STOCHASTIC PATH INTEGRAL APPROACH TO FCS. LECTURE 2: APPLICATIONS

EUGENE SUKHORUKOV

1. Cascade diagrammatics.
2. Electrical circuits: Environmental corrections.
3. Instant fluctuations.
4. Non-perturbative effects: super-Poissonian noise.
5. Mesoscopic threshold detectors.
6. Quantum-to-classical crossover in FCS.
7. Various applications.

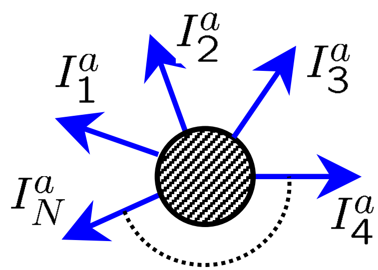
References:

Jordan, Sukhorukov, Pilgram, J. Math. Phys. **45**, 4386 (2004)
Jordan, Sukhorukov, Phys. Rev. Lett. **93**, 260604 (2004)

DIAGRAMMATICS FOR CURRENT CUMULANTS

- **large parameter ⇒ SP approximation ⇒ no loops!**
- **low-order ⇒ high-order cumulants**
- **finite number of diagrams**

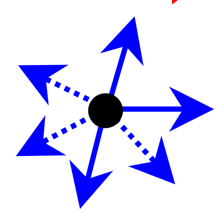
$$\langle\langle I_1^a I_2^a \cdots I_N^a \rangle\rangle =$$



One node example:

$$I = \frac{G_R I_L + G_L I_R}{G_L + G_R}$$

$$\langle\langle I^n \rangle\rangle_m = \frac{G_R^n \langle\langle I_L^n \rangle\rangle + G_L^n \langle\langle I_R^n \rangle\rangle}{(G_L + G_R)^n}$$



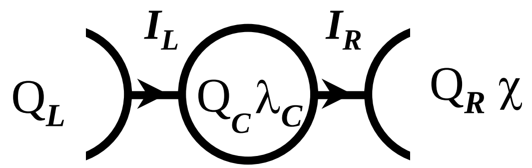
RULES:

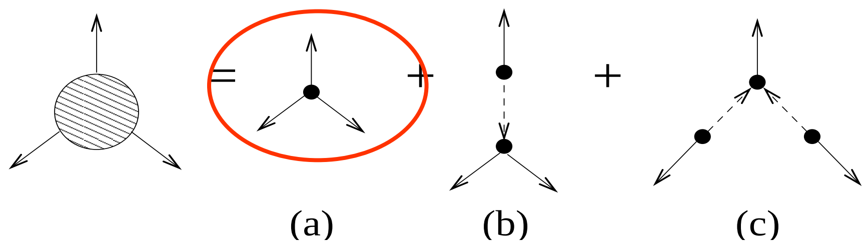
- **solve Langevin Eq., find** $\langle\langle \ldots \rangle\rangle_m$ **vertexes** ⇒
- **external lines:** ⟶ **for** I_α^a
- **internal lines:** Q_α^c ⋯⟶ $\partial(\ldots)/\partial Q_\alpha^c$
- **vanishing vertexes:** ⋯⟶●⟶

Nagaev, PRB (2002)

Jordan, *E.S.*, Pilgram, J. Math. Phys. (2004)

DIAGRAMMATICS: EXAMPLE





$$\langle\langle I^3 \rangle\rangle = \langle\langle I^3 \rangle\rangle_m + 3\langle\langle I\, Q_C \rangle\rangle_m \frac{\partial}{\partial Q_C}\langle\langle I^2 \rangle\rangle_m + 3\langle\langle I\, Q_C \rangle\rangle_m^2 \frac{\partial^2}{\partial Q_C^2}\langle\langle I \rangle\rangle_m$$

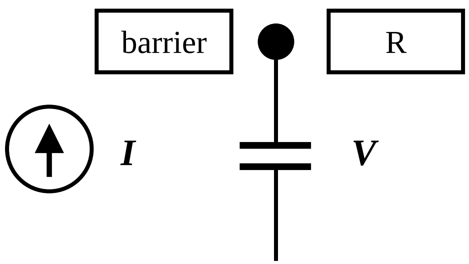
Nagaev, PRB (2002)

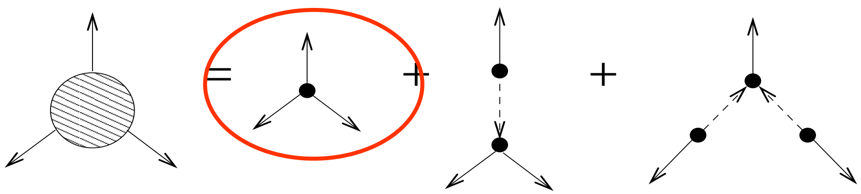
Jordan, *E.S.*, Pilgram, J. Math. Phys. (2004)

APPLICATION: TUNNEL JUNCTION

Third cumulant:

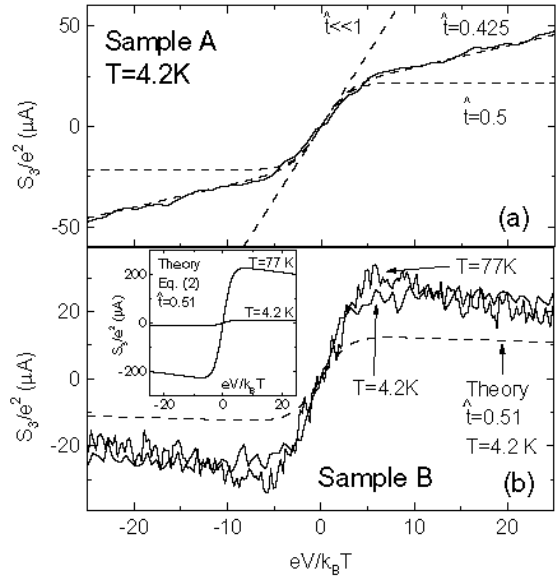
$$\langle\langle I^3 \rangle\rangle = e^2 I + \text{corrections}$$

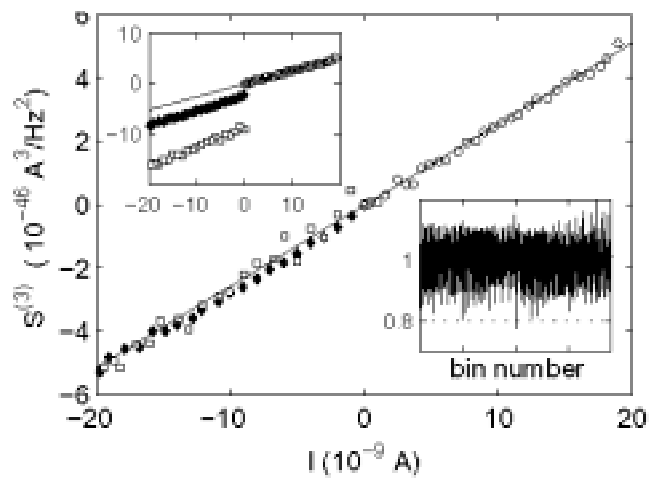




(a) (b) (c)

$$\langle\langle I^3 \rangle\rangle = \langle\langle I^3 \rangle\rangle_m + 3\langle\langle I\,V \rangle\rangle_m \frac{\partial}{\partial V}\langle\langle I^2 \rangle\rangle_m + 3\langle\langle I\,V \rangle\rangle_m^2 \frac{\partial^2}{\partial V^2}\langle\langle I \rangle\rangle_m$$

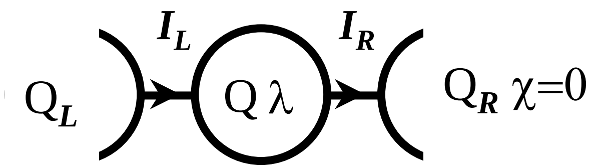


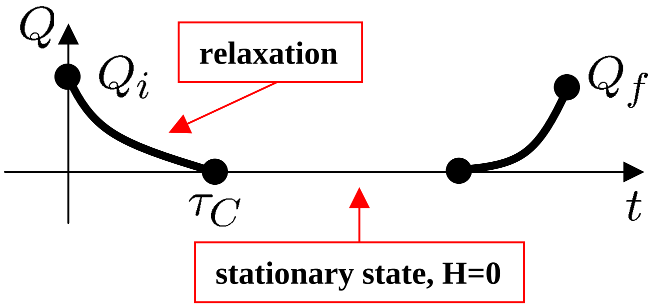


Reulet, et al., PRL (2003)

Reznikov, et al., cond-mat/0504382

INSTANT FLUCTUATIONS





Evolution for $t \gg \tau_C$:

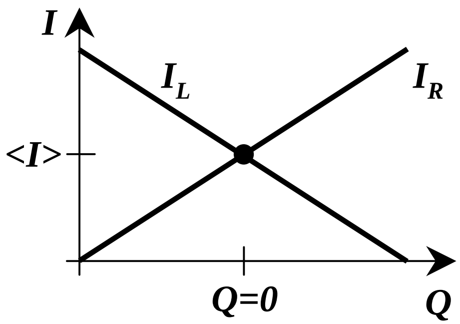
$$U(Q_f, Q_i) = U(Q_f)U(Q_i)$$

Instant fluctuations:

$$U(Q_f) = \exp[S_{\rm sp}(Q_f)].$$

Hamiltonian:

$$H = H_L(\lambda) + H_R(-\lambda)$$
$$= (I_L - I_R)\lambda + (1/2)(F_L + F_R)\lambda^2$$



Jordan, *E.S.*, Pilgram, J. Math. Phys. (2004)

ZERO “ENERGY” LINES

Integral of motion:

$$\dot{H} = \partial_\lambda H \dot{\lambda} + \partial_Q H \dot{Q} = 0$$

$$H_{\text{sp}}(\lambda, Q) = 0$$

Saddle-point action:

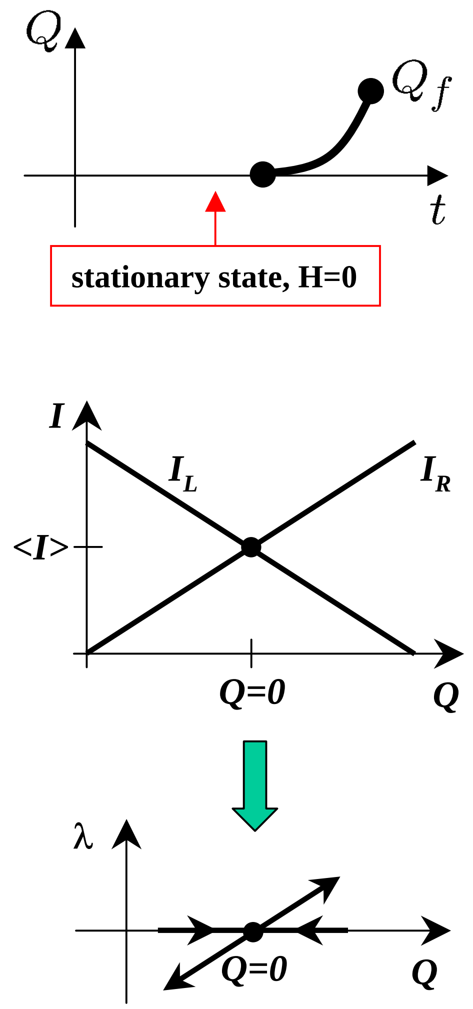
$$S_{\text{sp}} = \int dt[-\lambda \dot{Q} + H_{\text{sp}}] = -\int \lambda(Q) dQ$$

Zero “energy” lines:

$$-(G_L + G_R)Q\lambda + (1/2)(F_L + F_R)\lambda^2 = 0$$

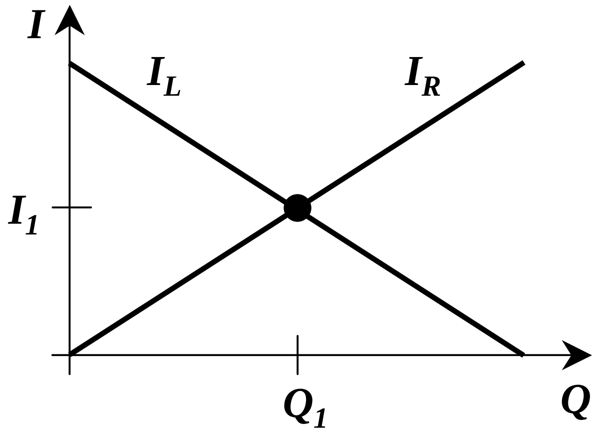
Result:

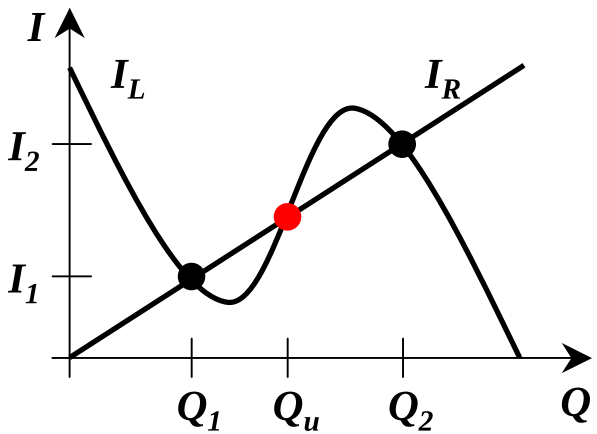
$$S_{\text{sp}} = -[(G_L + G_R)/(F_L + F_R)] \cdot Q^2$$



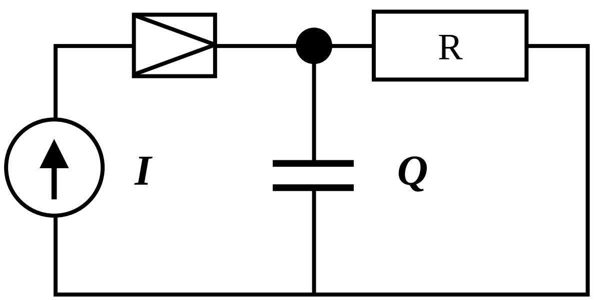
Jordan, *E.S.*, PRL 93, 260604 (2004)

NONLINEAR EFFECTS AND INSTABILITY

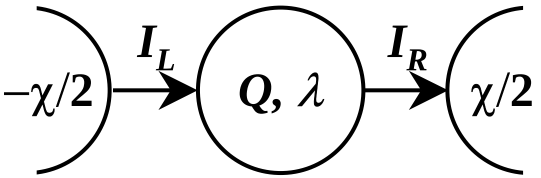




ELECTRICAL CIRCUIT



ABSTRACT NETWORK



DYNAMICS, FLUCTUATIONS =?

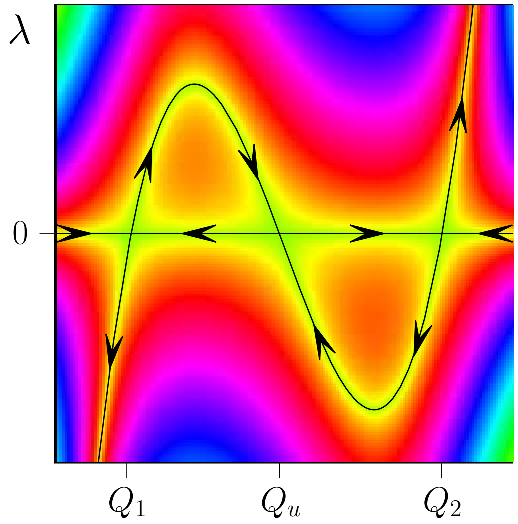
Landauer, J. App. Phys. (1962)

Hänggi and Thomas, Phys. Rep. (1982)

INSTANTON SOLUTION

Hamiltonian:

$$H(\lambda) = H_L(\lambda) + H_R(-\lambda)$$



Gaussian noise:

$$H = (I_L - I_R)\lambda + (1/2)(F_L + F_R)\lambda^2$$

$= E$, "*energy*" of trajectory

Instanton line:

$H_{\mathrm{sp}} = 0 \quad \Rightarrow \quad S = \int \lambda dQ$

$\lambda = 0$

$$\lambda_{\mathrm{in}} = -2(I_L - I_R)/(F_L + F_R)$$

General noise:

!

$$H_L(\lambda_{\mathrm{in}}) + H_R(-\lambda_{\mathrm{in}}) = 0$$

Poissonian processes:

$$H_0 = I_L(e^{\lambda} - 1) + I_R(e^{-\lambda} - 1)$$

$\Rightarrow \lambda_{\mathrm{in}} = -\ln[I_L/I_R]$. !

Jordan, *E.S.*, PRL 93, 260604 (2004)

Hänggi and Thomas, Phys. Rep. (1982)

INSTANTON RATES

Stability analysis:

$$\lambda = 0 \quad \Rightarrow \quad \dot{Q} = I_L - I_R,$$

$$\lambda_{\text{in}} \quad \Rightarrow \quad \dot{Q} = I_R - I_L$$

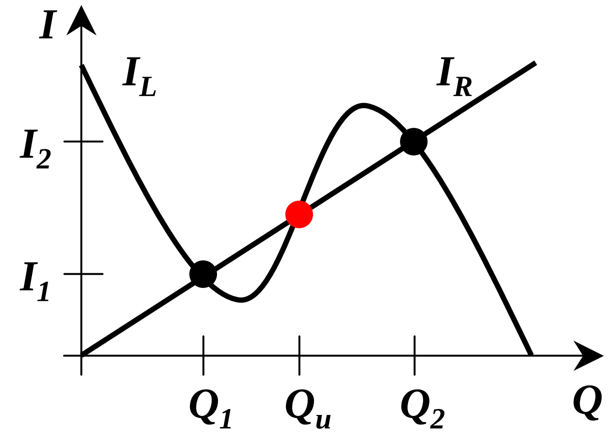
Instanton Action:

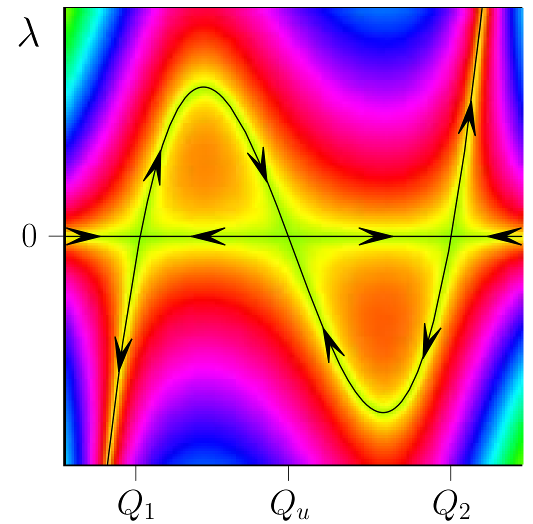
$$S_{1,2} = \int_{Q_{1,2}}^{Q_u} \lambda_{\text{in}}\, dQ$$

Switching rates:

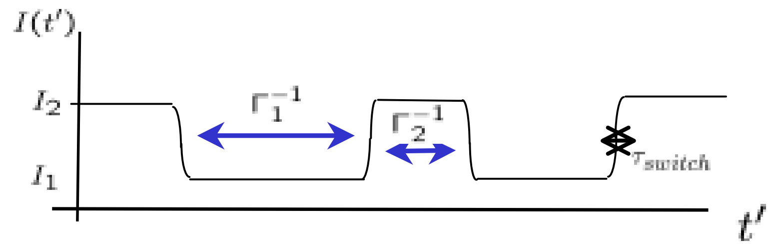
$$\Gamma_{1,2} = \omega_{1,2} \exp(-S_{1,2}) = small$$

$$\omega_{1,2} \sim 1/\tau_C \rightarrow \text{attempt frequency}$$





TELEGRAPH PROCESS

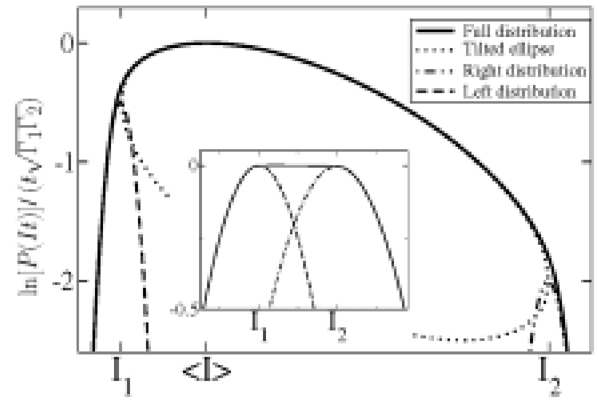


Effective Hamiltonian:

$$\dot{U} = \hat{H} U,$$

$$\hat{H} = \begin{pmatrix} H_1(\chi) - \Gamma_1 & \Gamma_2 \\ \Gamma_1 & H_2(\chi) - \Gamma_2 \end{pmatrix}$$

Current generator:



$$H(\chi) = (1/2)\sum_n (H_n - \Gamma_n) + \sqrt{(H_2 - H_1 - \Delta\Gamma)^2/4 + \Gamma_1\Gamma_2}$$

Current cumulants:

$$\langle I \rangle = \sum_{n=1,2} I_n P_n,$$

$$\langle\langle I^2 \rangle\rangle = \sum_n F_n P_n + 2(\Delta I)^2 \Gamma_1 \Gamma_2 / \Gamma_S^3,$$

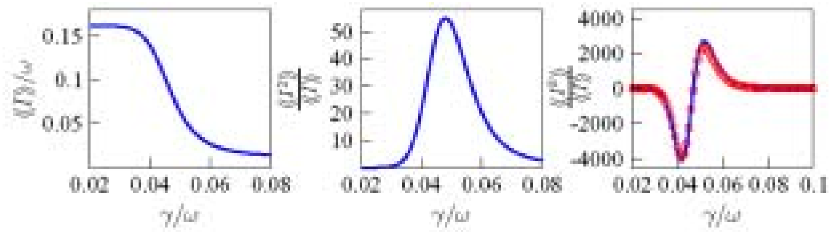
$$\langle\langle I^2 \rangle\rangle = \sum_n C_n P_n + 6(\Delta I)^3 \Gamma_1 \Gamma_2 \Delta\Gamma / \Gamma_S^5$$

Jordan, *E.S.*, PRL 93, 260604 (2004)

TELEGRAPH PROCESS, THRESHOLD DETECTORS

Third cumulant:

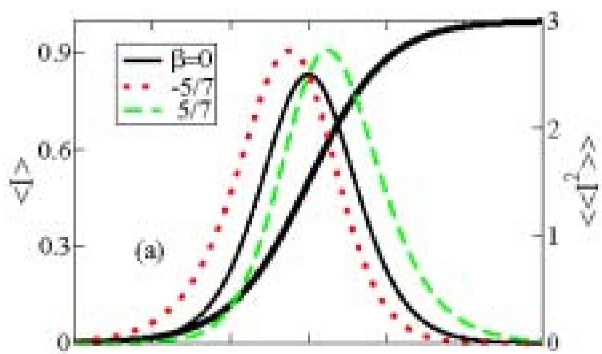
$$\langle\langle I^3\rangle\rangle_{tel} = 3\langle\langle I^2\rangle\rangle_{tel}^2 \frac{(I_1+I_2)/2-I}{(I_2-I)(I-I_1)}$$



Universal I-V:

NOISE OF SOURCE

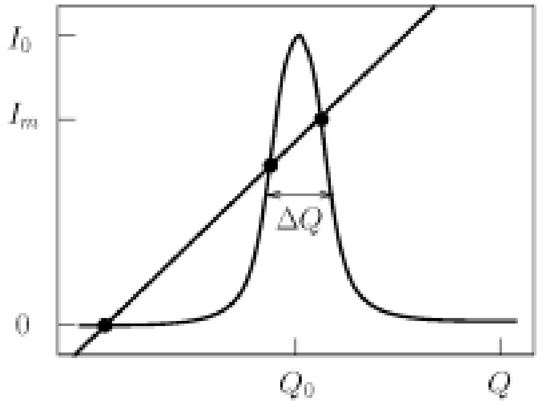
$$I(V) = (I_1+I_2)/2 + (\Delta I/2)\tanh[(C\,\Delta I/F)\,(V-V_0)]$$

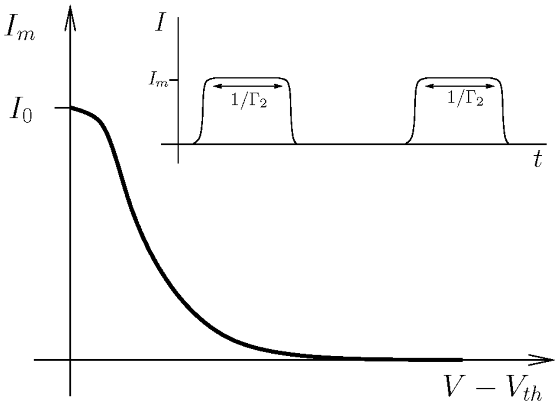


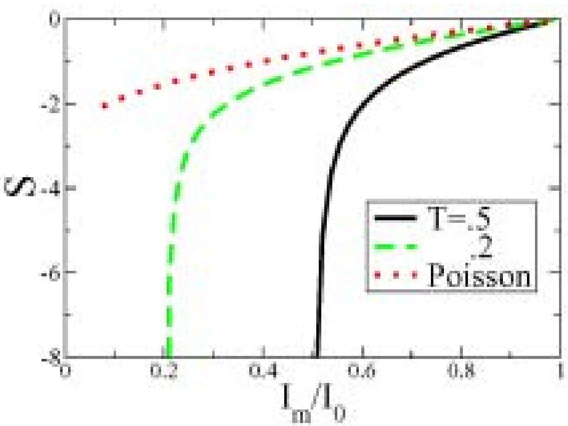
Jordan, *E.S.*, PRL 93, 260604 (2004)

Flindt et al., Europhys. Lett. (2005)

PAULI STABILIZATION







Action from experiment:

$$S(\mathcal{R}) = \log \Gamma/(V - V_{\text{th}}), \quad \mathcal{R} = I_m/I_0$$

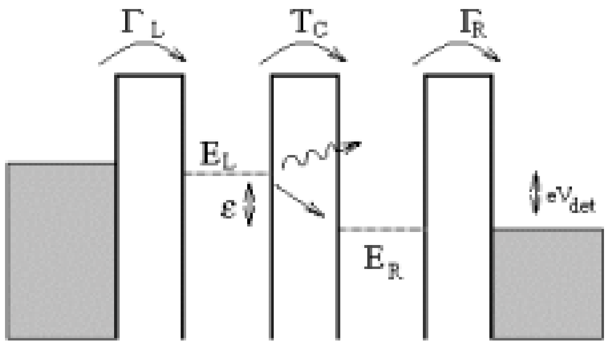
Examples:

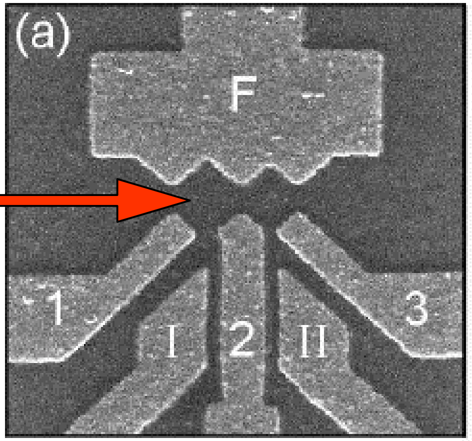
$$\lambda_{\text{in}} = \begin{cases} -1/\mathcal{R}, & \text{Gaussian} \\ \log \mathcal{R}, & \text{Poissonian} \\ (T \log T)/(\mathcal{R} - T), & \text{Binomial} \end{cases}$$

Jordan, *E.S.*, cond-mat/0503751

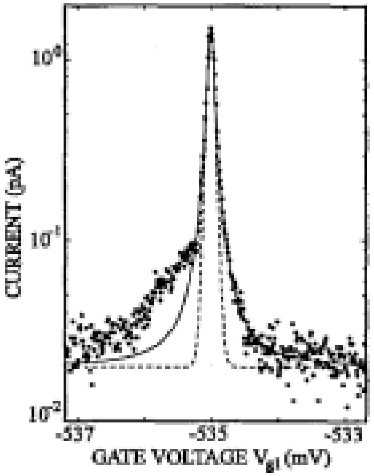
DOUBLE QUANTUM DOT DETECTOR

Double quantum dot:

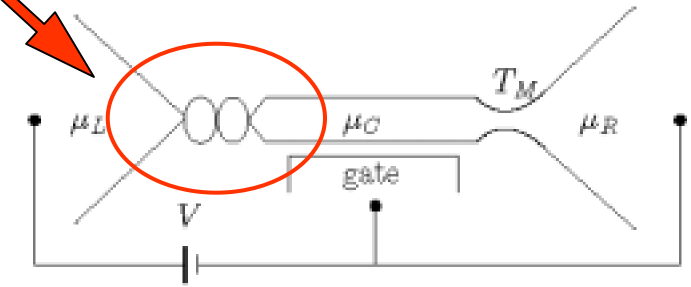




Current peak:



Threshold detector:



Kouwenhoven et al., Rev. Mod. Phys. (2003)

QUANTUM TO CLASSICAL CROSSOVER

Noise temperature:

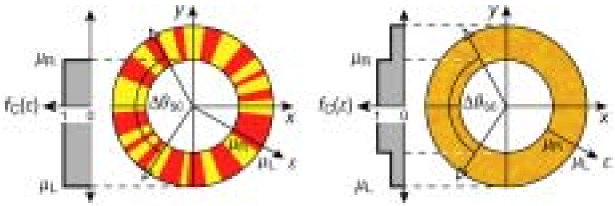
$S = GT_C$, G is conductance

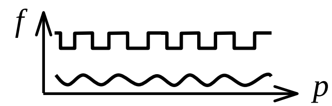
$T_C = \Delta\mu \langle f_p(1 - f_p)\rangle_p.$

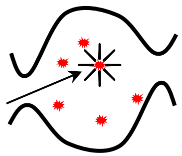
$T_C = \Delta\mu f_C(1 - f_C)$

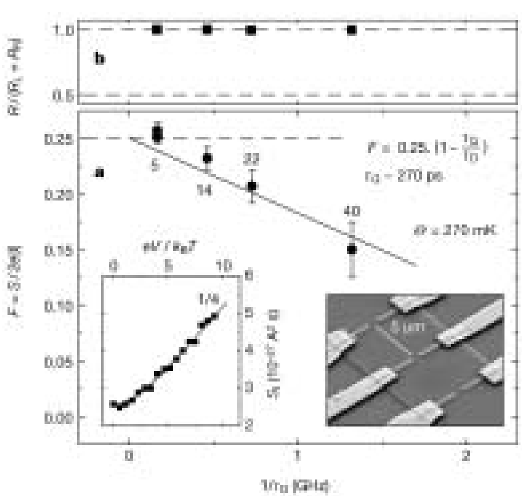
$\Rightarrow$ quantum limit,

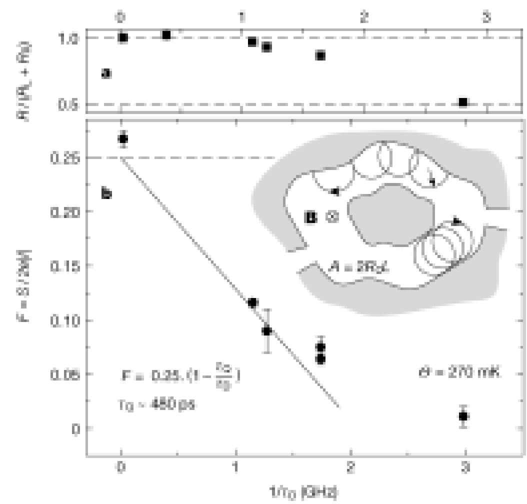
$T_C = 0 \Rightarrow$ classical limit.











Oberholzer, *E.S.* & Schönenberger, Nature 415, 2114 (2002)

CROSSOVER, SPI

Kinetic equation:

$$v\nabla f_p = -(f_p - f_C)/\tau_Q$$

where τ_Q is scattering time

$$v\nabla (f_p - f_C)^2 = -(2/\tau_Q)(f_p - f_C)^2$$

Result:

$$T_C = (\Delta\mu/4)(1+\tau_Q/\tau_D)^{-1}$$

$$\rightarrow \boxed{F = (1/4)(1+\tau_Q/\tau_D)^{-1}}$$

Solution, SPI:

$$\exp[S(\lambda,t)] = \iint \mathcal{D}\lambda_C \mathcal{D} f_C \exp[\Delta\mu n_F \lambda_C \dot{f}_C + H(\lambda_C, f_C)]\,,$$

where $H(\lambda_C) = H_L(\lambda_C - \lambda/2) + H_R(\lambda_C + \lambda/2)$

Binomial generators:

$$H_{L,R}(\lambda) = \Omega N_{L,R}\left\{\langle \ln[1 + f_p(e^{\lambda} - 1)]\rangle_p - \lambda f_{L,R}\right\}$$

$\Rightarrow$ semiclassics for Levitov result

E.S., Bulashenko, PRL 94, 116803 (2005)

CROSSOVER: LIMITS

Classical limit, $\tau_Q \to \infty$:

$\mathbf{v}_p \nabla f_p = 0 \Rightarrow f_p = 0, 1,$

$H_{L,R} = \Omega N (f_C - f_{L,R}) \lambda,$

$\Rightarrow I = \Omega N/2, \langle\langle I^n \rangle\rangle = 0$

Quantum limit, $\tau_Q \to 0$:

$\Rightarrow f_p = f_C$

Saddle Point: $f_C = 1/2, \lambda_C = 0$

$\mathcal{H} = 2\ln(1 + e^{\lambda/2}) - 2\ln 2$

$\Rightarrow$ RMT result with $F = 1/4$

E.S., Bulashenko, PRL 94, 116803 (2005)

CROSSOVER: DERIVATION

Step 1: $\nabla[\mathbf{v}_p(f_p - f_C)^k] + (k/\tau_Q)(f_p - f_C)^k = 0$

Step 2: $\int d\mathbf{r} \nabla \mathbf{v}_p(\ldots) = \int d\mathbf{s}\mathbf{v}_p(\ldots) \Rightarrow \boxed{\gamma = \tau_Q/\tau_D}$

$$\langle (f_p - f_C)^k \rangle = \tfrac{\gamma}{k+2\gamma}[(1-f_C)^k + (-f_C)^k]$$

Step 3: $\langle \ln[1 + f_p(e^\lambda - 1)] \rangle = \sum_k (c_k/k!) \langle (f_p - f_C)^k \rangle$

Step 4: minimization $\Rightarrow$ $f_C = 1/2$ and $\lambda_C = 0$

Result:

$$\boxed{\mathcal{H} = \lambda/2 + 2\int_0^1 \frac{du\, u^{2\gamma+1}}{\coth^2(\lambda/4) - u^2}}$$

E.S., Bulashenko, PRL 94, 116803 (2005)

CROSSOVER: TRANSPORT STATISTICS

Normalized cumulants:

$$F_n = \langle\langle I^n \rangle\rangle / I = \partial_\lambda^n \mathcal{H}|_{\lambda=0}$$

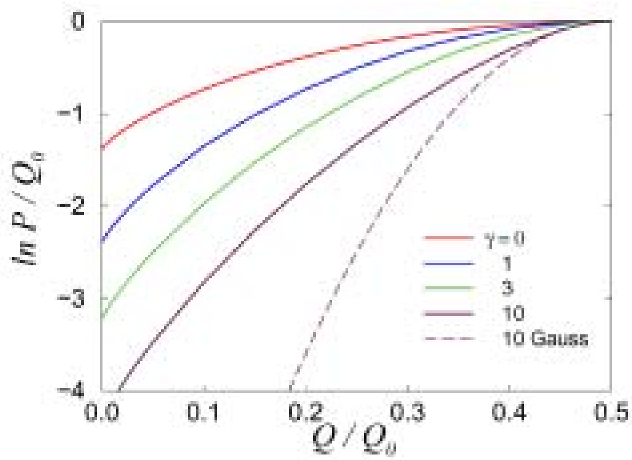
$$F = \frac{1}{4}\frac{1}{1+\gamma}, \qquad F_3 = 0, \qquad F_4 = \frac{1}{16}\frac{\gamma - 1}{(1+\gamma)(2+\gamma)}$$

Extreme value statistics:

$$\ln P(\mathcal{Q})/(\Omega N t) = \min_\lambda \{\mathcal{H}(\lambda) - \mathcal{Q}\lambda\}$$

$$= -2|\mathcal{Q} - 1/2| \ln(8\gamma|\mathcal{Q} - 1/2|)$$

Why only $\ln(\gamma)$?



E.S., Bulashenko, PRL 94, 116803 (2005)

CROSSOVER: EIGENVALUE DISTRIBUTION

Analytical continuation:

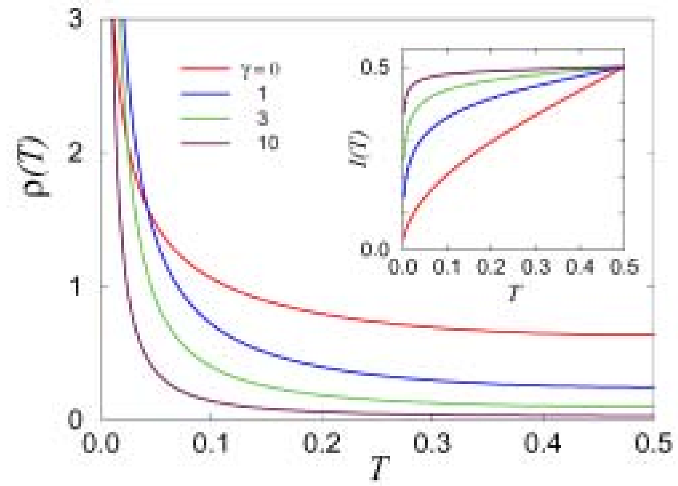
$\mathcal{H}(\lambda) \Rightarrow \rho(T)$, $\forall\ \gamma = \tau_Q/\tau_D$

$$\rho(T) = \frac{\gamma}{\pi\sqrt{T(1-T)}} \int_{-1}^{1} \frac{(1-u^2)|u|^{2\gamma-1} du}{(1+u)^2 - 4Tu}$$

$$\rho(T)_{\gamma=0} = \frac{1}{\pi\sqrt{T(1-T)}}$$

$$\rho(T)_{\gamma\to\infty} = \frac{1}{8\pi\gamma\,[T(1-T)]^{3/2}}$$

$\Rightarrow$ cutoff at $T, 1-T \sim 1/\gamma^2$

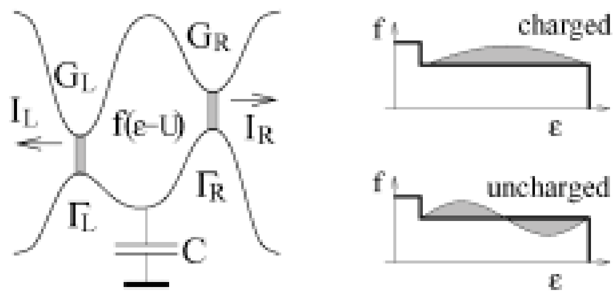


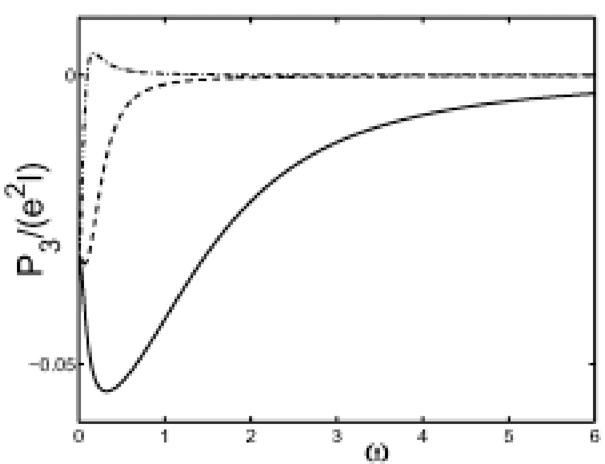
Formation of almost open/closed channels!

E.S., Bulashenko, PRL 94, 116803 (2005)

Frequency Scales for Current Statistics of Mesoscopic Conductors

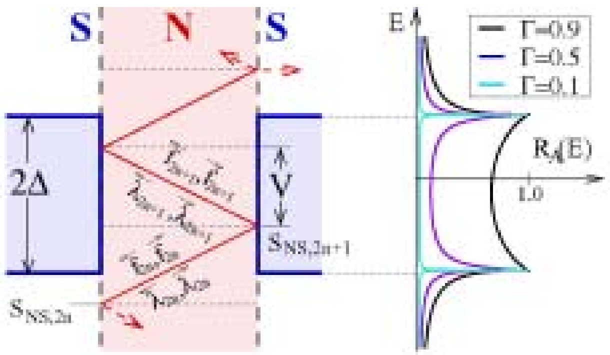
K. E. Nagaev,[1,2] S. Pilgram,[2] and M. Büttiker[2]

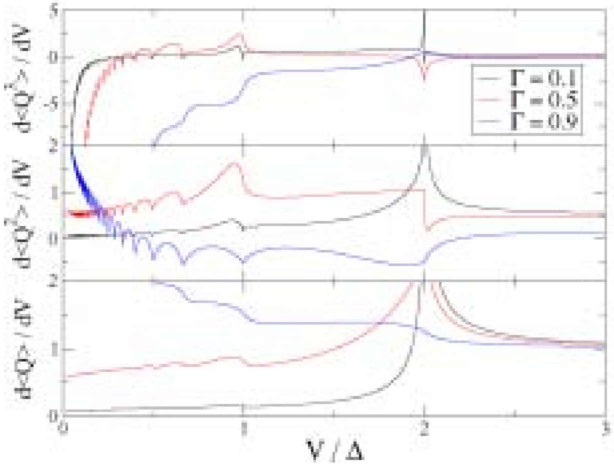




Noise and Full Counting Statistics of Incoherent Multiple Andreev Reflection

S. Pilgram and P. Samuelsson





Full counting statistics for voltage and dephasing probes

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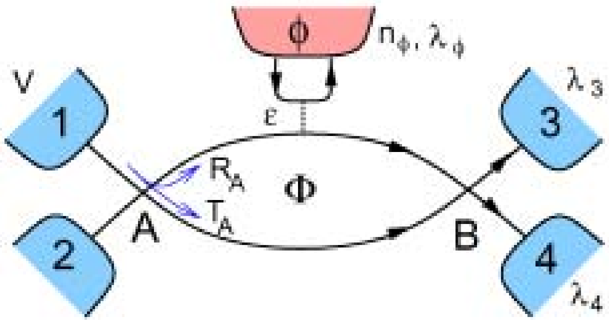

SUMMARY OF SPI APPROACH

The theory:

1. SPI $\Rightarrow$ saddle point of noise generator
2. Diagrammatics $\Rightarrow$ "cascade corrections"
3. Field theory $\Rightarrow$ universality
4. Non-perturbative solutions

$\Rightarrow$ super-Poissonian noise

$\Rightarrow$ threshold noise detectors

Recent results on FCS:

1. Hot electrons
2. ω dependence
3. SNS system
4. Qunatum-to-classical crossover

Applications in:

1. Mesoscopics
2. Statistical mechanics
3. Biology
4. Econophysics