\varepsilon(\hbar q)}{\hbar\, dq}$$

$$I = I_{\rightarrow} + I_{\leftarrow} = \frac{2e}{2\pi\hbar}\left(\varepsilon_R - \frac{\hbar\omega_c}{2}\right) - \frac{2e}{2\pi\hbar}\left(\varepsilon_L - \frac{\hbar\omega_c}{2}\right)$$

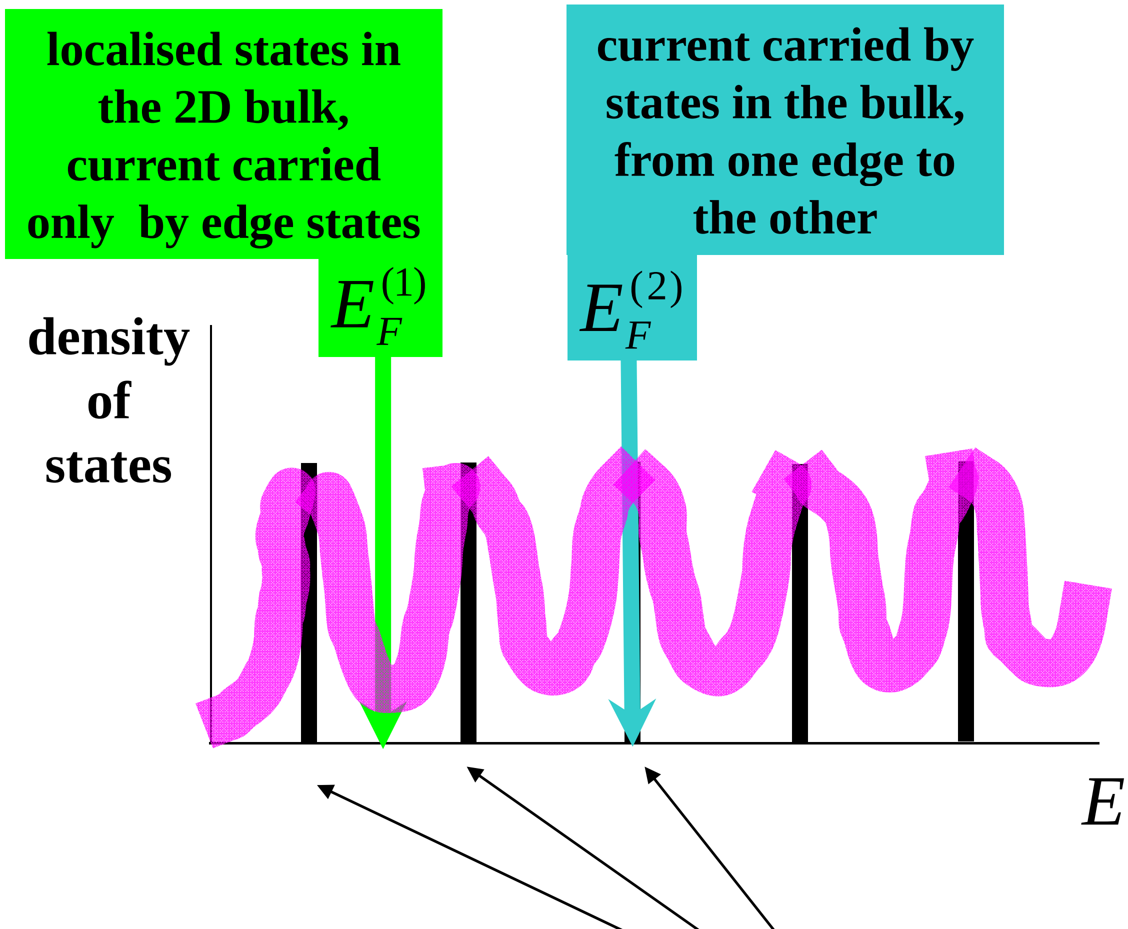
spin degeneracy

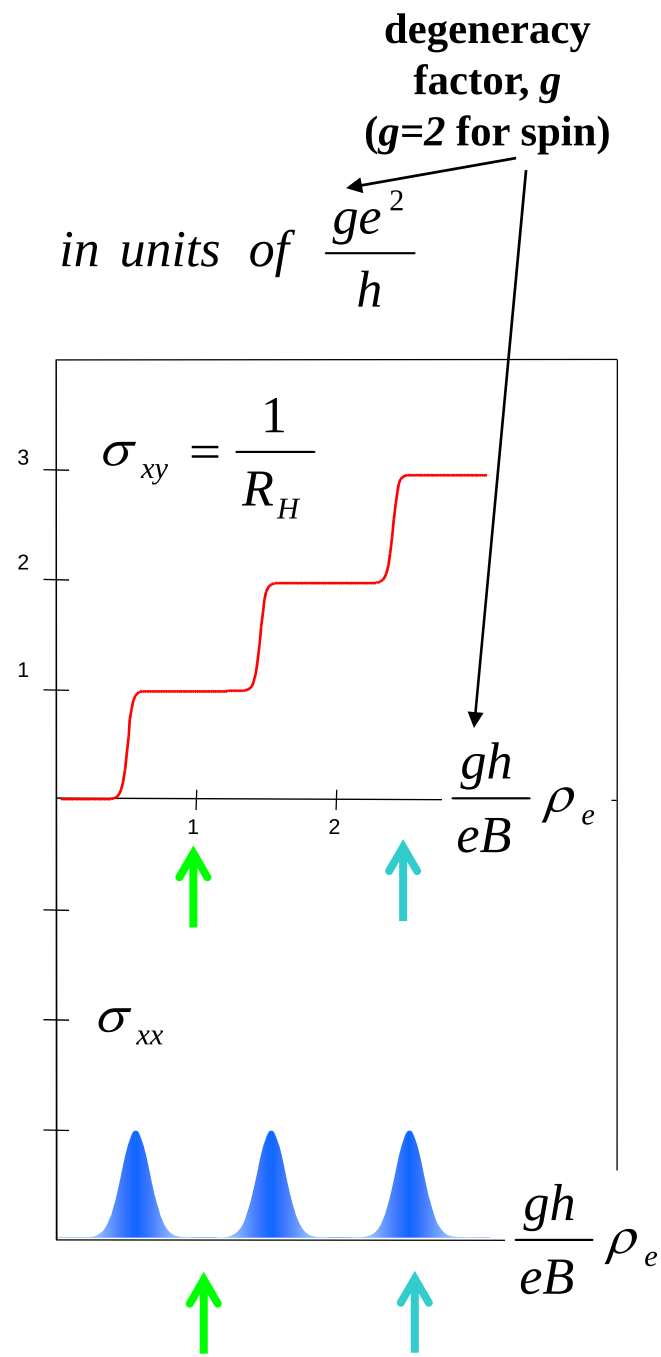
$$I = \frac{2e}{2\pi\hbar}\left(\varepsilon_R - \varepsilon_L\right) = \frac{2e}{2\pi\hbar}\cdot eV_H = \frac{2e^2}{h} V_H$$



Quantum resistance standard (1 Klitzing)

Integer quantum Hall effect





$$H = \frac{(-i\hbar\nabla - \frac{e}{c}\vec{A})^2}{2m} \Rightarrow E_n = (n + \tfrac{1}{2})\hbar\omega_c$$

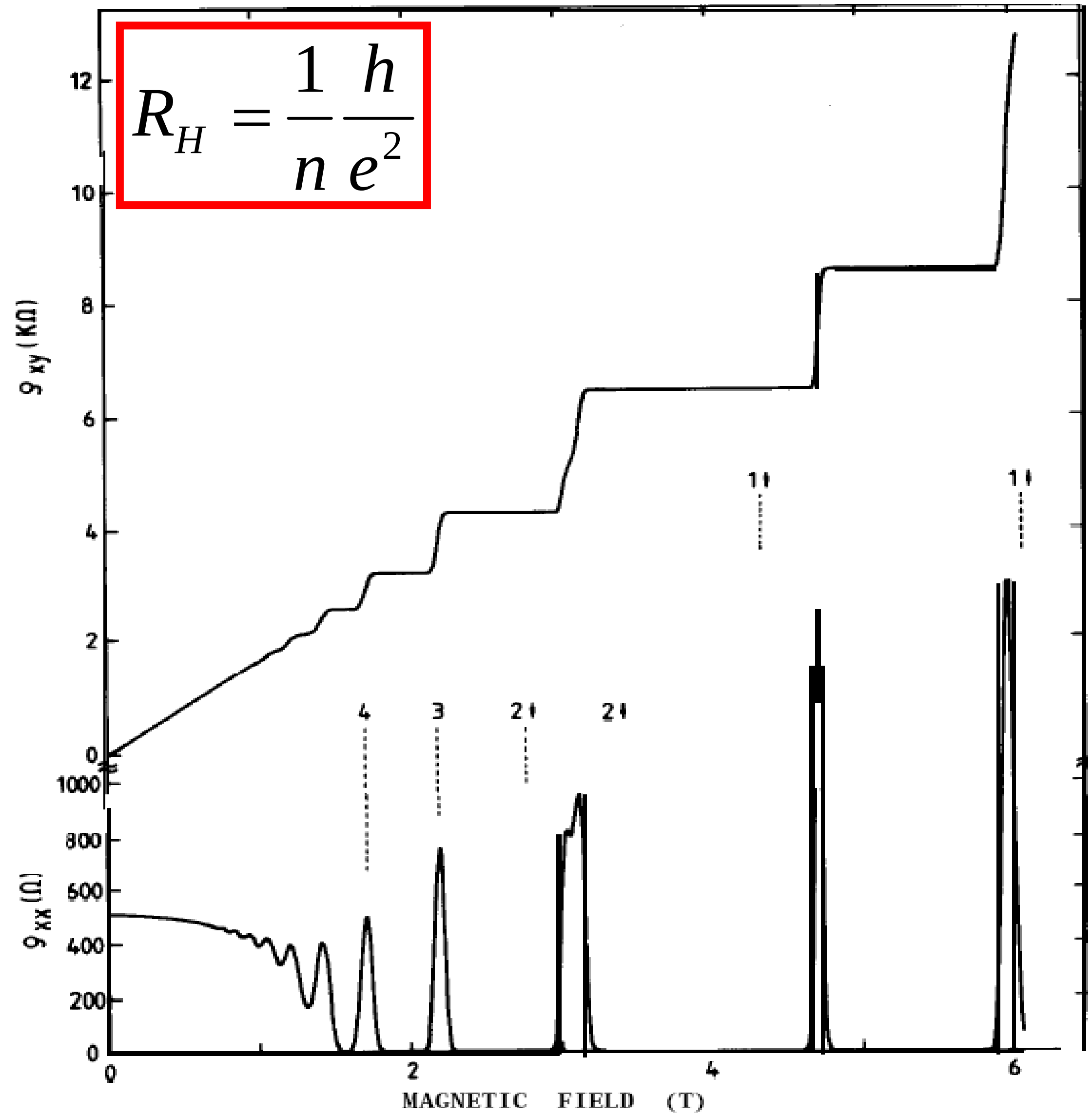
The Nobel Prize in Physics 1985

"for the discovery of the quantized Hall effect"

(1980)

Klaus von Klitzing

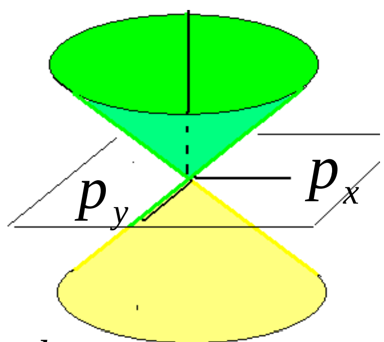
$$R_H = \frac{1}{n}\frac{h}{e^2}$$



n **= number of filled spin-polarised Landau levels**

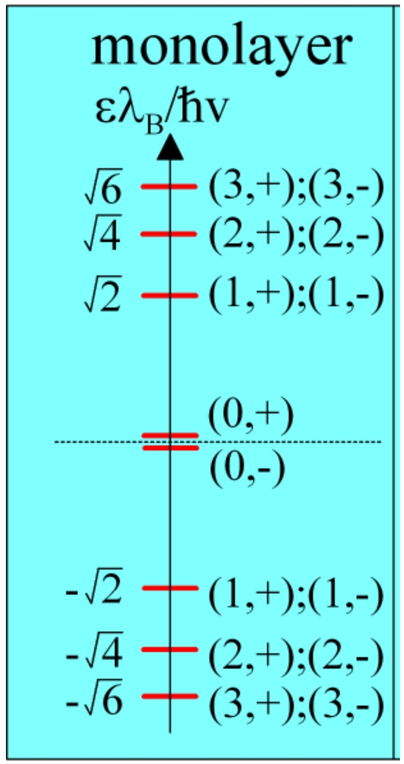
‘relativistic-type’
Landau level spectrum

$$\varepsilon^{cond} = vp$$



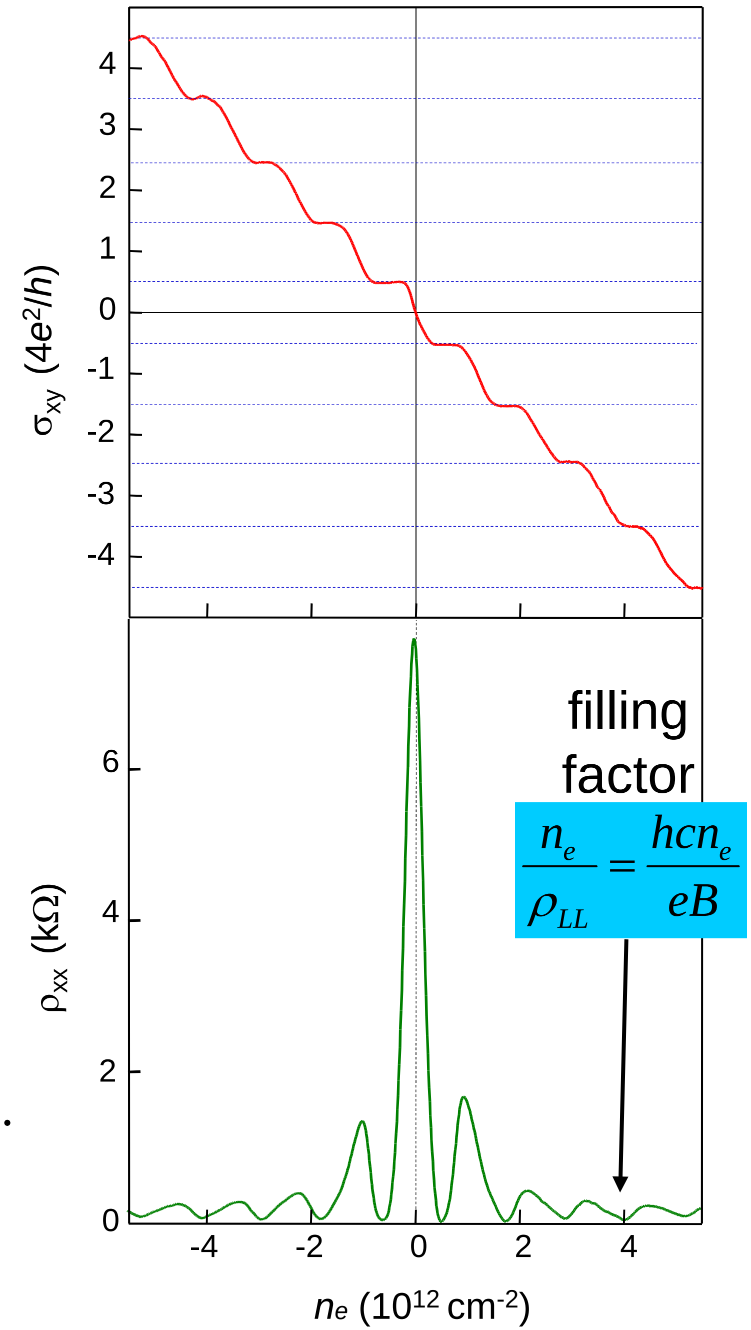
$$\varepsilon^{valence} = -vp$$

$$v \sim 10^8\, cm/\sec$$



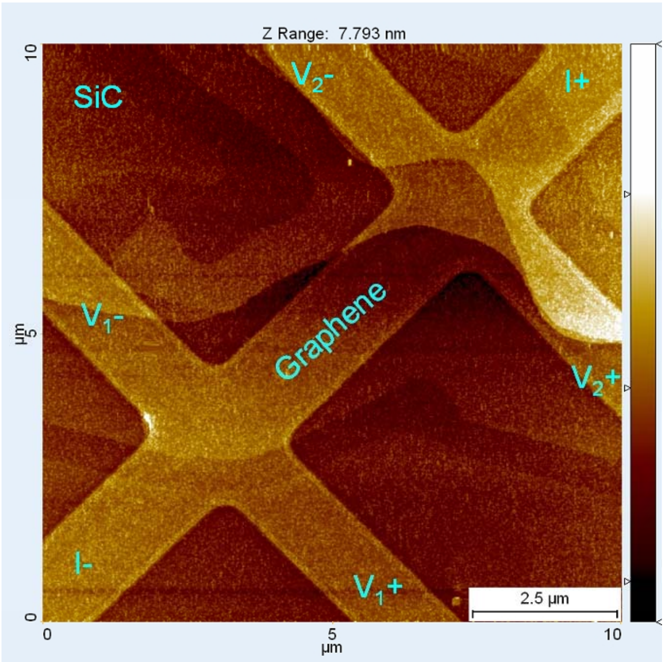
$$\varepsilon_n^{\pm} = \pm\sqrt{2n}\,\frac{\hbar v}{\lambda_B} \qquad n = 0,1,2,3\ldots$$

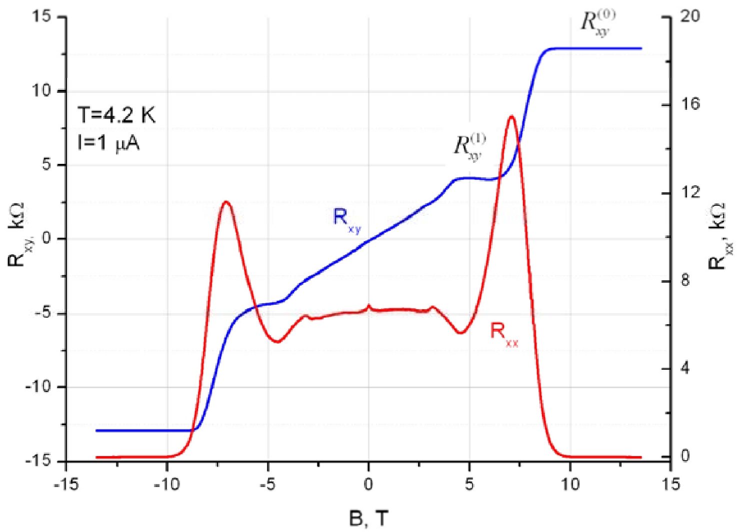
$$\lambda_B \equiv r_c^{(0)} = \sqrt{\frac{\hbar c}{eB}} \qquad \text{magnetic length}$$



Graphene synthesised on SiC
flake growth and lies as a carpet over SiC substrate.

Seyller (Erlangen), Yakimova (Linkoping)





Development of QHE resistance standard using SiC-synthesised graphene.
Currently: 9 digit accuracy
(Chalmers, NPL-UK, Lancaster)