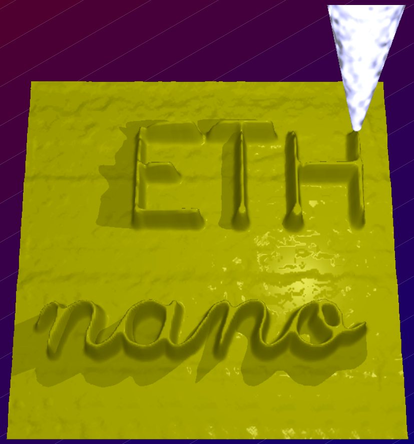
Counting statistics of single electron transport in a quantum dot



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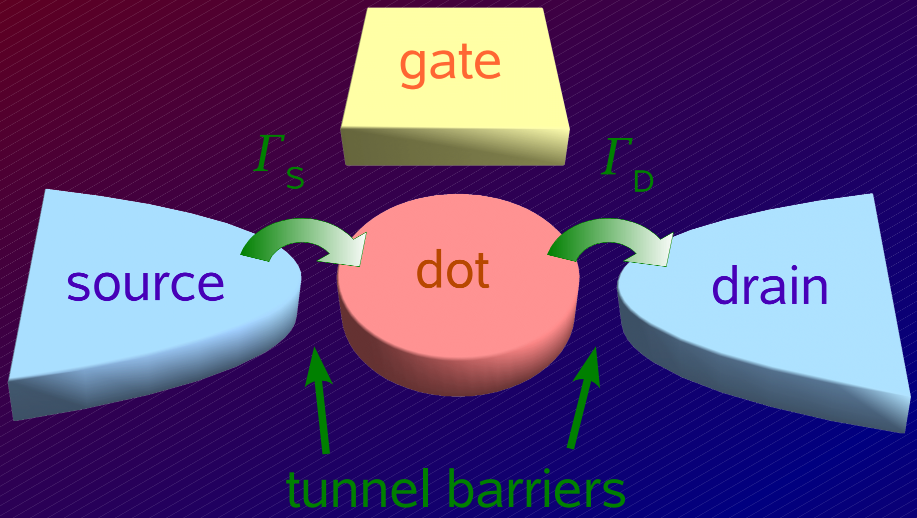
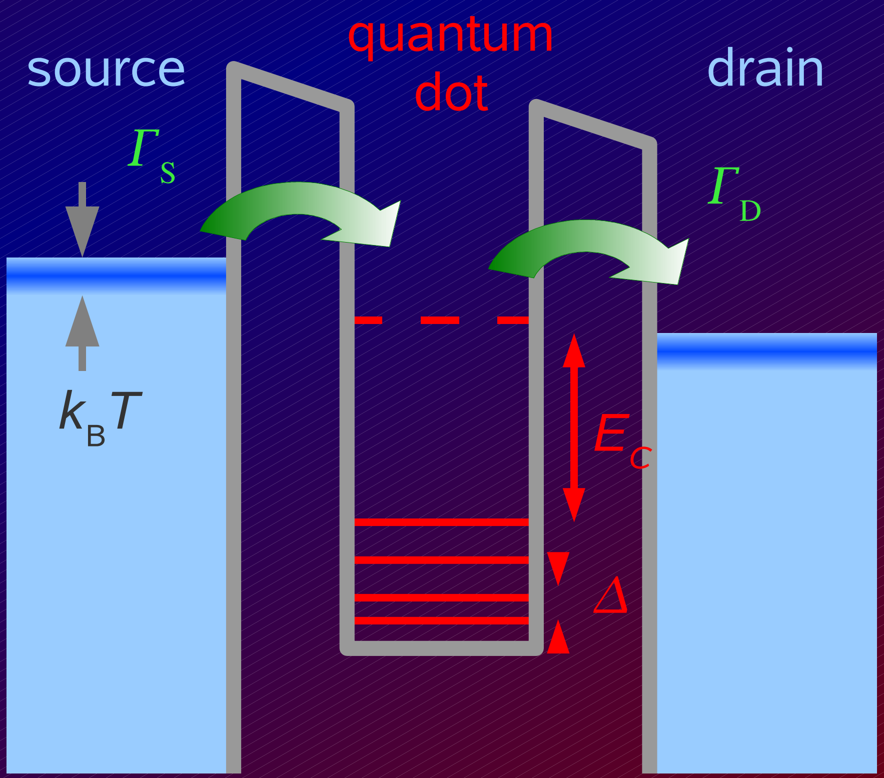
Nanoelectronics 2006, Lancaster University, January 8th, 2006

Outline

- Introduction
- Detection of single electron transport
- Current fluctuations and full counting statistics in a semiconductor quantum dot
- Tunneling through multiple states: bunching
- Conclusion

Semiconductor quantum dots

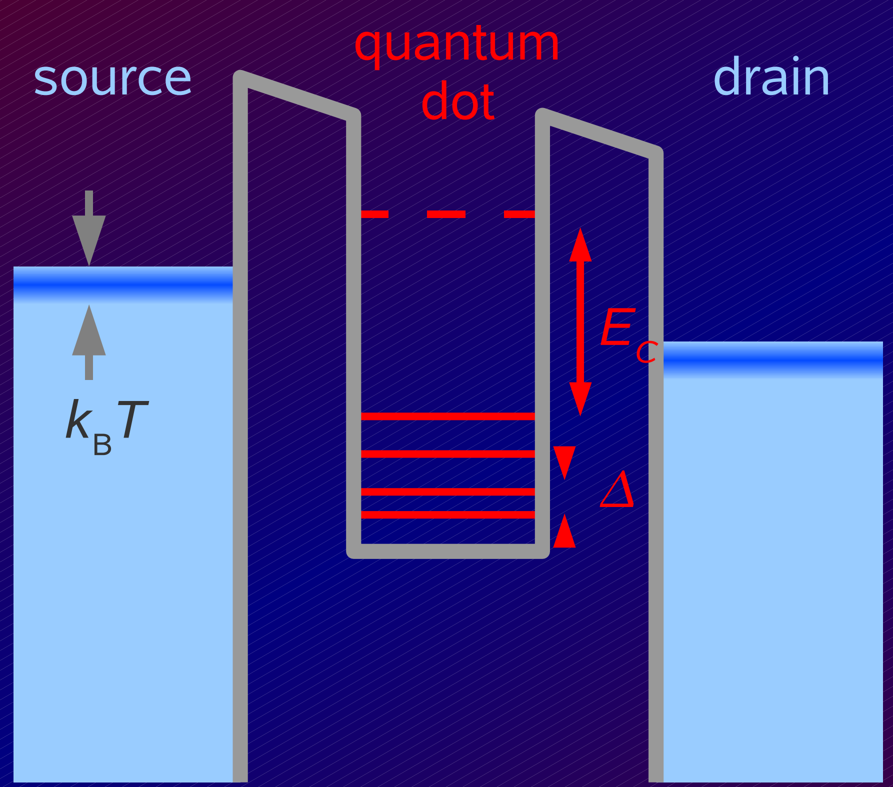




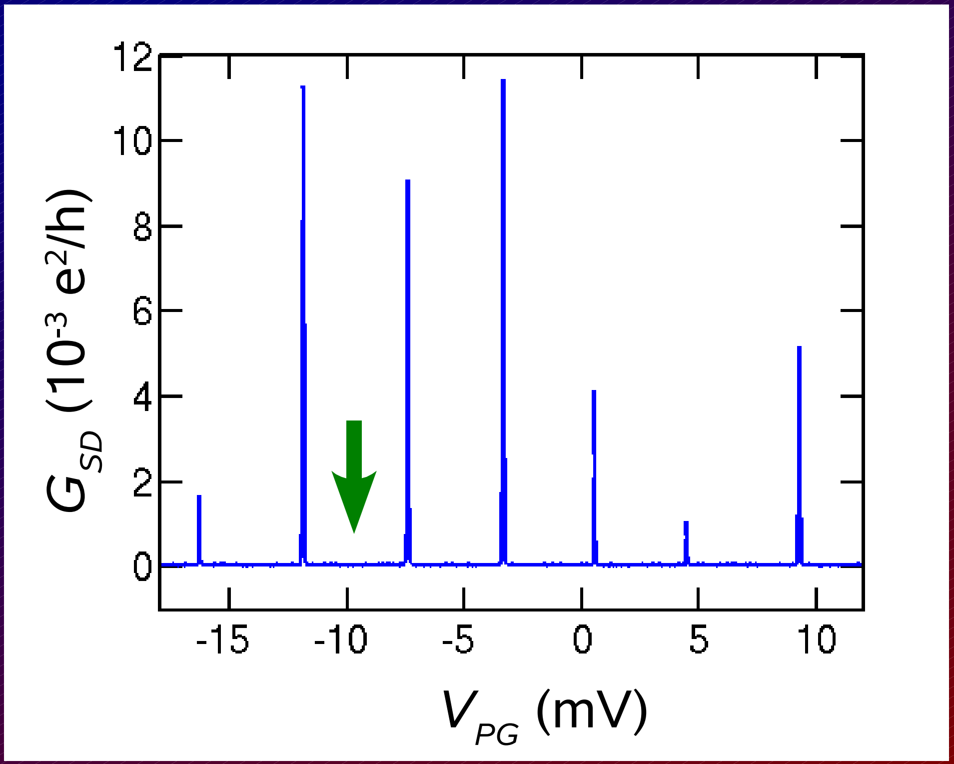
- Electrostatic energy E_C
- Quantum level spacing Δ

DC current measurement in a quantum dot

- Spectroscopy of electronic states



$$k_B T \ll \Delta < E_C$$



DC current measurement in a quantum dot

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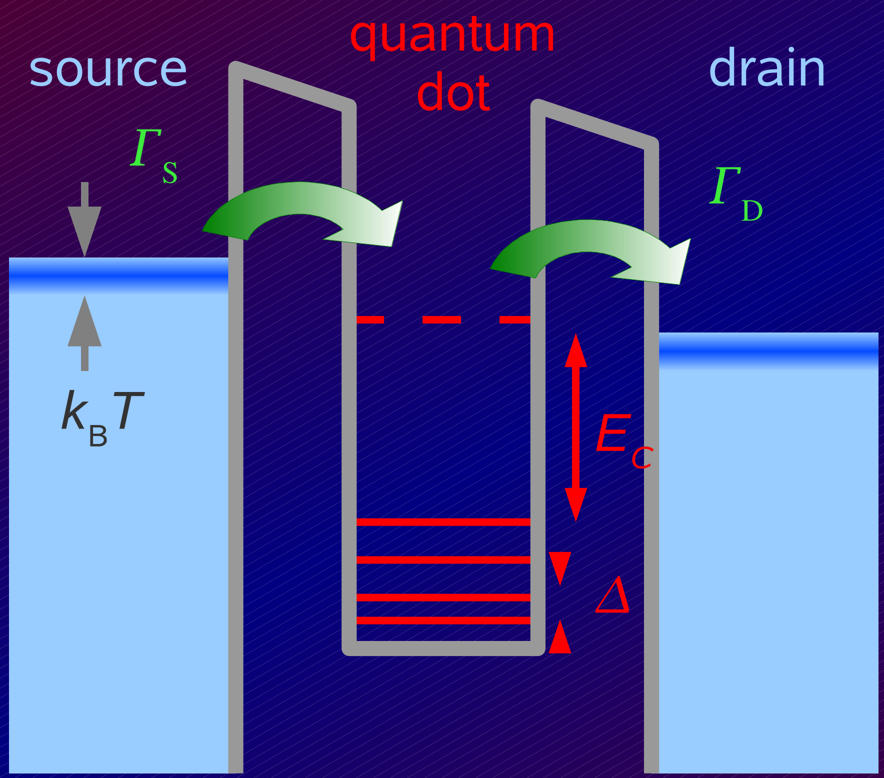

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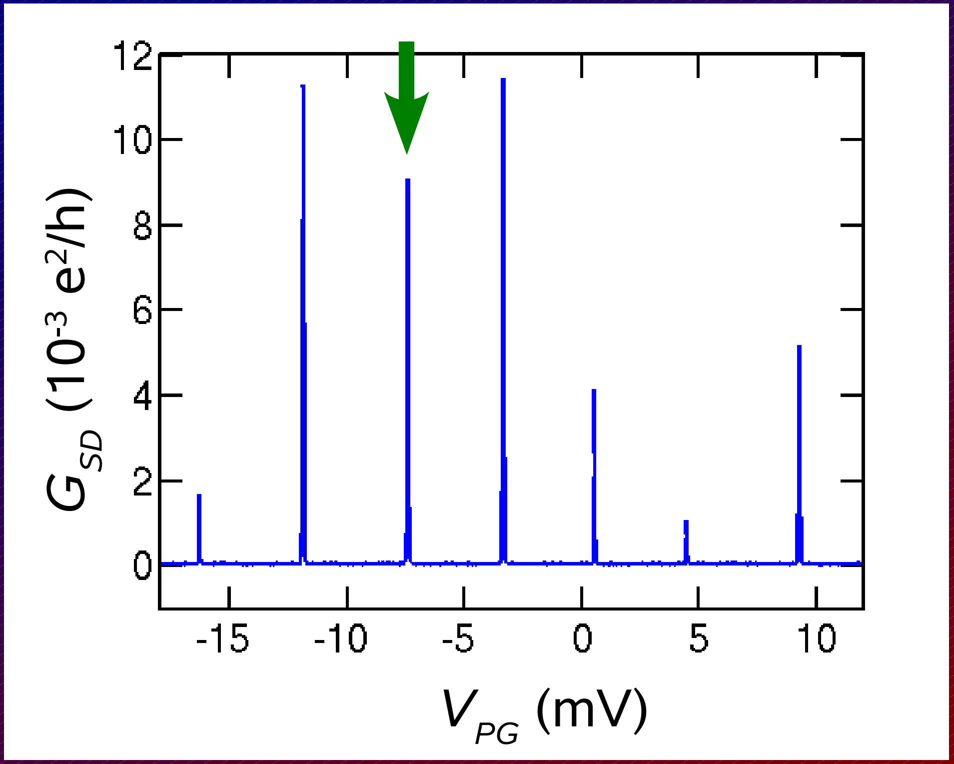

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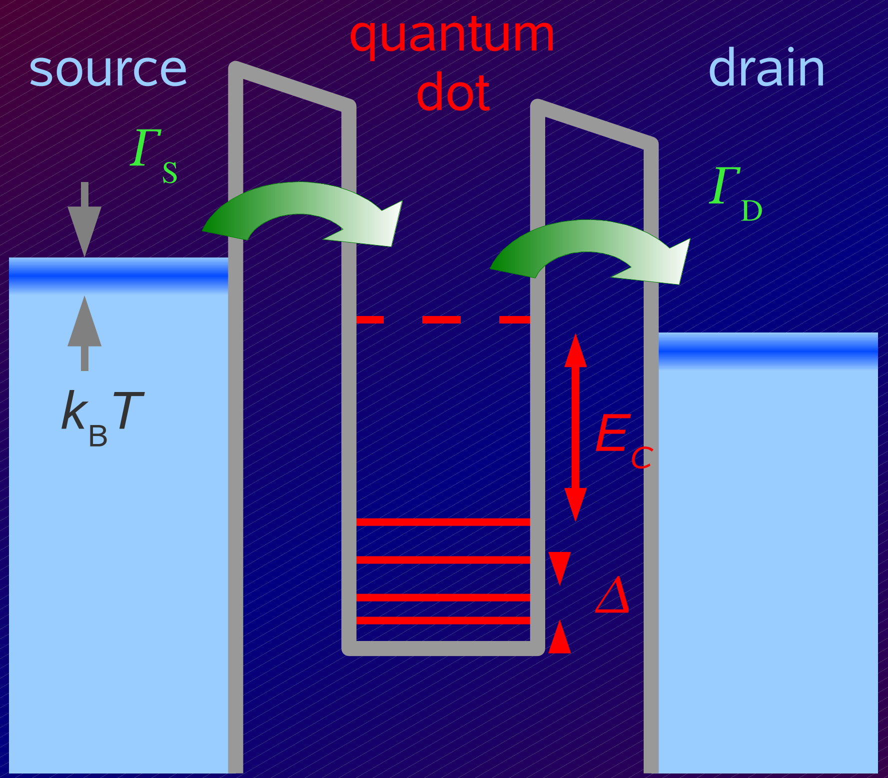

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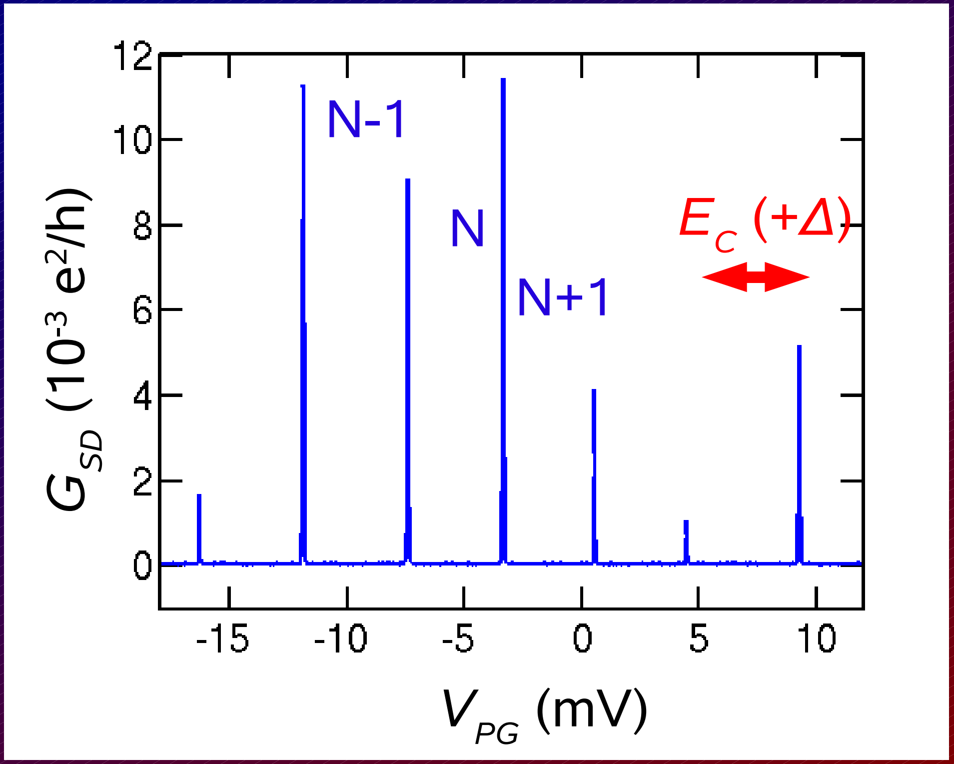
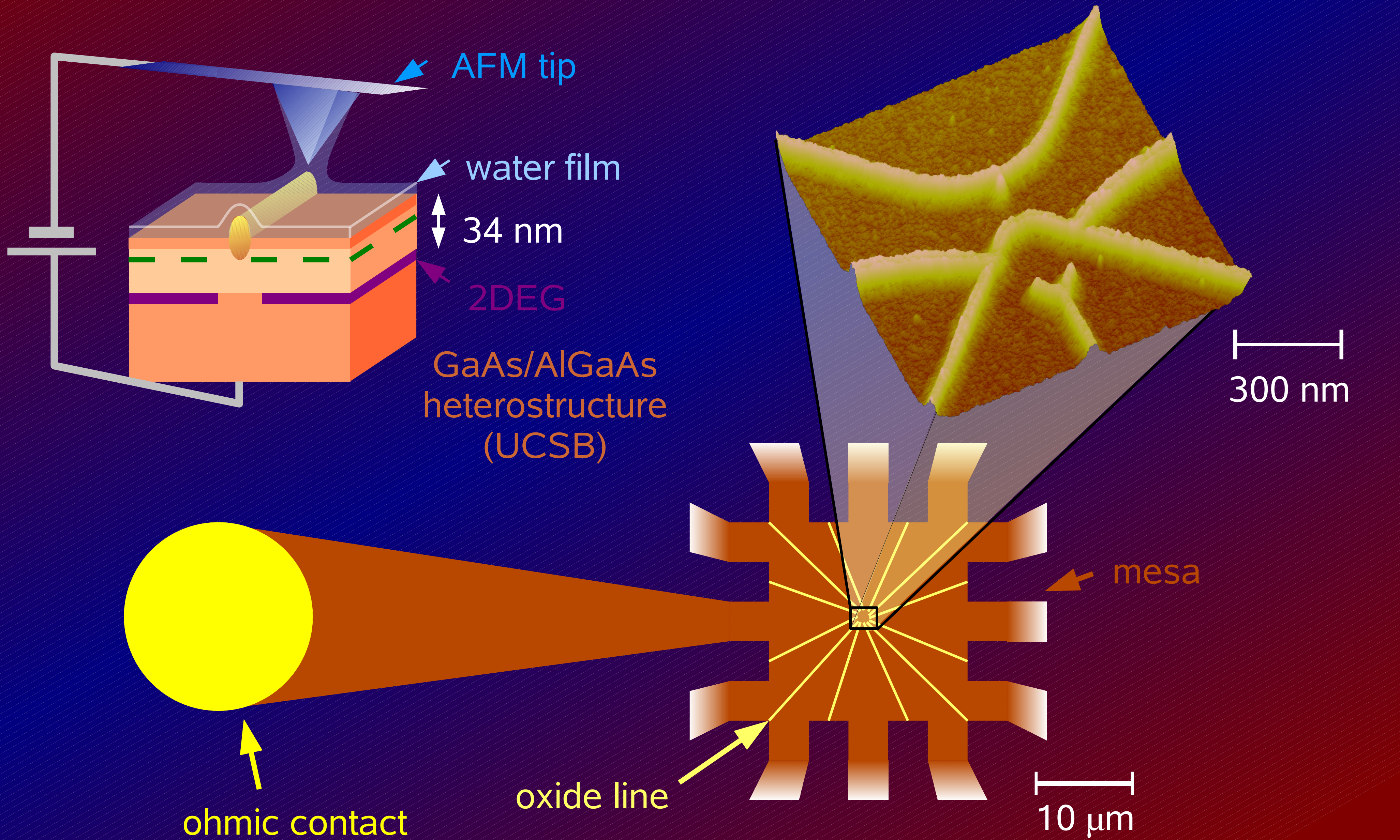
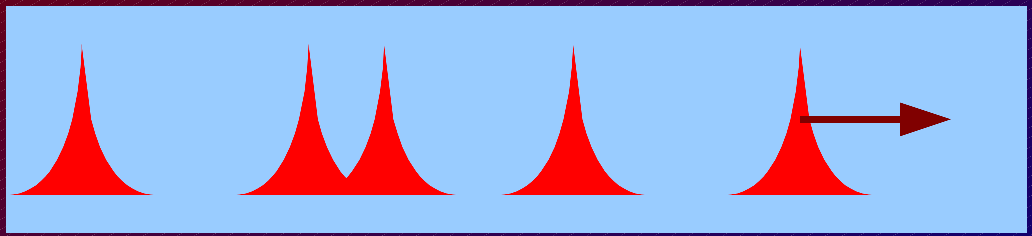


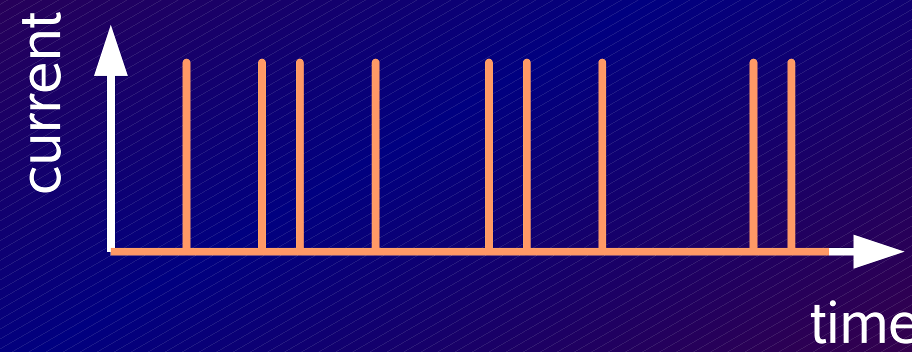
Quantum dots realized by AFM lithography



Measurement of current fluctuations



conductor



- Shot noise due to discreteness of charges
  - classical shot noise for independent particles (Poissonian noise): $S_I = 2eI$
- Usual measurement limited by noise of the current-meter ⇒ $S_I^{min} \approx 10^{-29}$ A^2/Hz

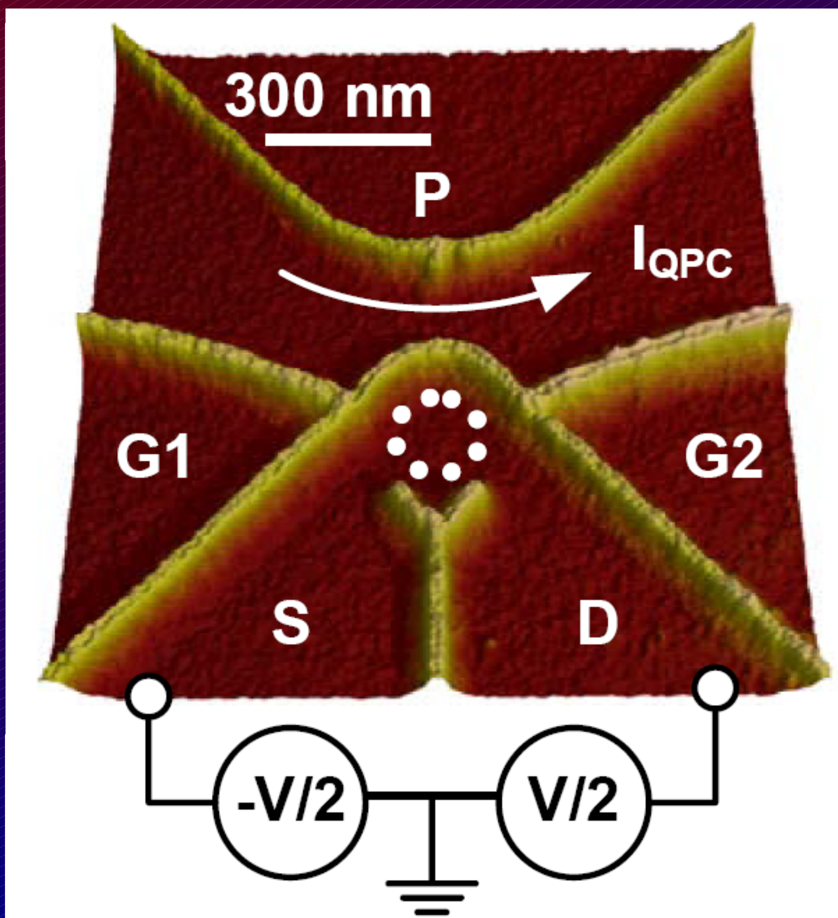
Noise in quantum dots

- Sub- vs. super-Poissonian shot noise in quantum dots
- Noise correlations in multi-terminal quantum dots
- Probing entanglement with the shot noise in quantum dot systems
- Kondo effect, spin blockade, etc...

Noise in quantum dots

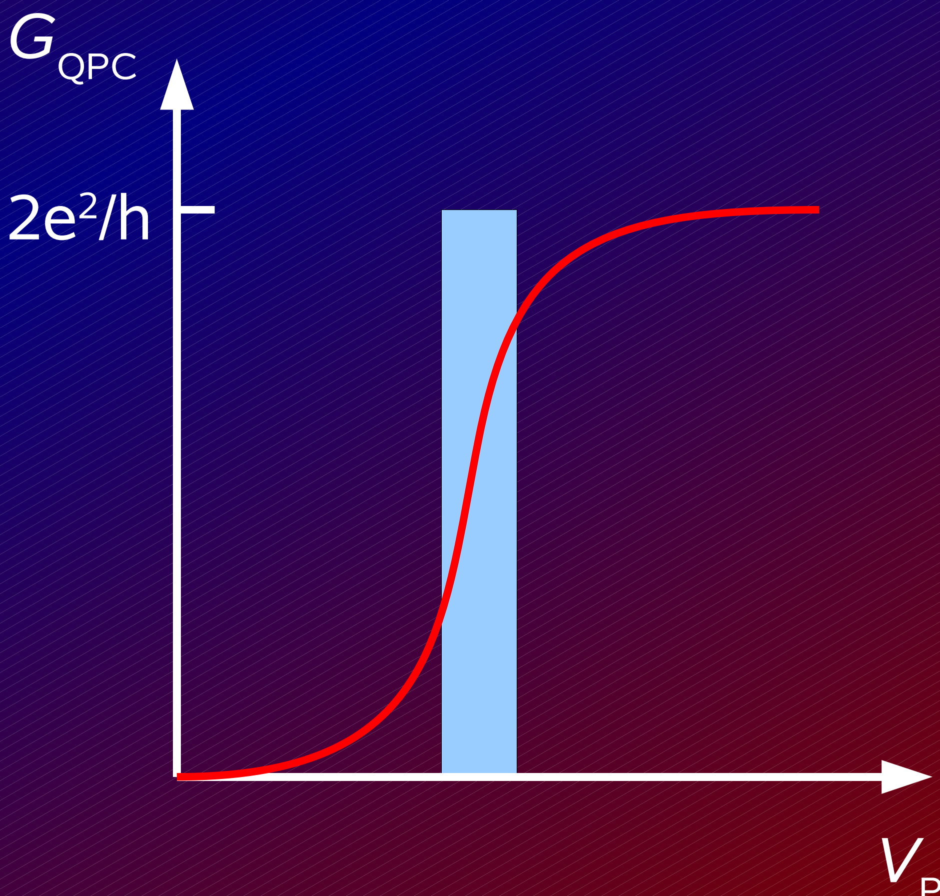
- Noise in interacting systems
  - deviations from Poissonian shot noise
- Early experiments in non-tunable quantum dots showed reduction of the shot noise: $S_I < 2eI$
  - Birk *et al.*, PRL **75,** 1610 (1995)
  - Nauen et al., PRB **70,** 033305 (2004)
- Challenge in lateral quantum dots
  - very low noise level: $I < 1$ pA $\Rightarrow S_I < 10^{-31}$ A^2/Hz !
  - strongly non-linear systems

Detection of single electron transport



T_e = 350 mK

- Quantum point contact as a charge detector



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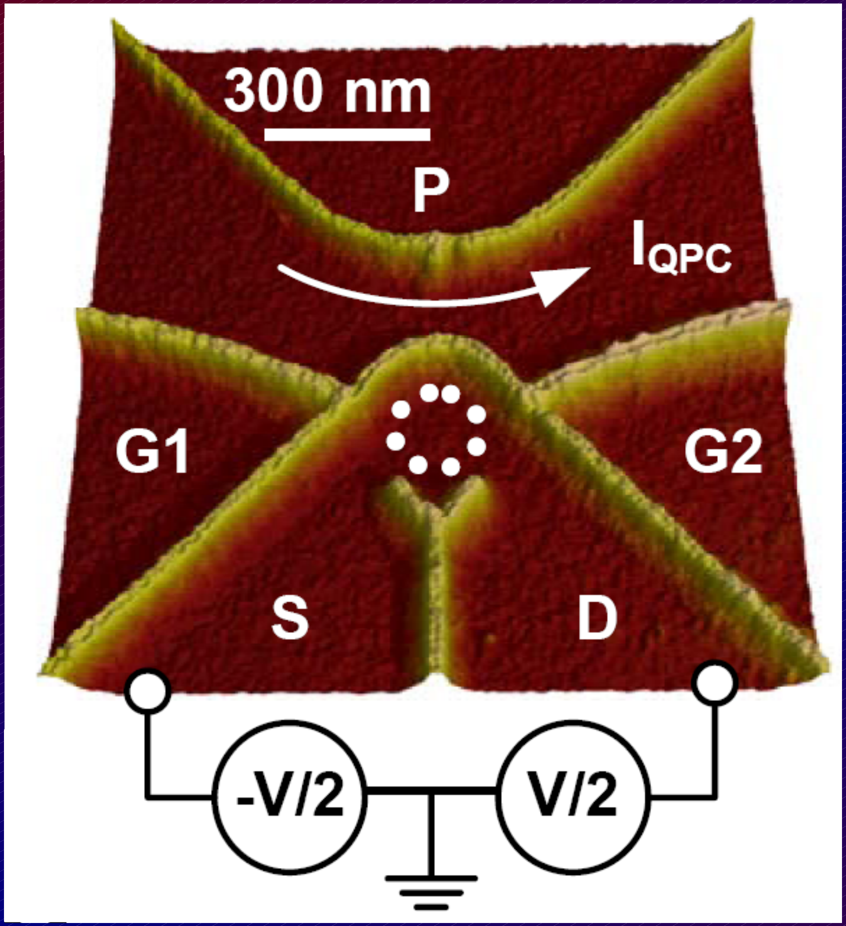
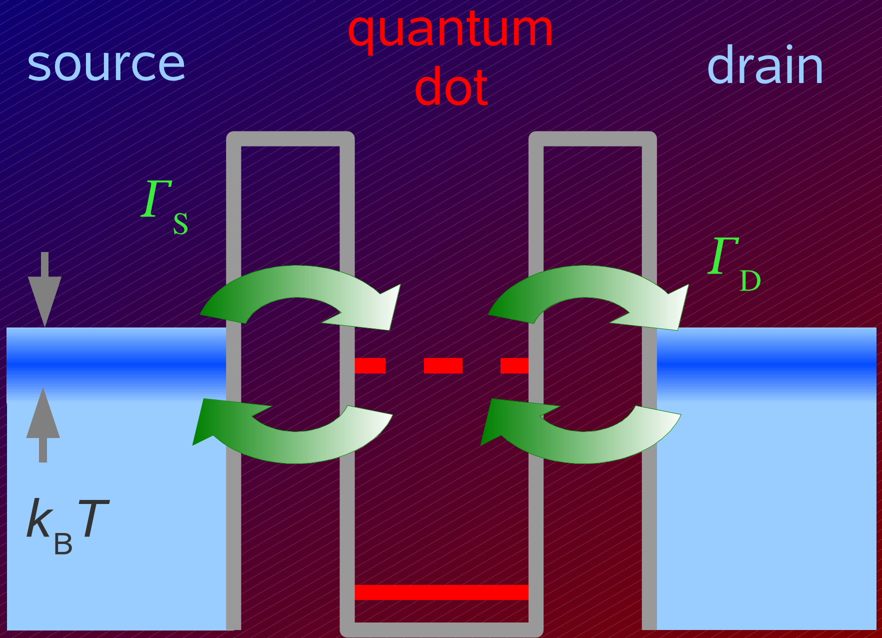


T_e = 350 mK

- Quantum point contact as a charge detector
- Low bias voltage on the quantum dot



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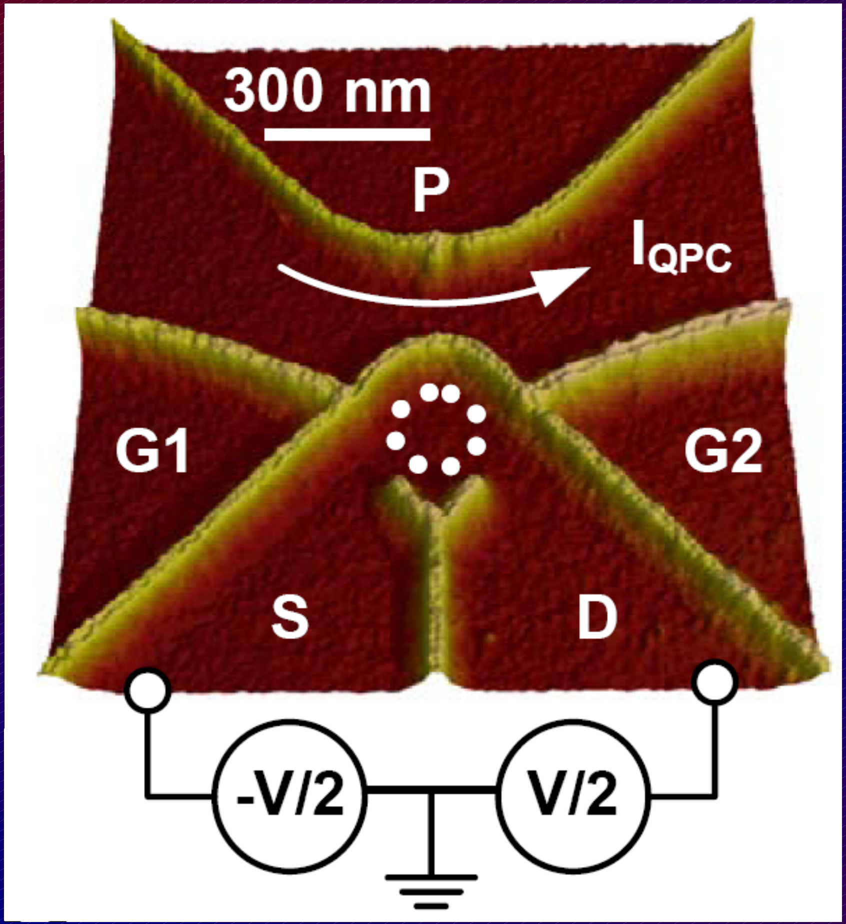

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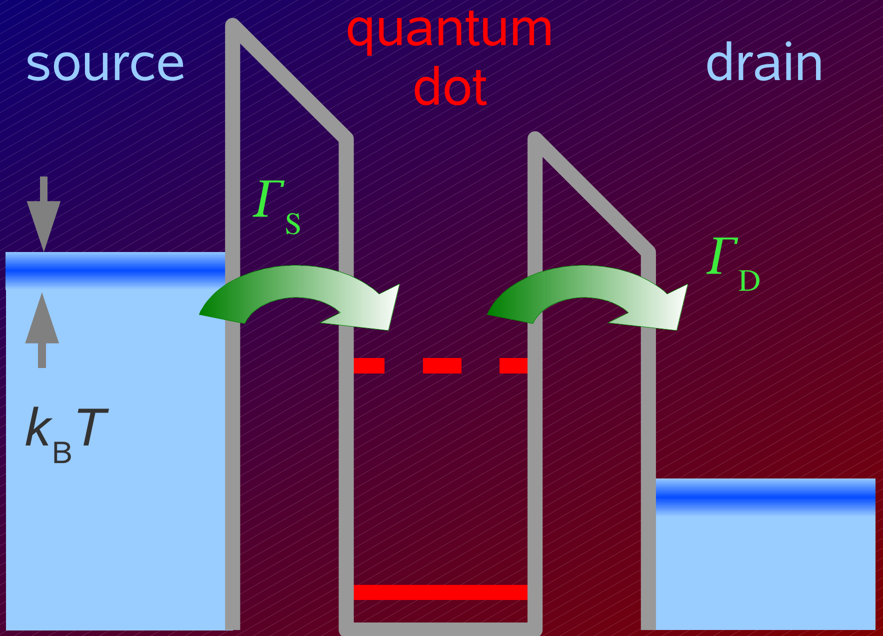

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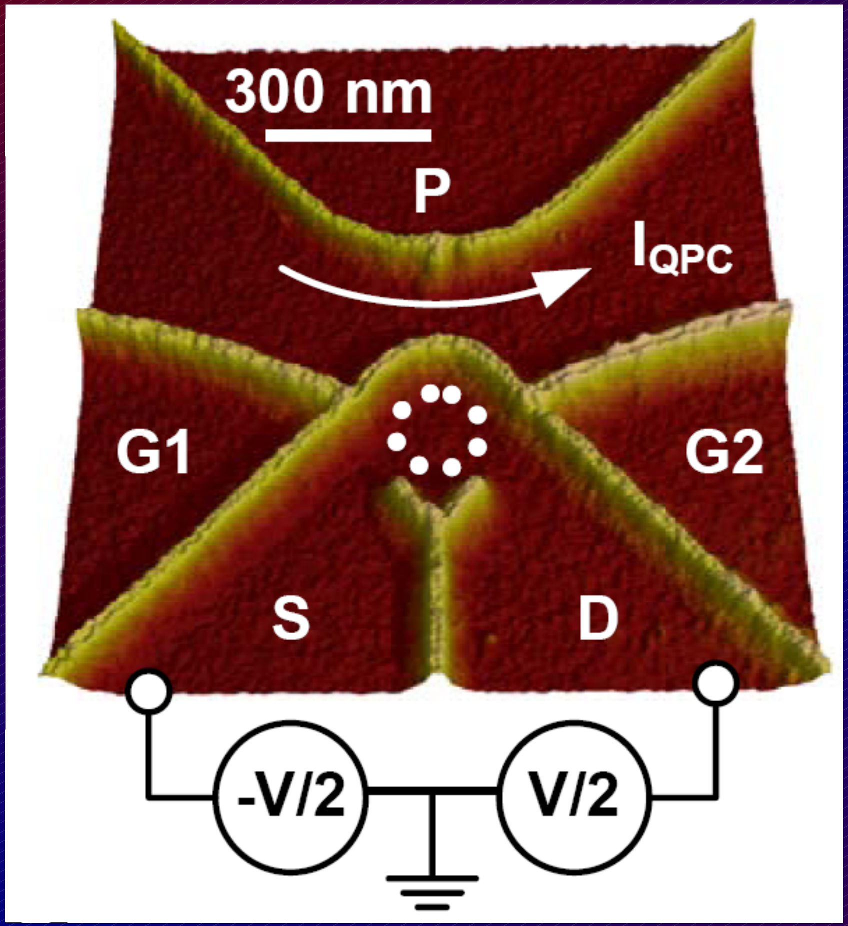

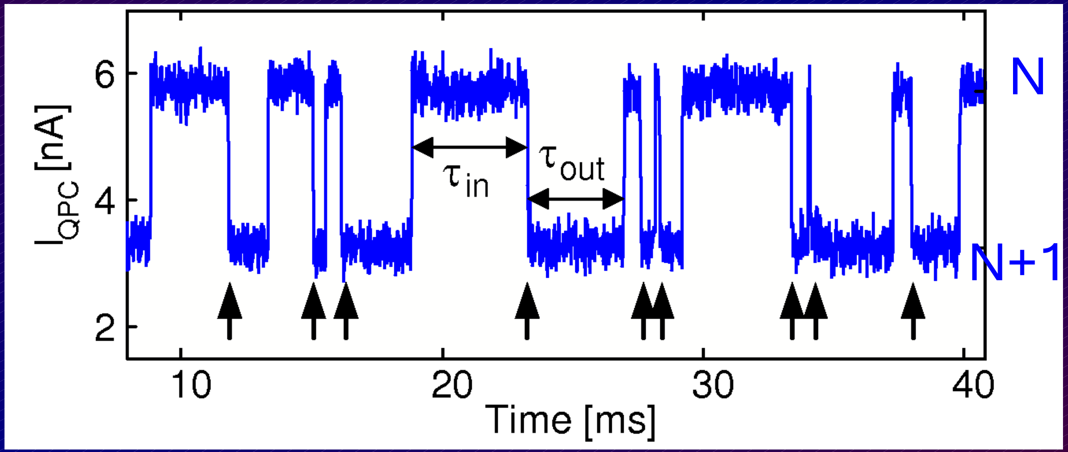

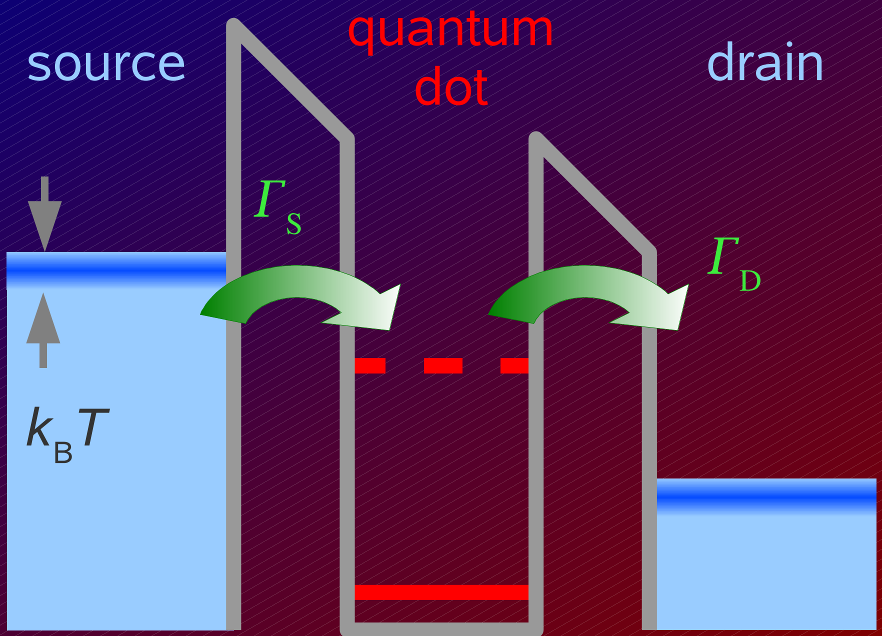

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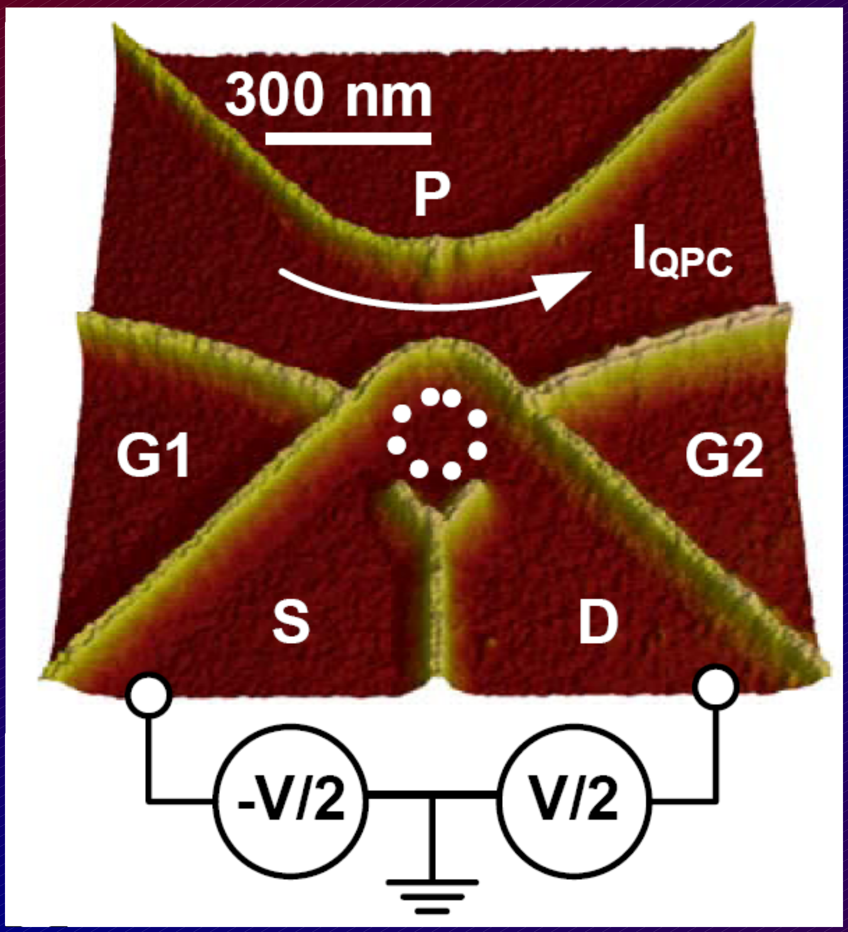

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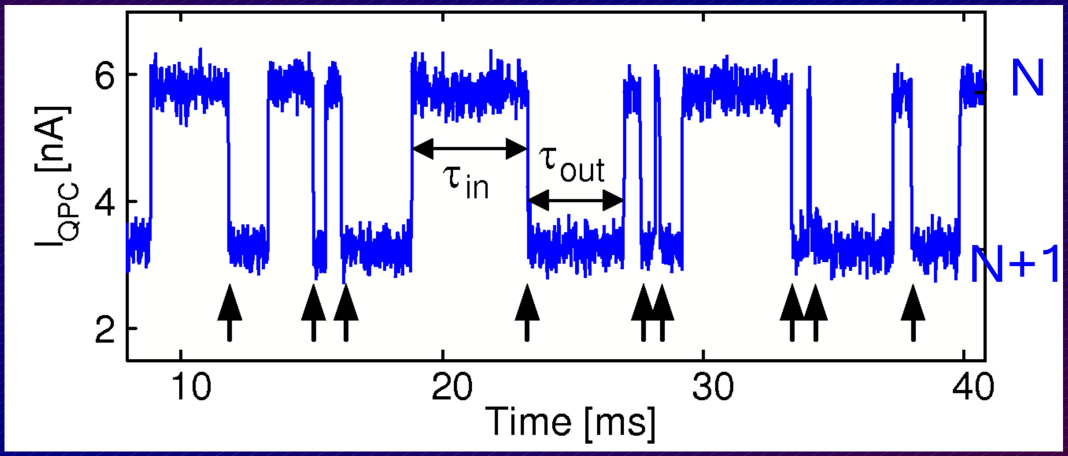
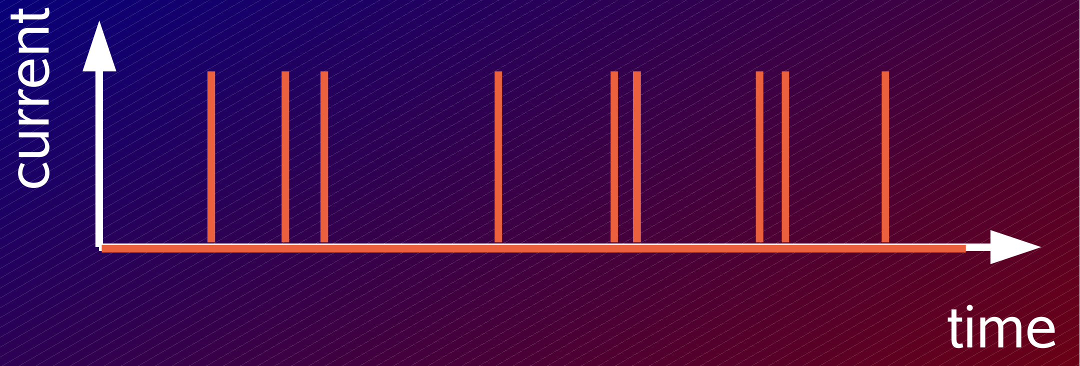
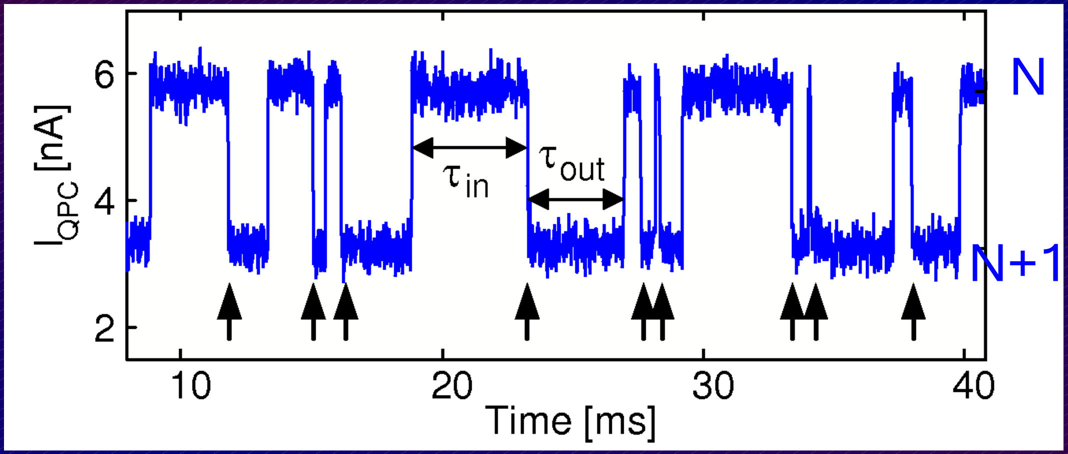
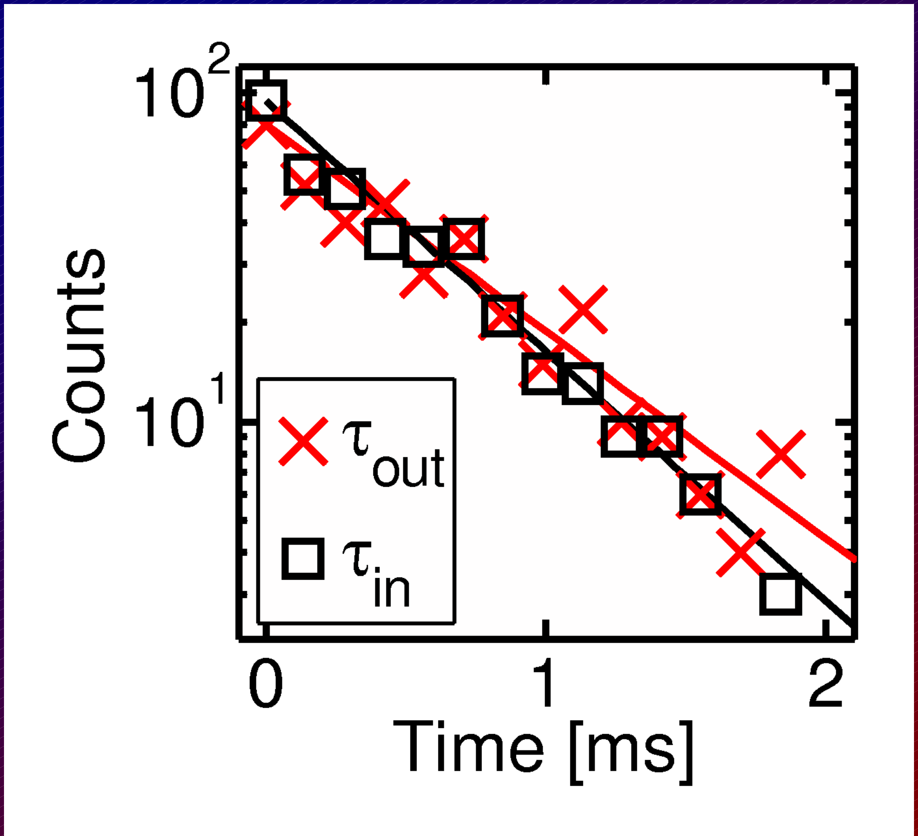




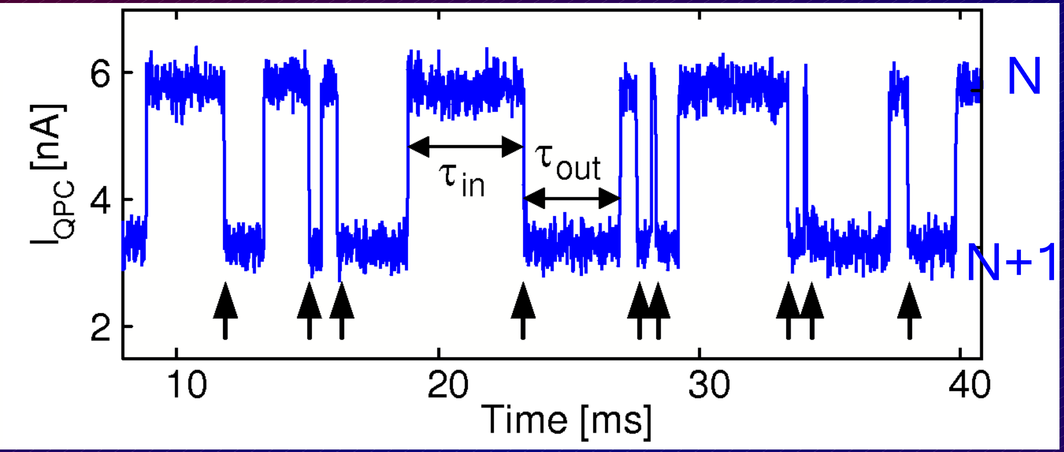
Determination of the individual tunneling rates

- Exponential distribution of waiting times for independent events
- $\Gamma_S = <\tau_{in}>, \Gamma_D = <\tau_{out}>$





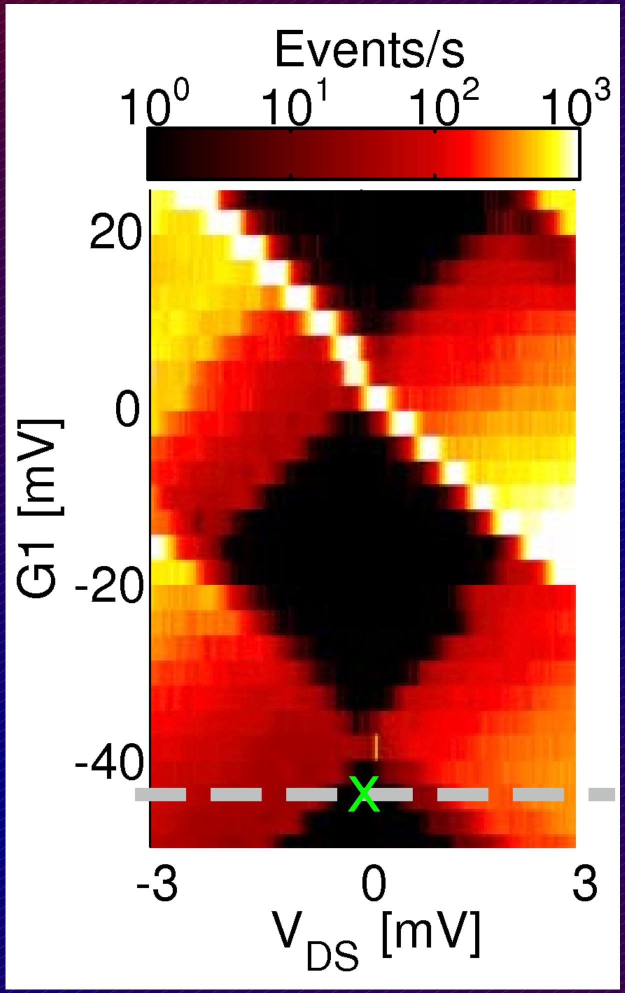
Measuring the current by counting electrons

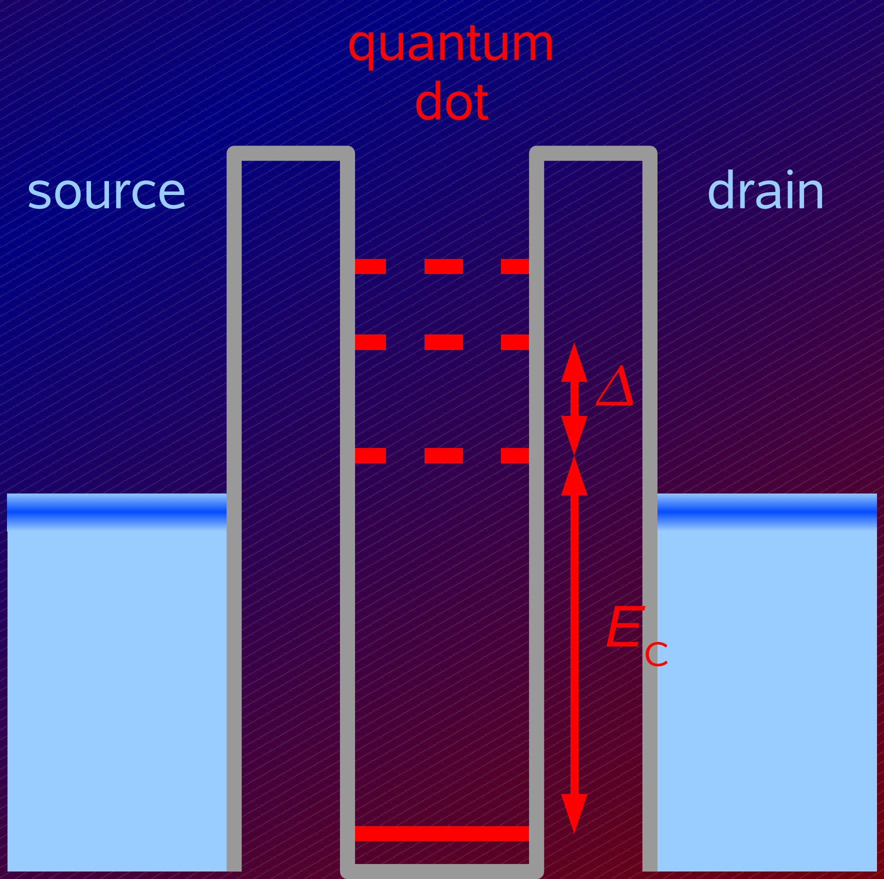


- Count number *n* of electron entering the dot within a time t_0: $I = e\langle n\rangle/t_0$
- Max. current = few fA (bandwidth = 30 kHz)
- BUT no absolute limitation for low current and noise measurements
  - we show here: $I \approx$ few aA, $S_I \approx 10^{-35}$ A^2/Hz

Coulomb diamond measured by electron counting

- $I = e\langle n \rangle / t_0$





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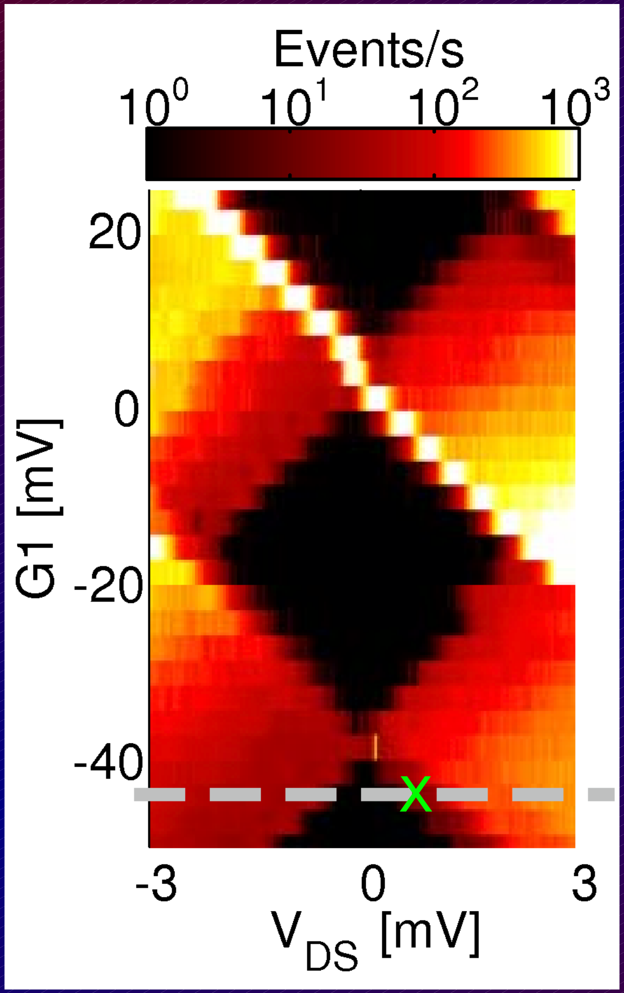

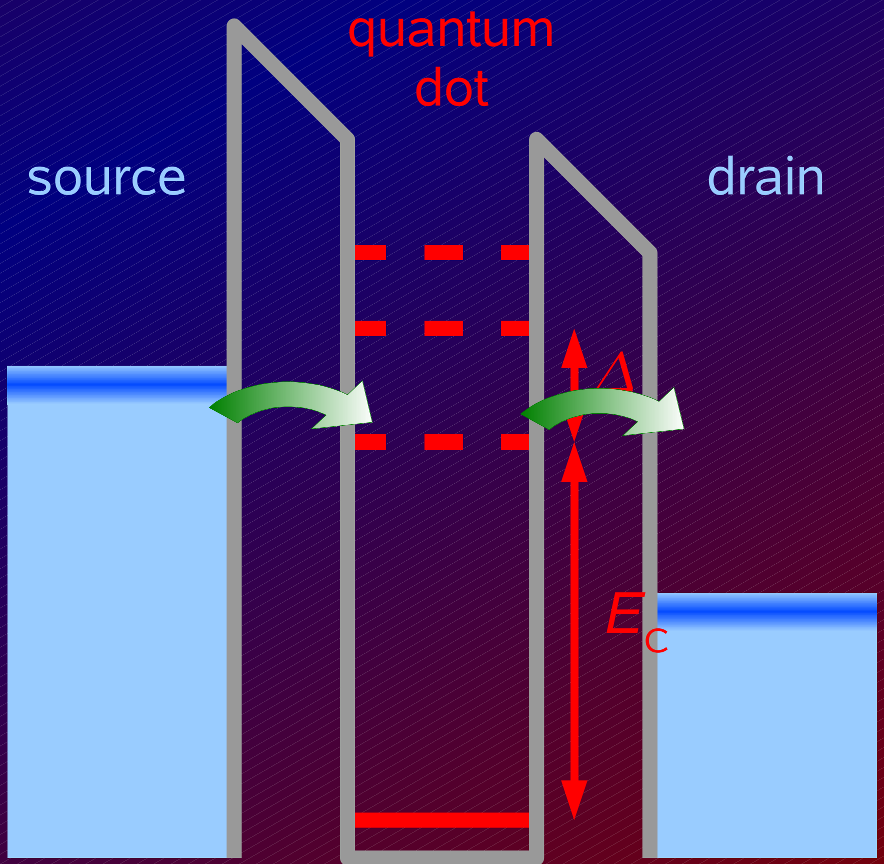

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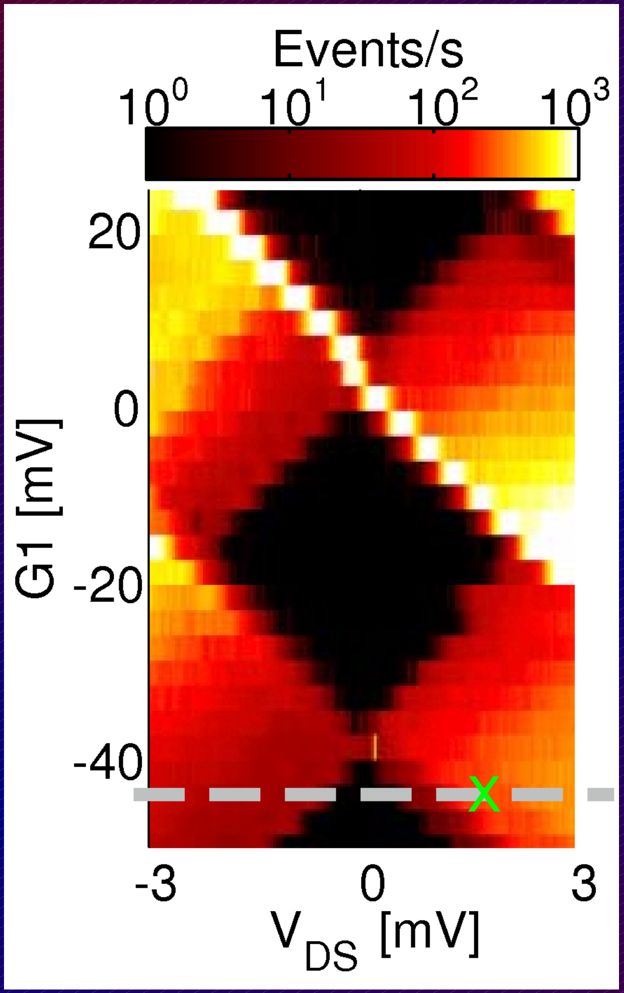

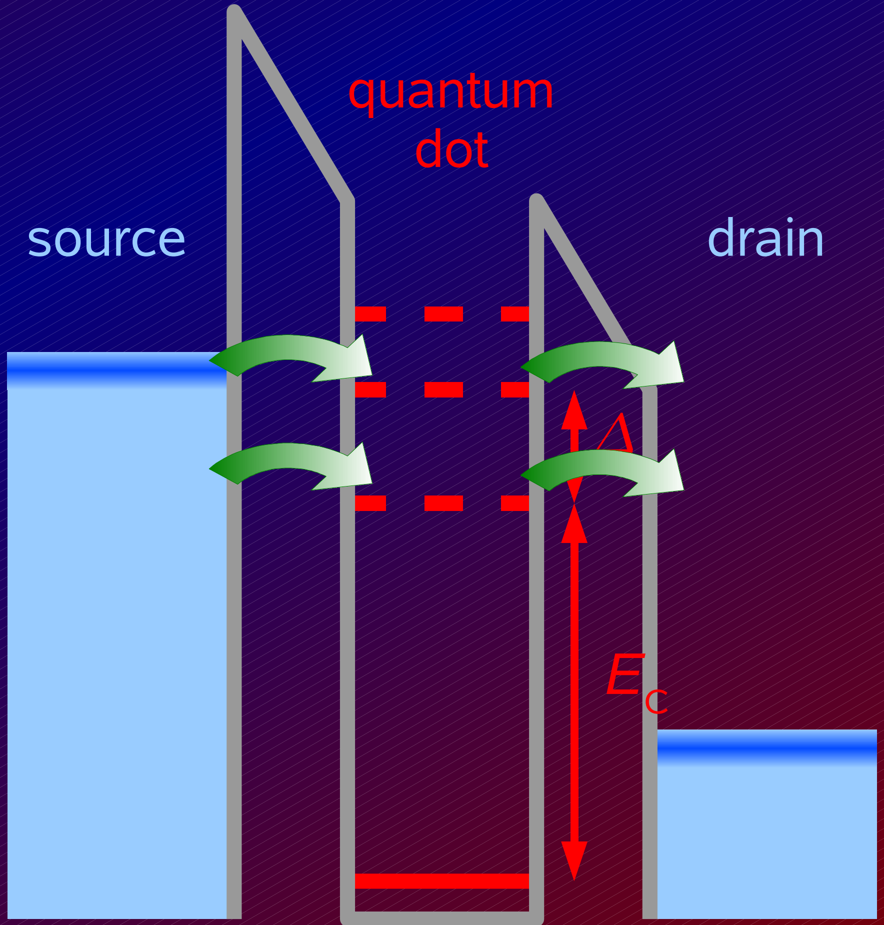

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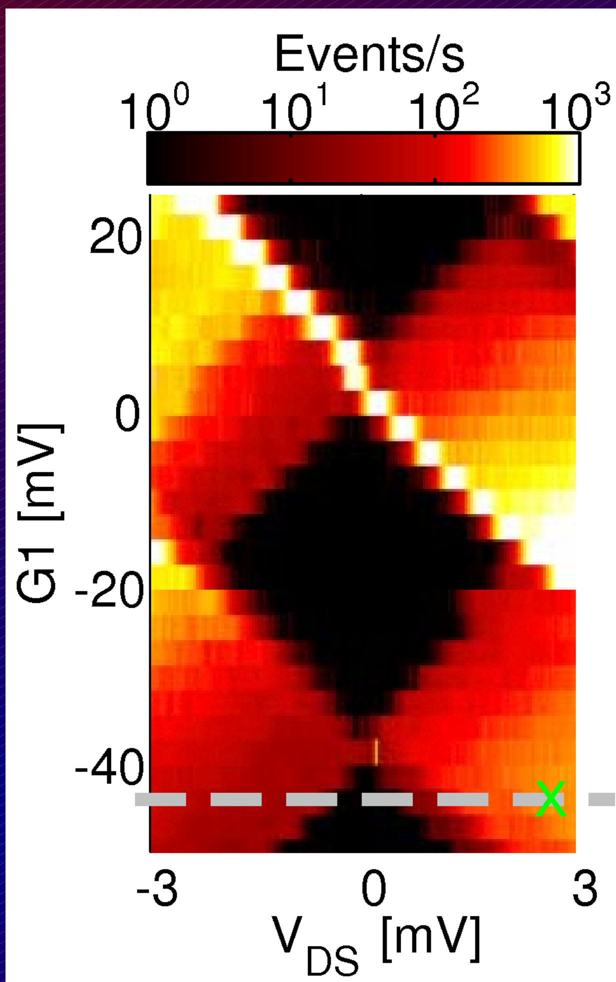

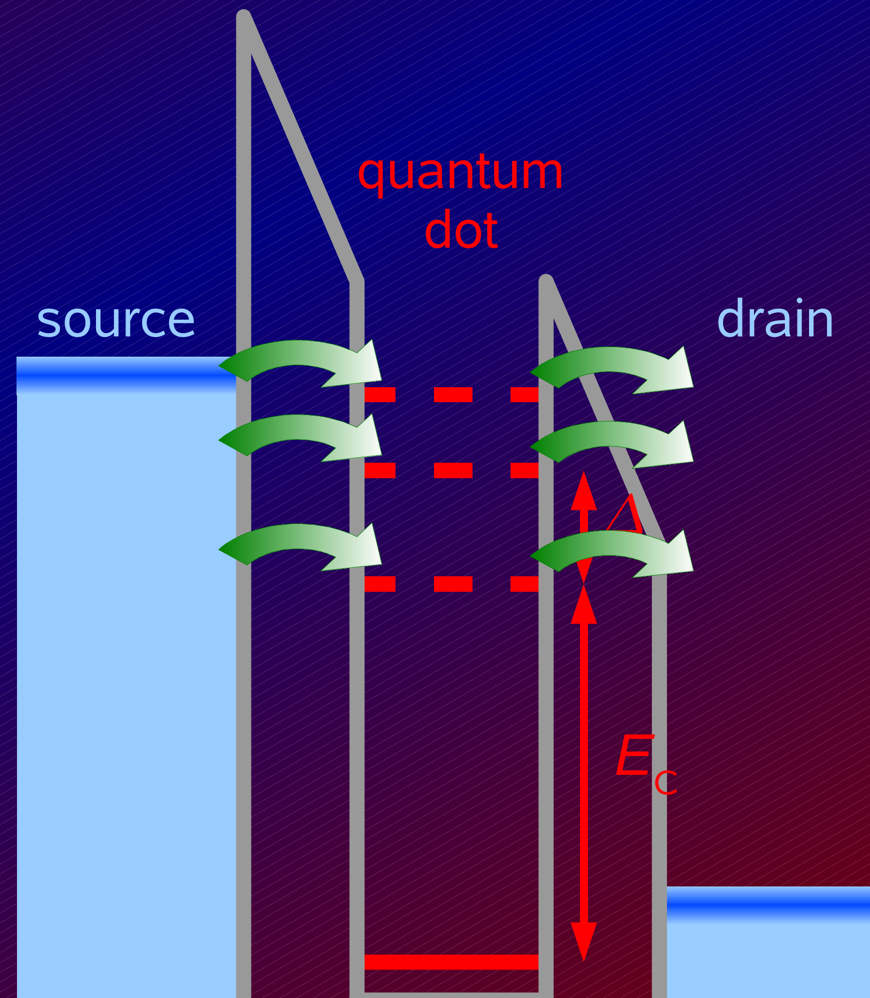

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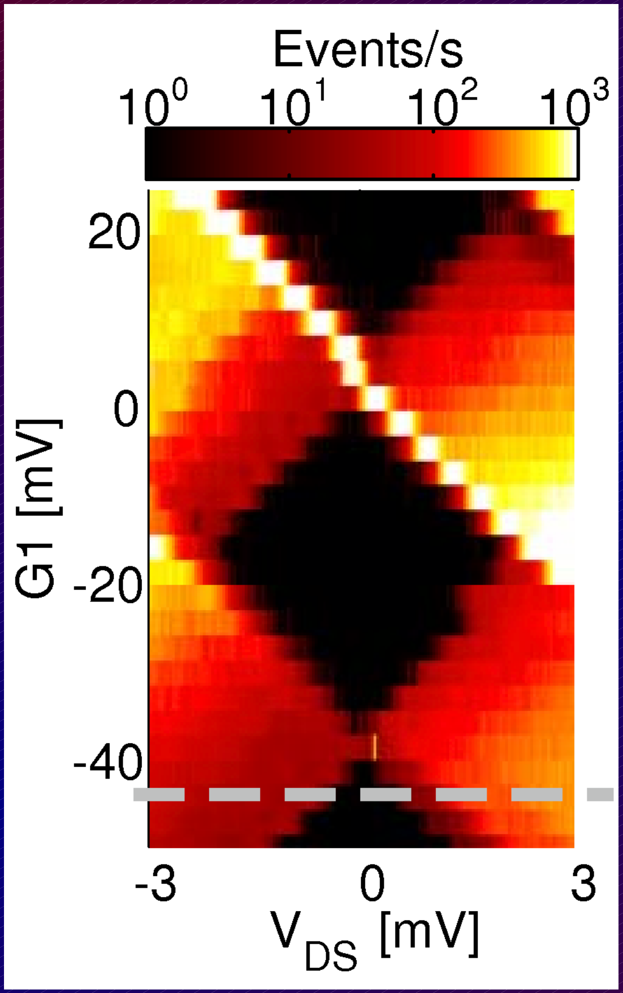
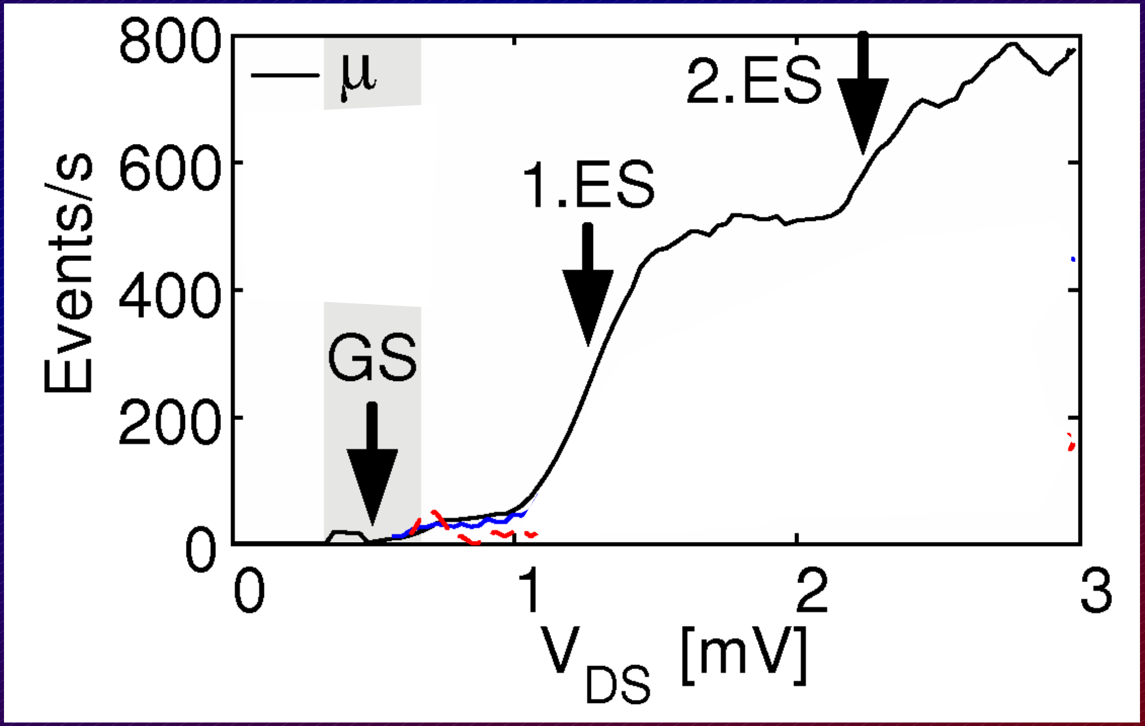
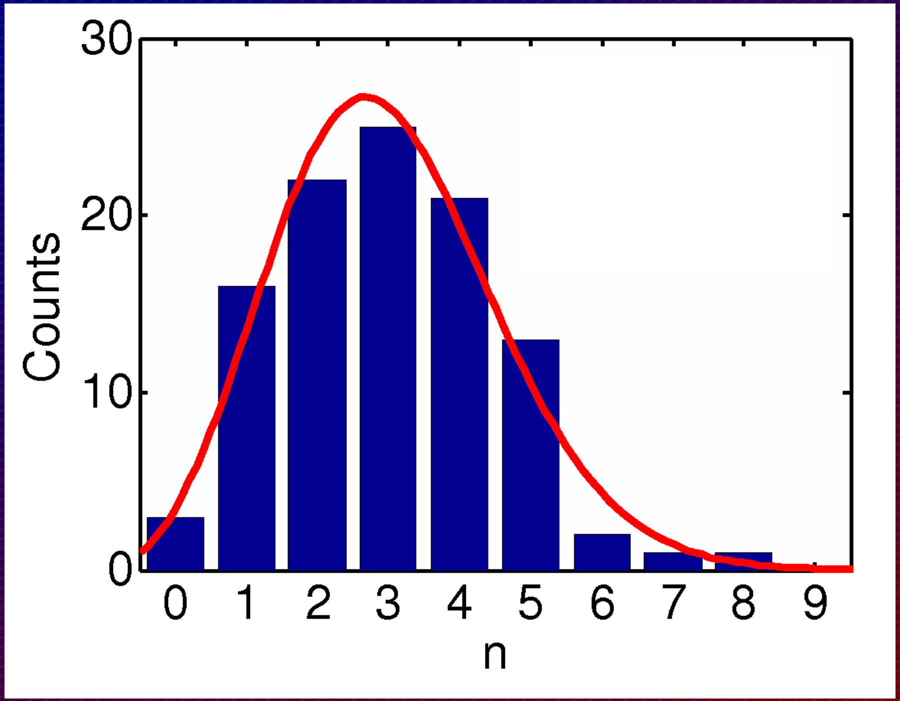




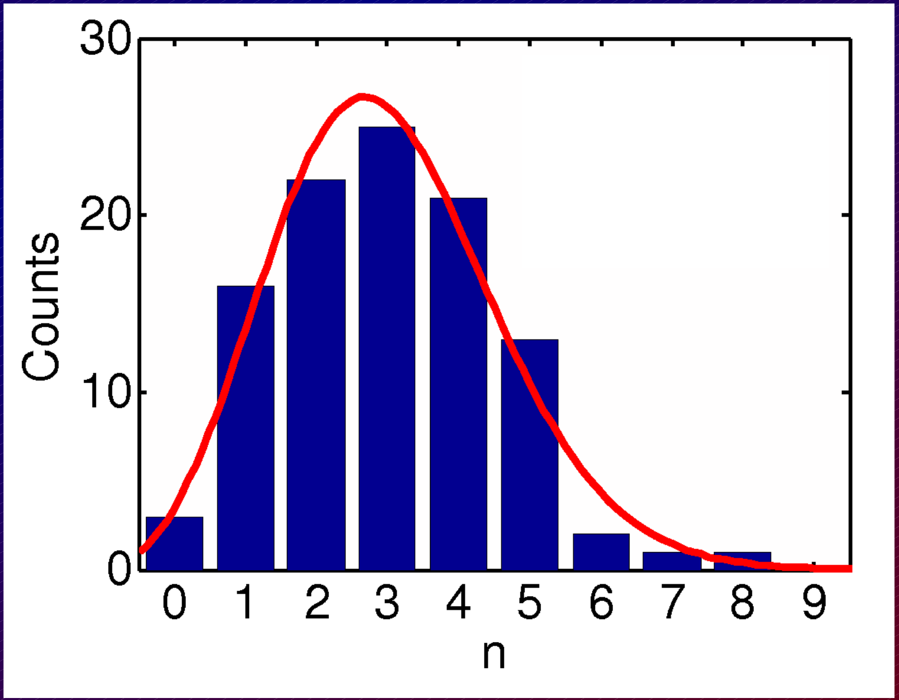
Current fluctuations measured by electron counting

- More than noise: access to the full counting statistics (distribution function)
  - $I = e\mu/t_0$,
    $\mu = \langle n \rangle$
  - $S_I = 2e^2\mu_2/t_0$,
    $\mu_2 = \langle (n - \langle n \rangle)^2 \rangle$
  - $S_I^3 = e^3\mu_3/t_0$,
    $\mu_3 = \langle (n - \langle n \rangle)^3 \rangle$
  - and many more...

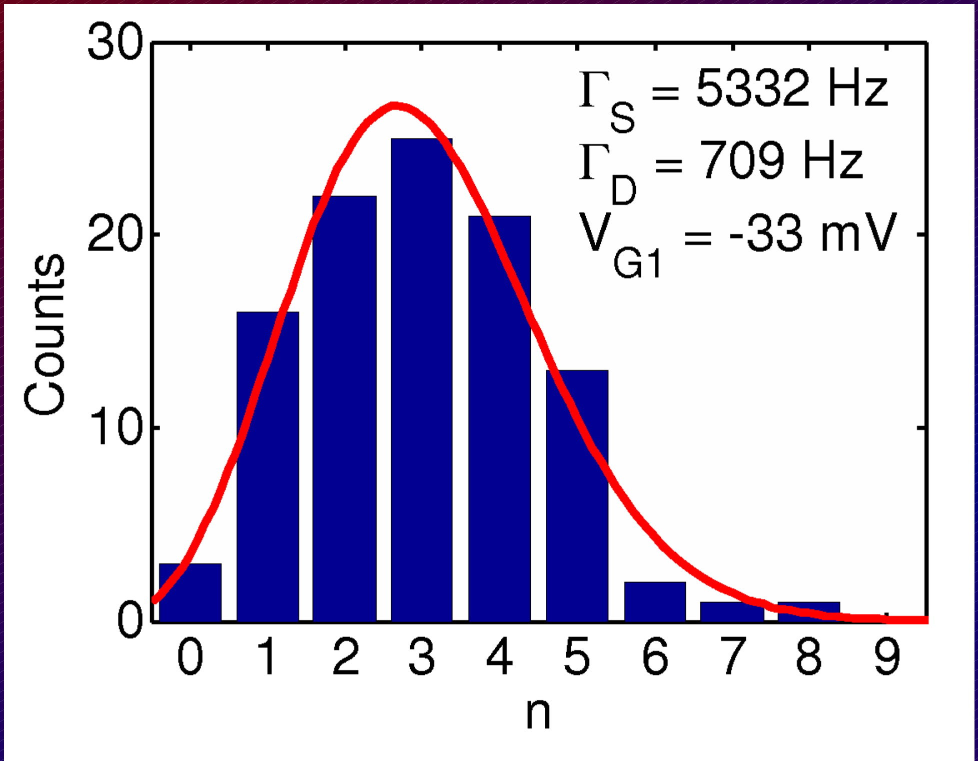


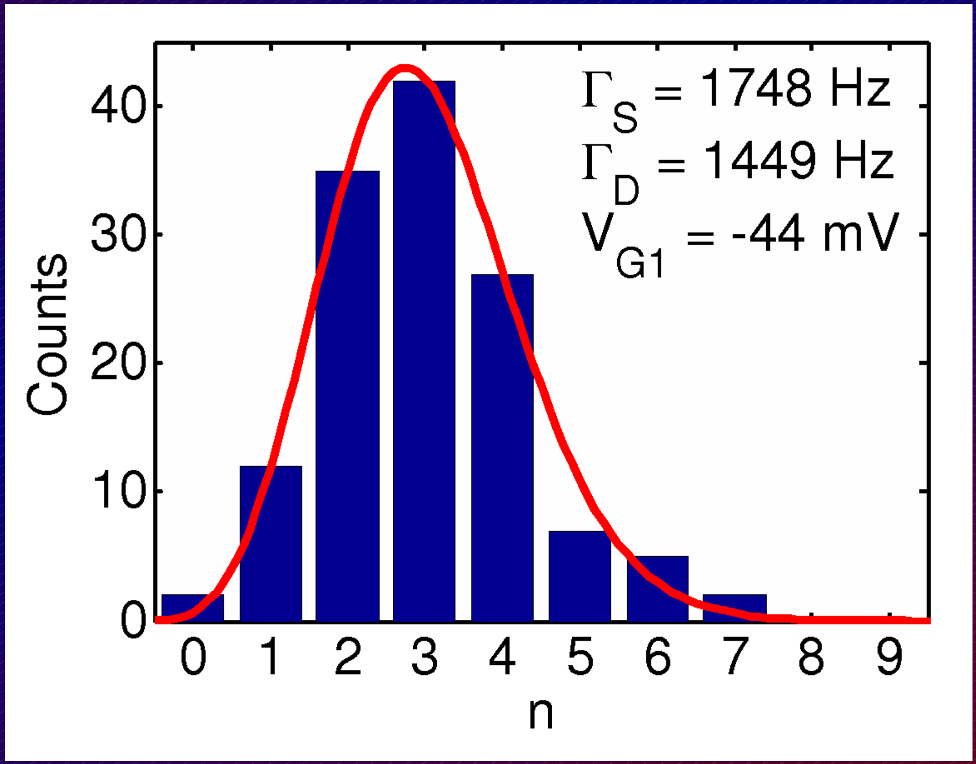
Distribution function for electrons in a conductor

- Classical noise for independent particles
  $\Rightarrow$ Poisson distribution: $\mu = \mu_2 = \mu_3$
- Particles with repulsive interaction $\Rightarrow$ sub-Poissonian distribution: $\mu_2 < \mu$, $\mu_3 < \mu$,...



Histogram of current fluctuations





- Poisson distribution for asymmetric coupling
- Sub-Poisson distribution for symmetric coupling

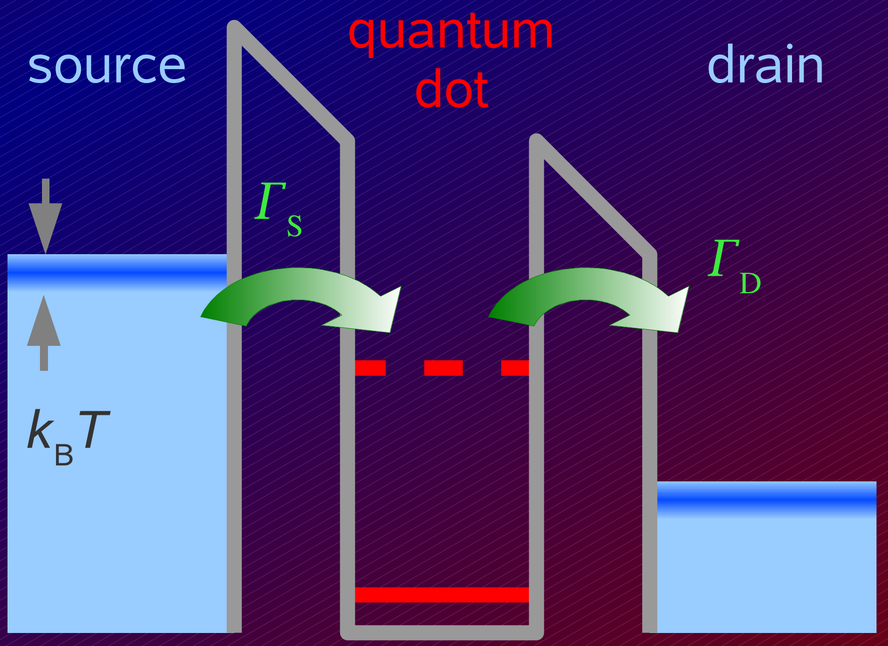
Theory: Hershfield *et al.*, PRB **47**, 1967 (1993)
Bagrets & Nazarov, PRB **67**, 085316 (2003)

Counting statistics in a single-level quantum dot

Bagrets & Nazarov, PRB **67**, 085316 (2003)

$$\frac{d}{dt}\begin{pmatrix} p_0 \\ p_1 \end{pmatrix} = M \begin{pmatrix} p_0 \\ p_1 \end{pmatrix}$$

$$M(\chi) = \begin{pmatrix} -\Gamma_D & \Gamma_D \\ \Gamma_S e^{i\chi} & -\Gamma_S \end{pmatrix}$$



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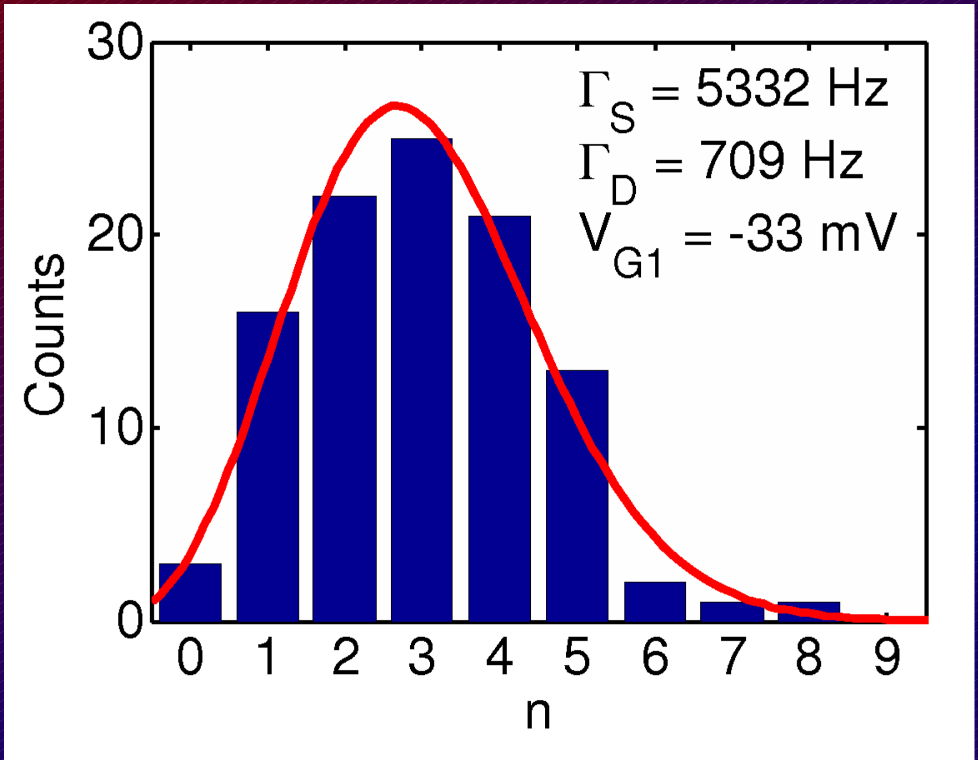

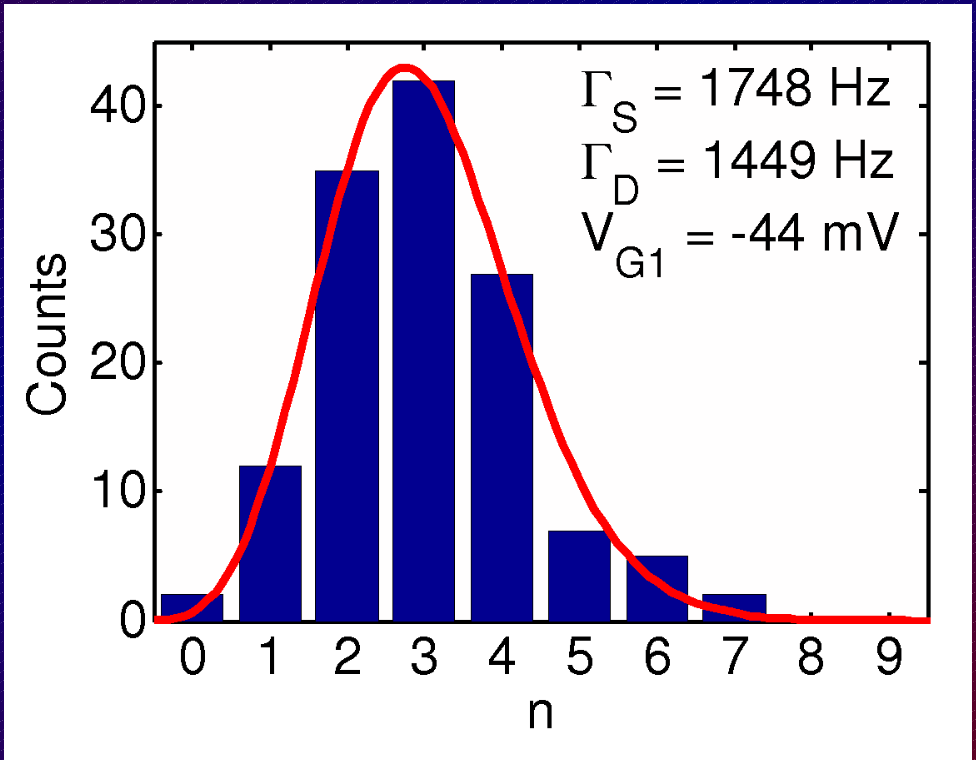

- Poisson distribution for asymmetric coupling
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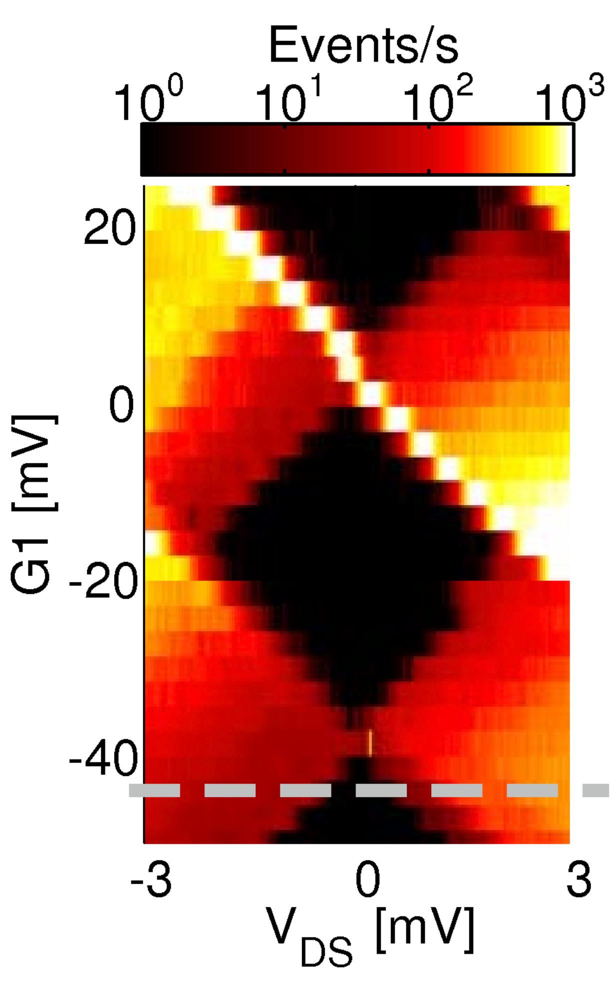
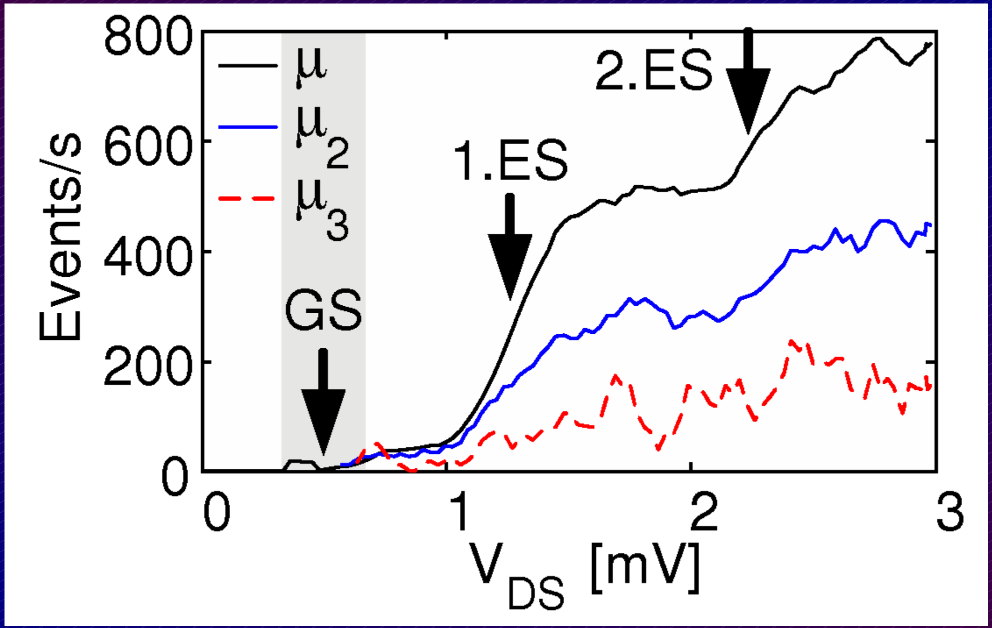
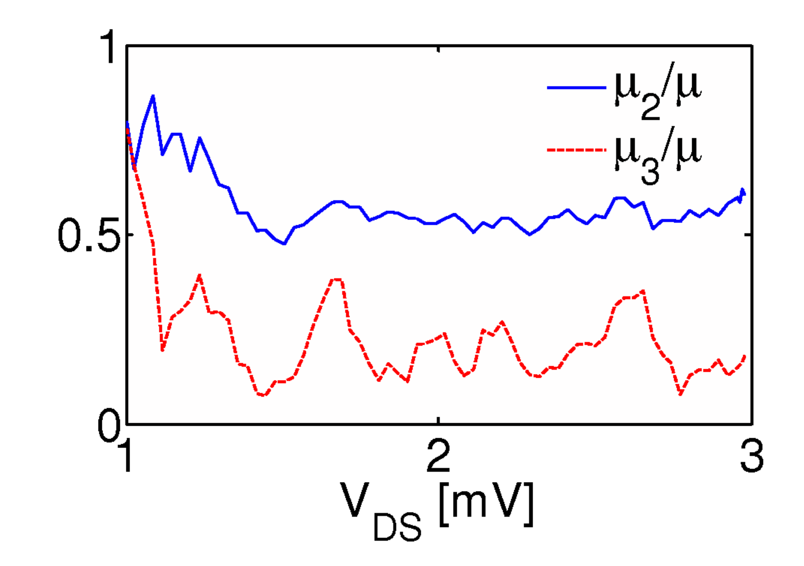
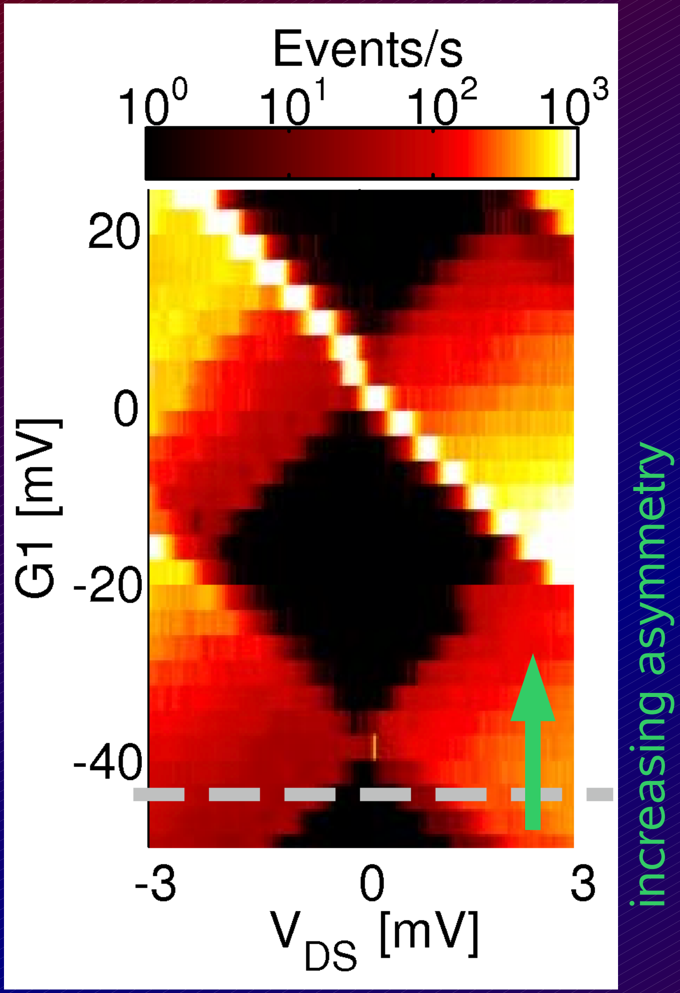
Bias dependence of the fluctuations



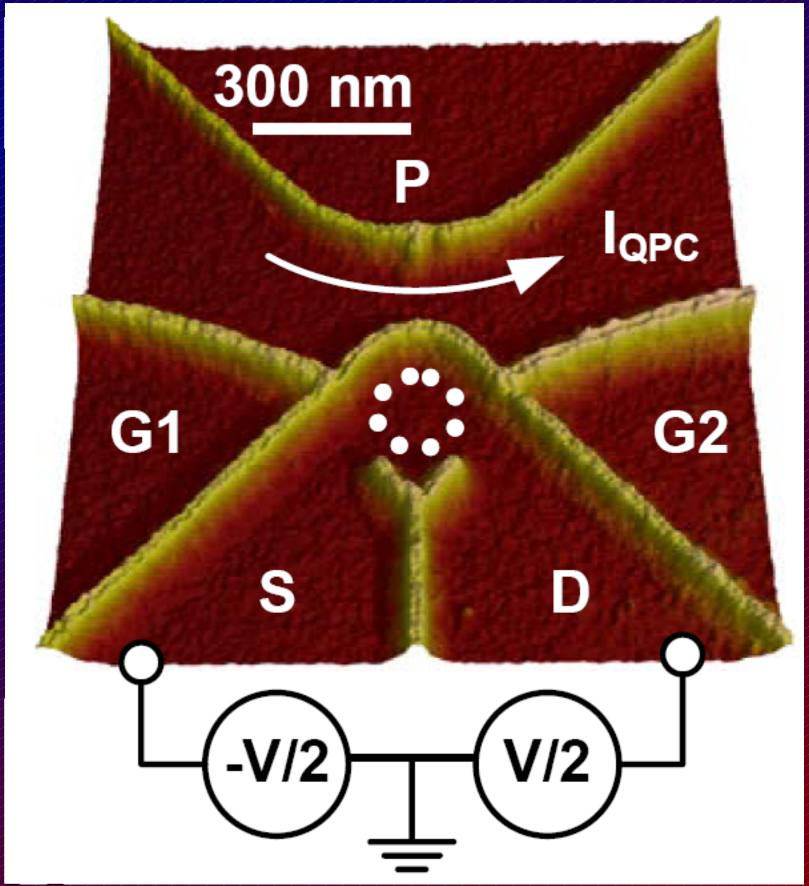




Adjustable asymmetry of the tunneling rates



$$a = \frac{\Gamma_S - \Gamma_D}{\Gamma_S + \Gamma_D}$$



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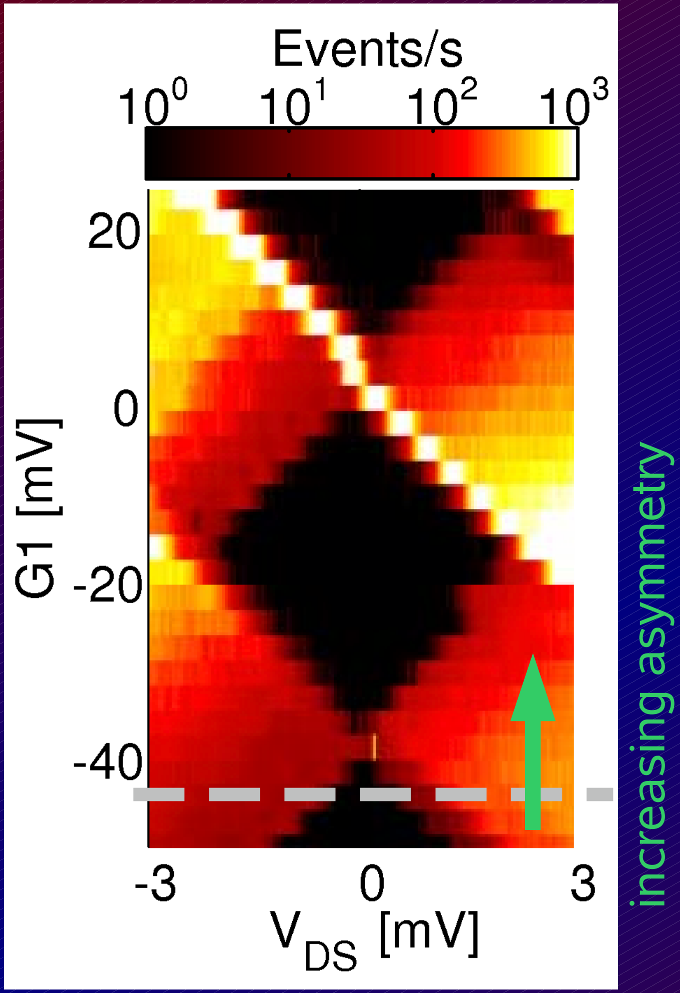

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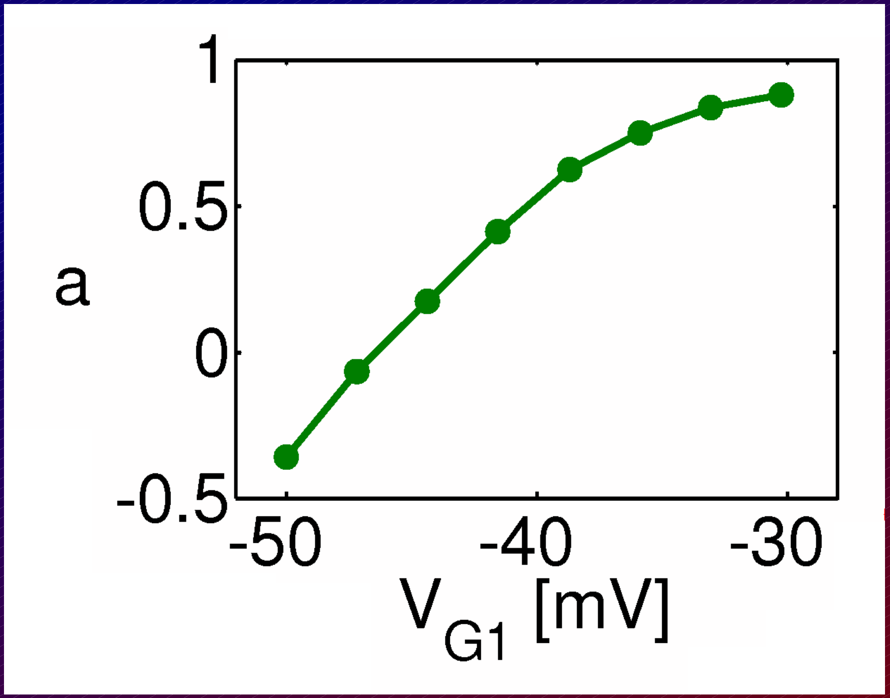
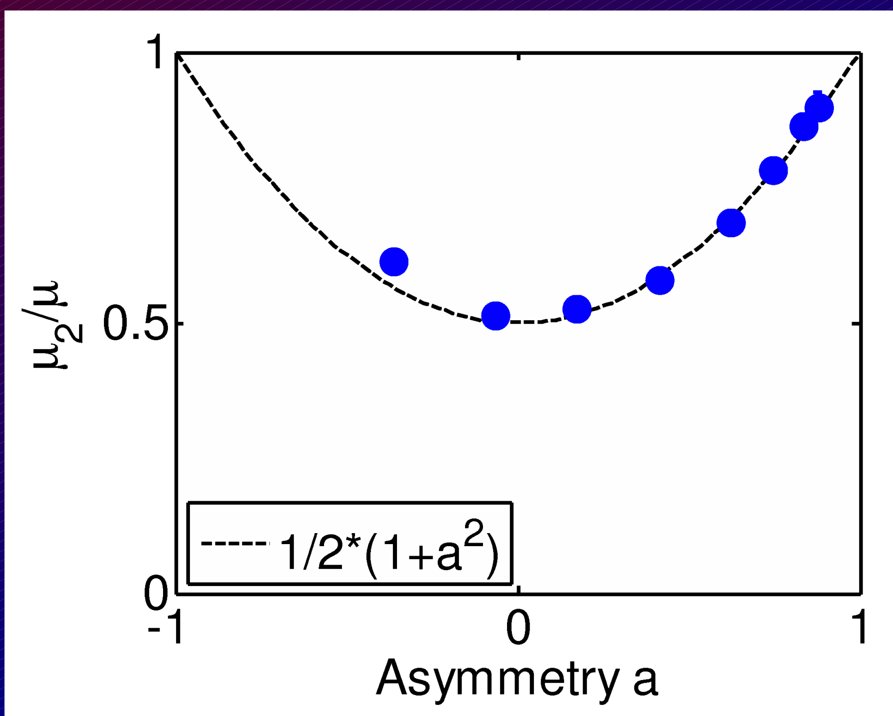
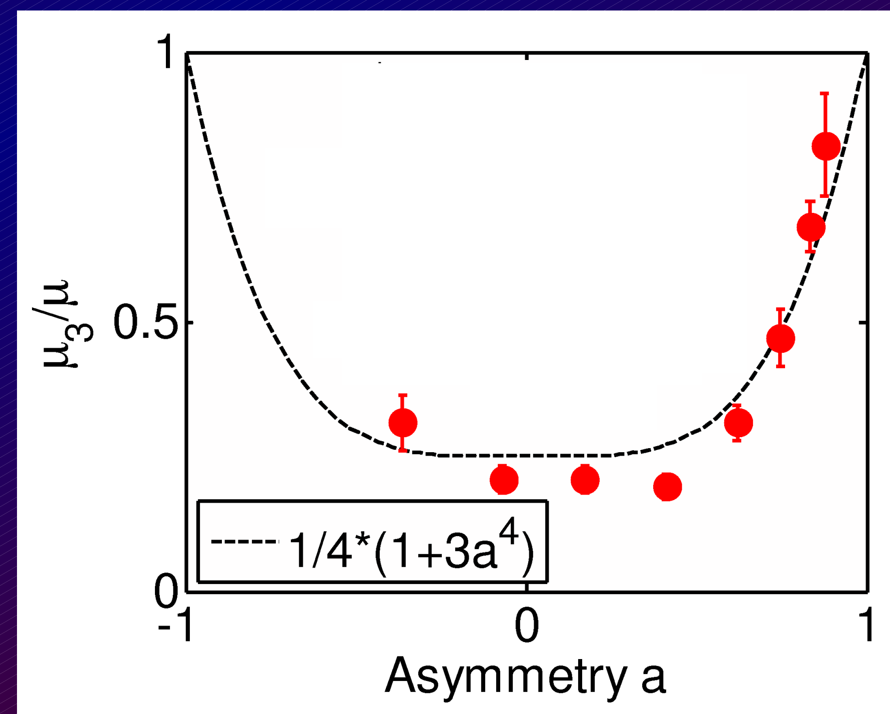


Current fluctuations vs. asymmetry

- Reduction of the second and third moments for symmetric coupling

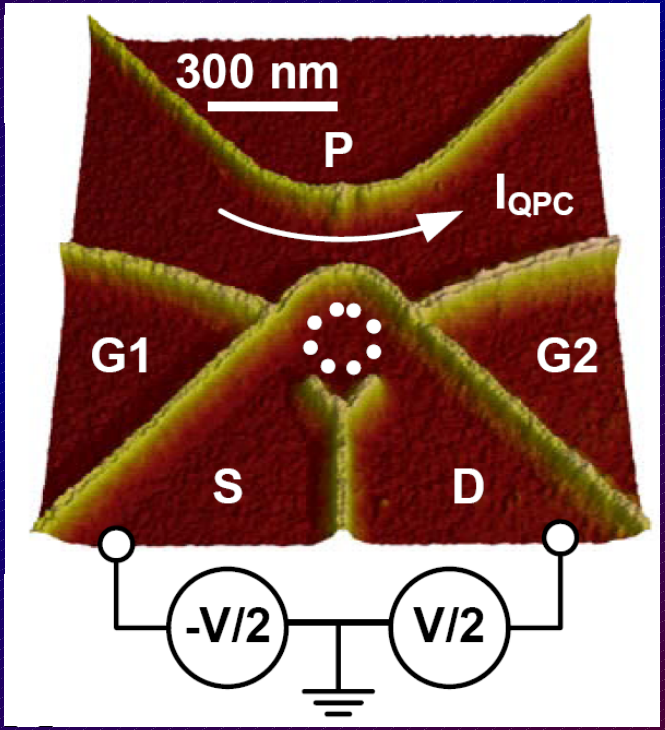


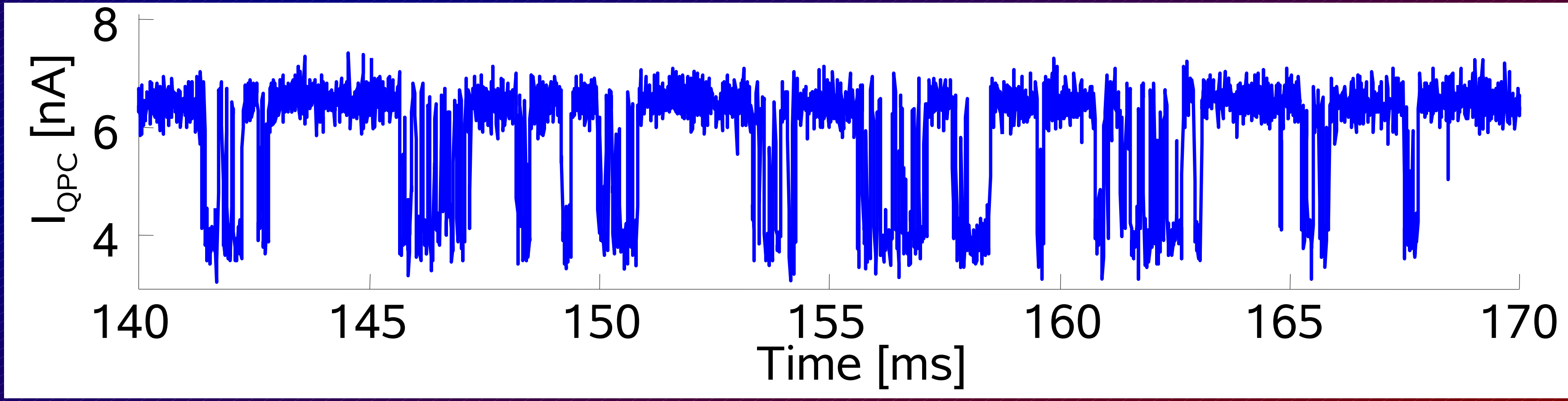


Theory: Hershfield *et al.*, PRB **47**, 1967 (1993)
Bagrets & Nazarov, PRB **67**, 085316 (2003)

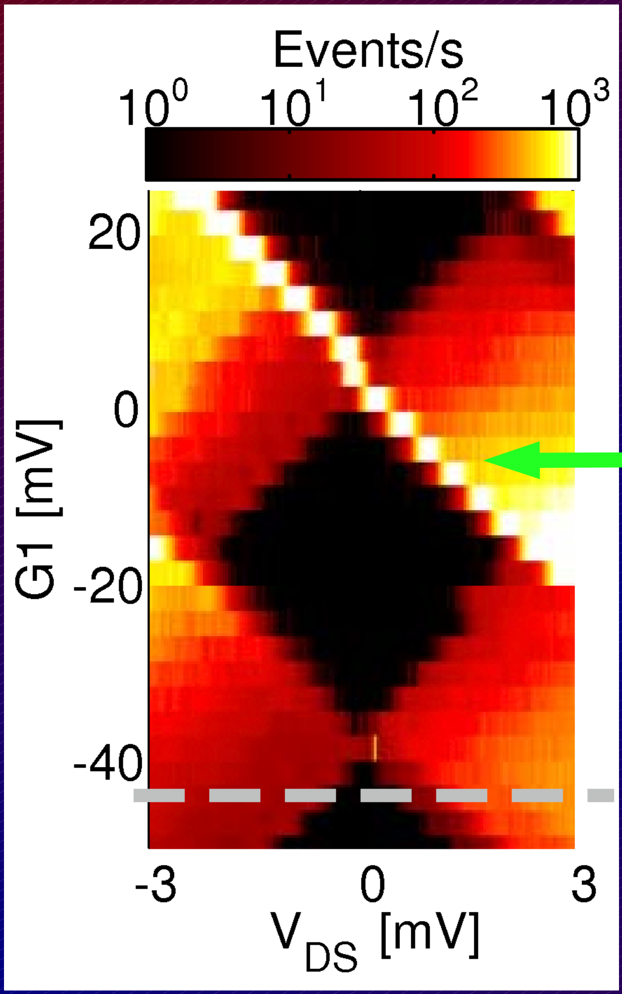
Bunching of electons

- Two time scales
  - $\Gamma_1 \sim 20$ kHz, $\Gamma_0 \sim 1.5$ kHz
- Fast tunneling sometimes blocked by a slow tunneling

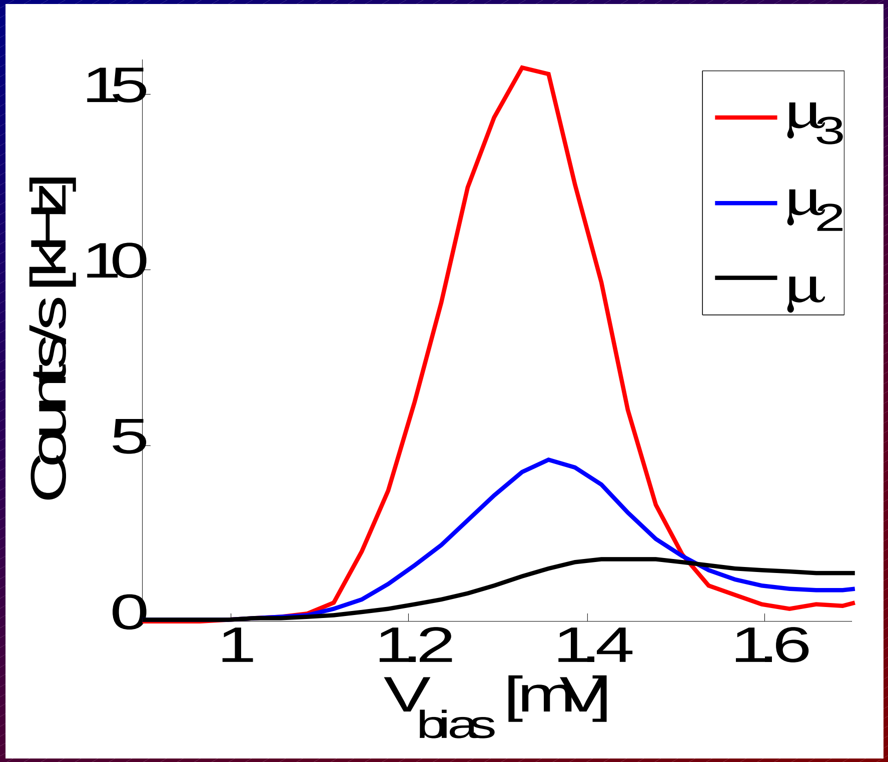




Bunching of electrons: super-poissonian shot noise

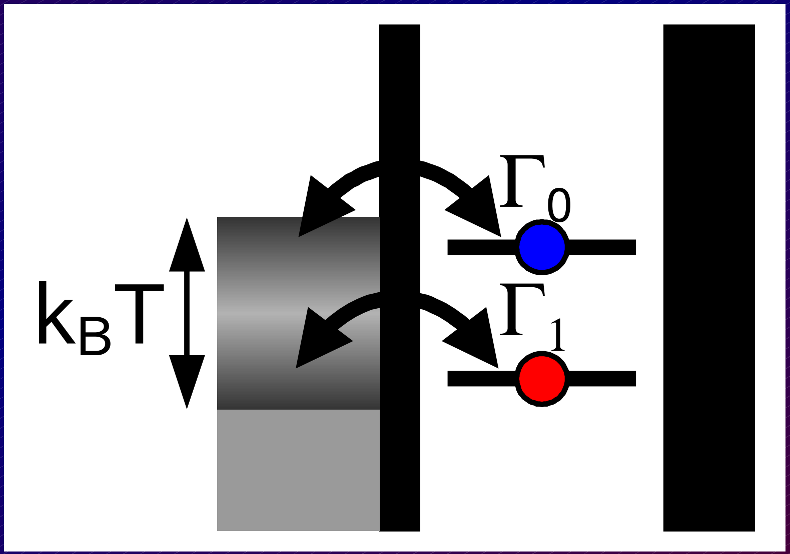


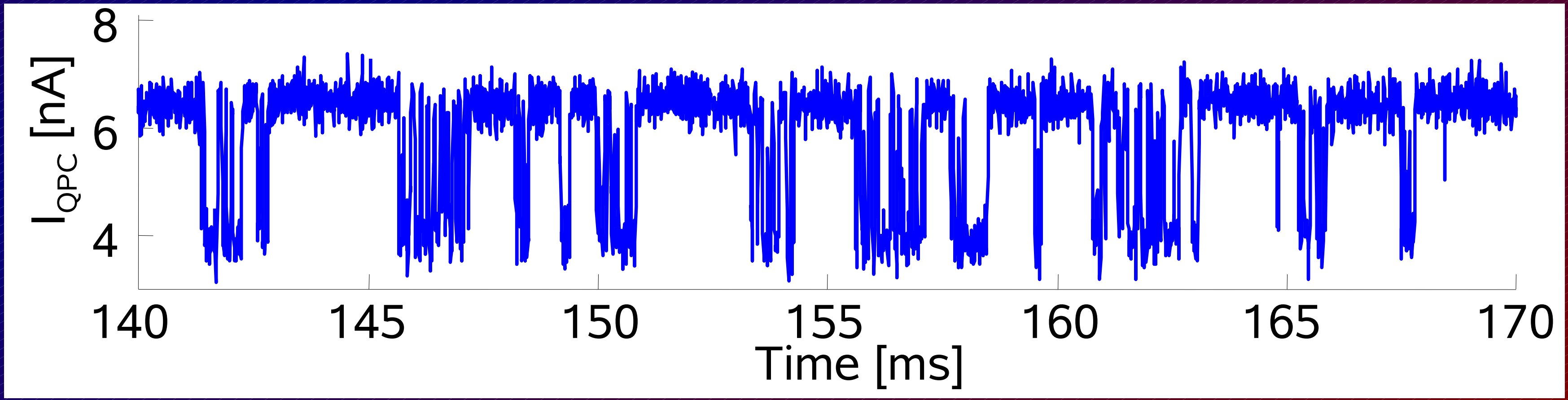
- super-poissonian noise occurs at the edge of conductance steps



Bunching of electrons: the model

- Needs two states with different coupling to the leads
  - the slow state blocks the conduction, due to Coulomb blockade
  - similar to Belzig, PRB **71,** 161301(R) (2005)



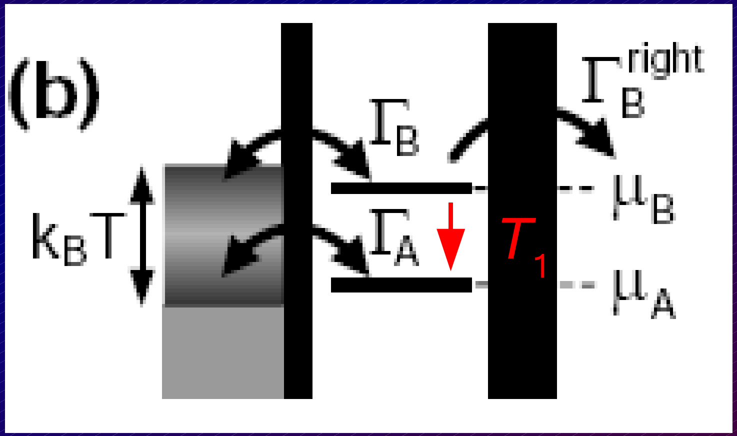


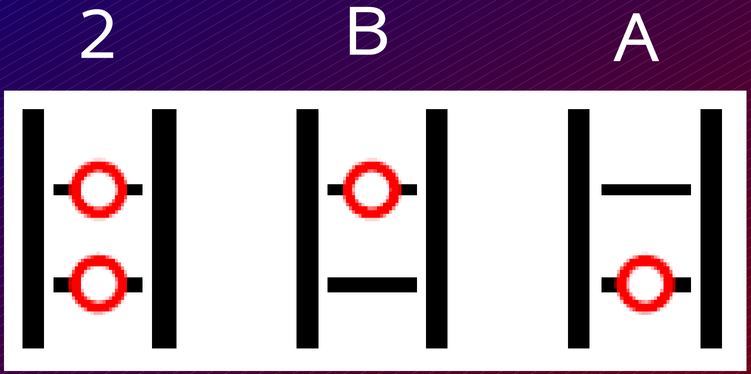
Bunching of electrons: the model

- Master equation

$$\frac{d}{dt}\begin{pmatrix} p_A \\ p_B \\ p_2 \end{pmatrix} = M \begin{pmatrix} p_A \\ p_B \\ p_2 \end{pmatrix}$$

$$M(\chi) = \begin{pmatrix} -\Gamma_B^i & \frac{1}{T_1} & (\Gamma_B^o + \Gamma_B^r) e^{i\chi} \\ 0 & -\Gamma_A^i - \frac{1}{T_1} & \Gamma_A^o e^{i\chi} \\ \Gamma_B^i & \Gamma_A^i & -\Gamma_A^o - \Gamma_B^o - \Gamma_B^r \end{pmatrix}$$



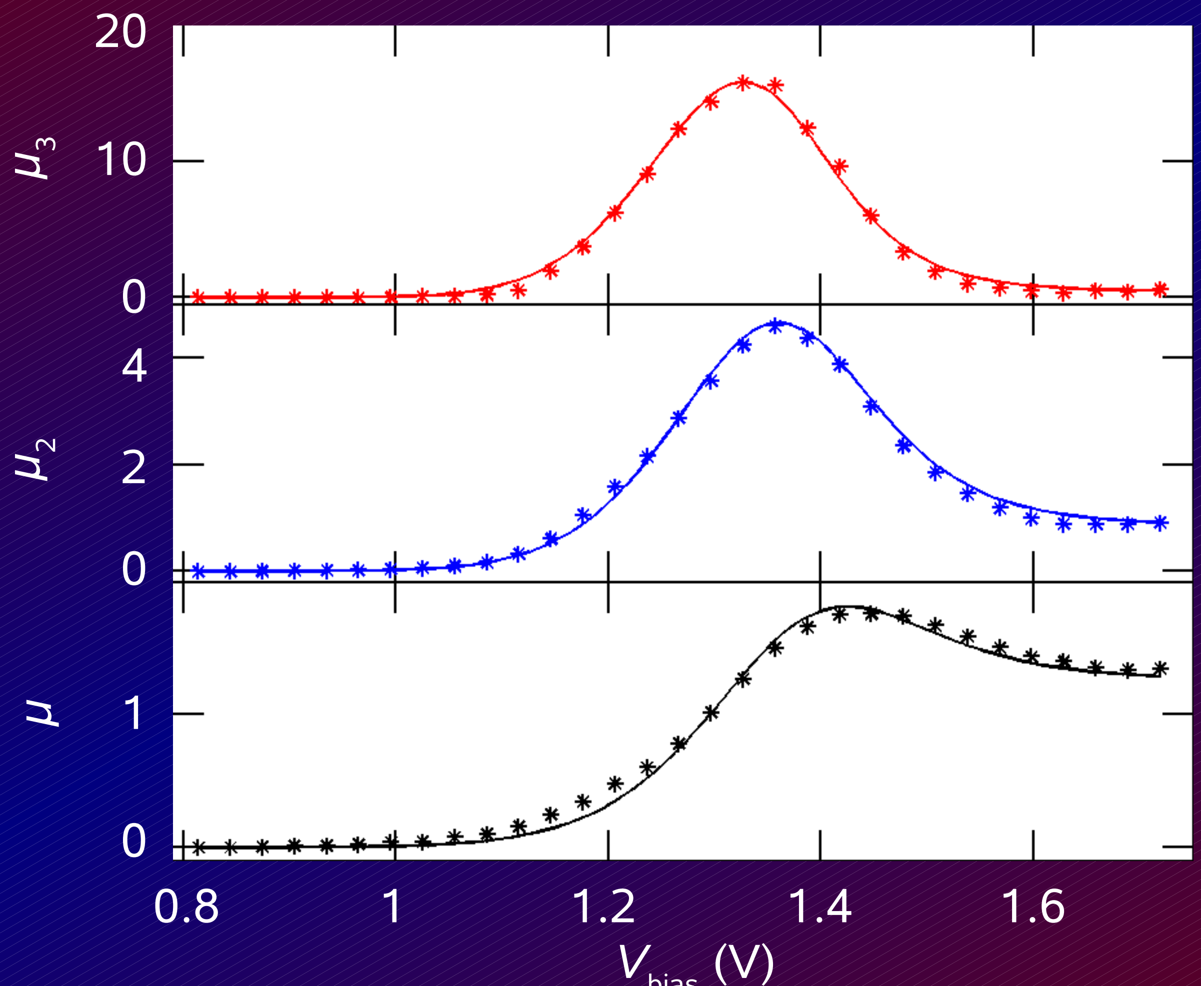


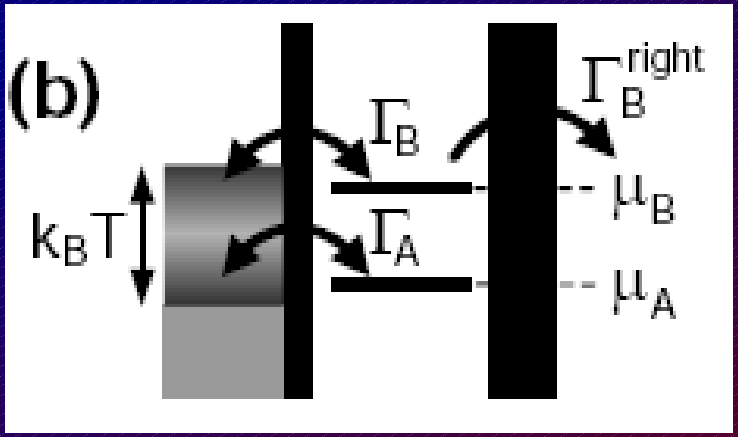
  - Cumulant generating function determined from the eigenvalues of $M(\chi)$

Bagrets & Nazarov, PRB **67,** 085316 (2003)
Belzig, PRB 71, 161301(R) (2995)

Bunching of electrons: super-poissonian shot noise

- Well described by our model





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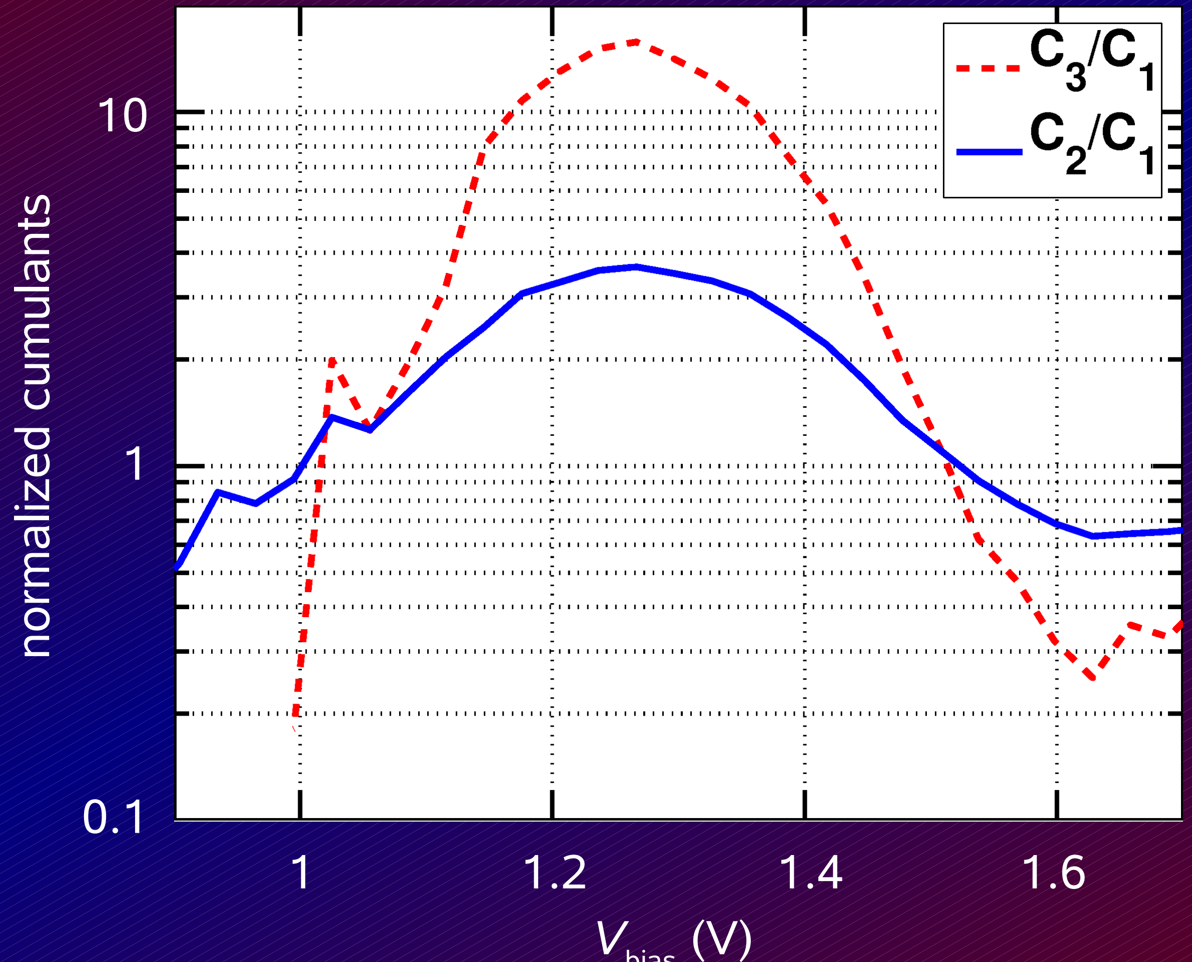
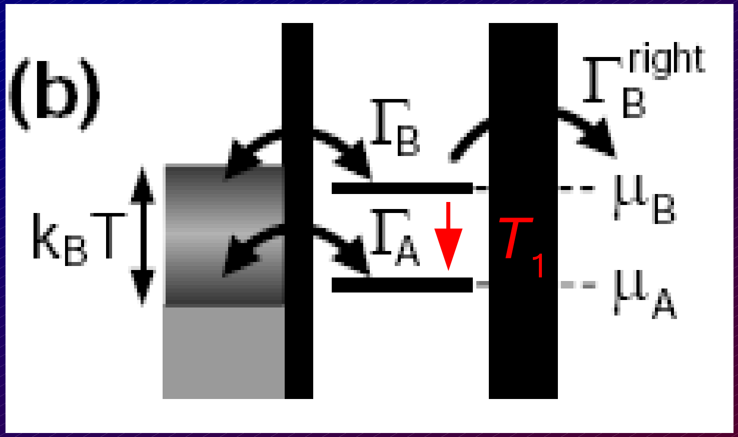
- Well described by our model, but requires a long relaxation rate: T_1 > 1ms... spin effect?





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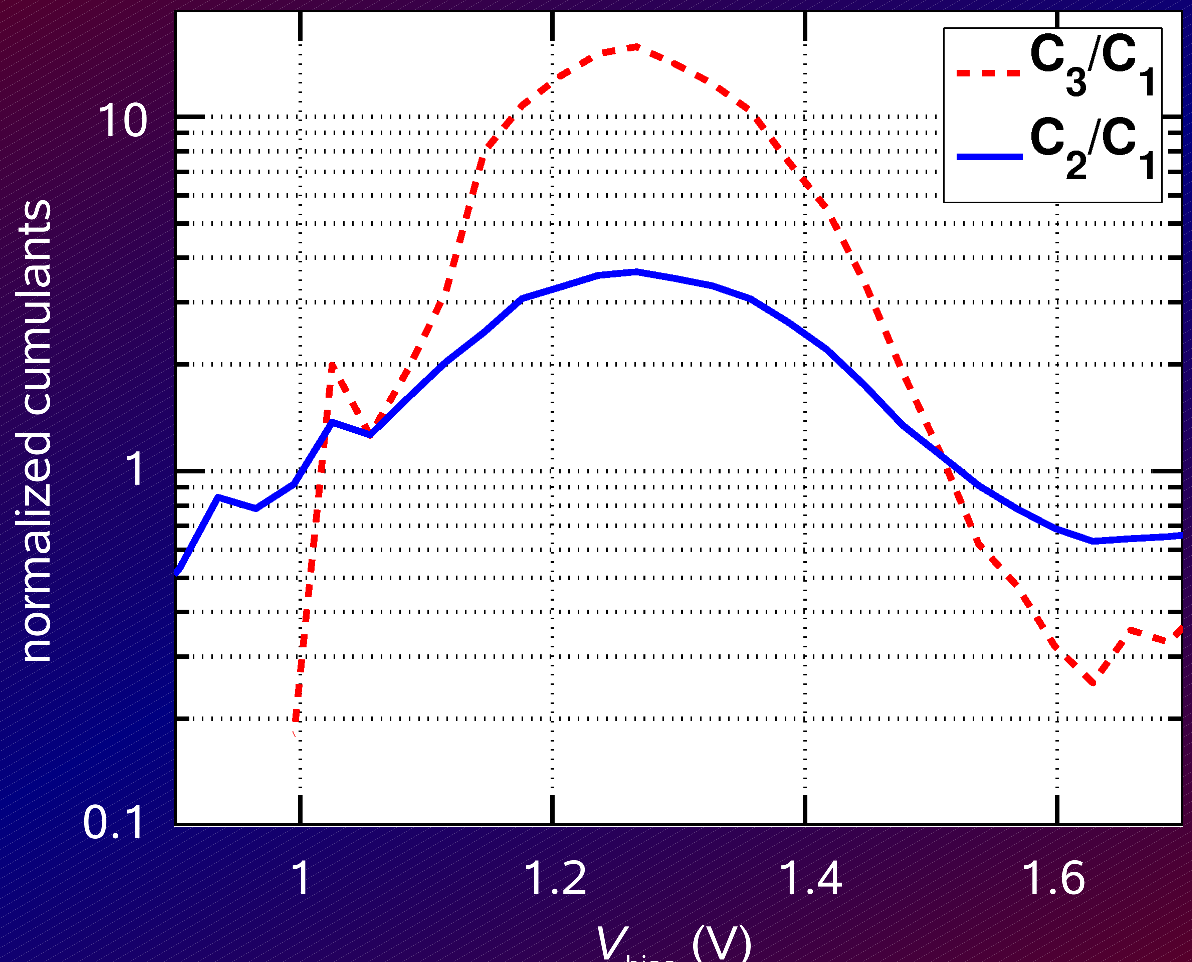

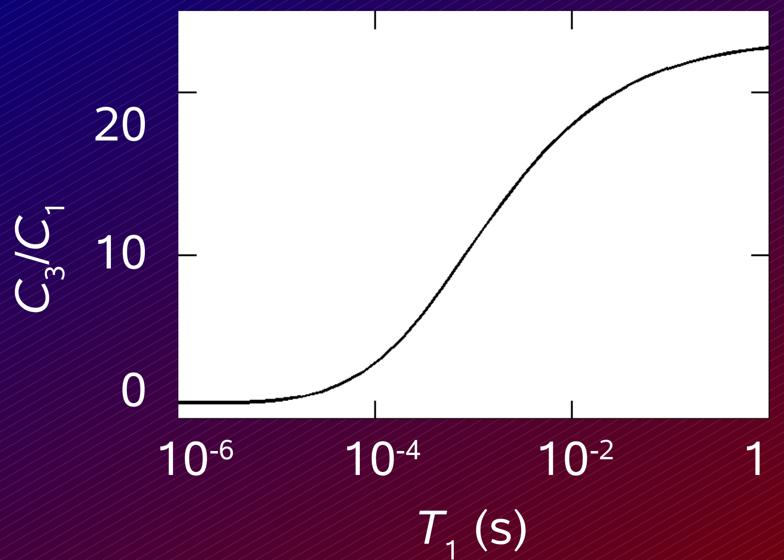

Conclusion

- Real-time detection of single electron traveling through a semiconductor quantum dot
- Measurement of current fluctuations
- Reduction of both the second and the third moments for symmetric coupling
- Bunching of electrons due to Coulomb blockade (information about relaxation time)
- Noise measurements are now available in lateral quantum dots

  (even full counting statistics)