Finite-frequency quantum noise in an interacting mesoscopic conductor

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International Conference

NANOELECTRONICS 2006

Novel Nanomaterials, Quantum Transport, and Noise of Electrons and Photons

8 - 11 January 2006, Lancaster University, UK

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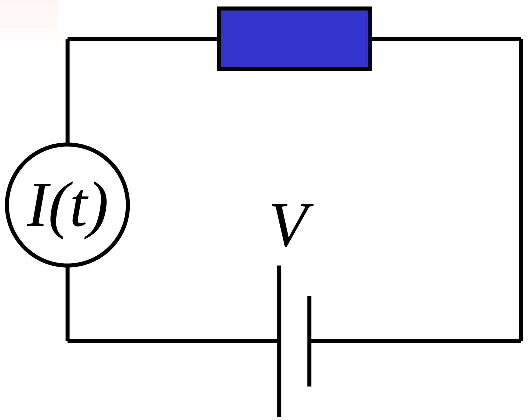
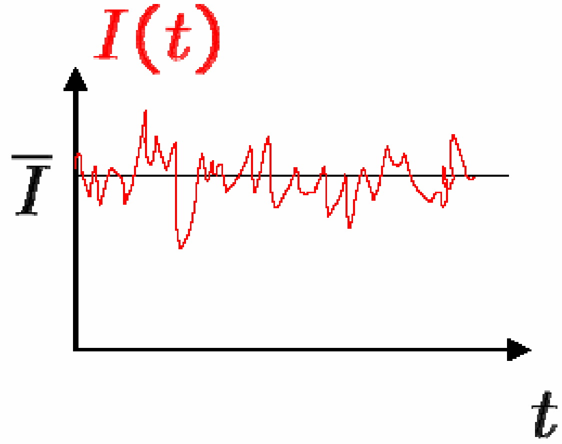
Current noise: the concept



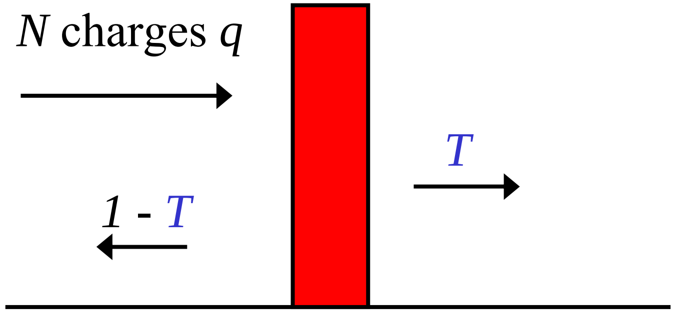


- Average current $\bar{I}$ $\longrightarrow$ conductance $G = \frac{d\bar{I}}{dV}$

- Noise $\delta I(t) = I(t) - \bar{I}$ $\longrightarrow$ noise spectral function

$$S(\omega) = \int dt e^{i\omega t} \overline{\delta I(t) \delta I(0)}$$

"Noise is the signal!" *(R. Landauer)*



$$\overline{Q} = qTN$$

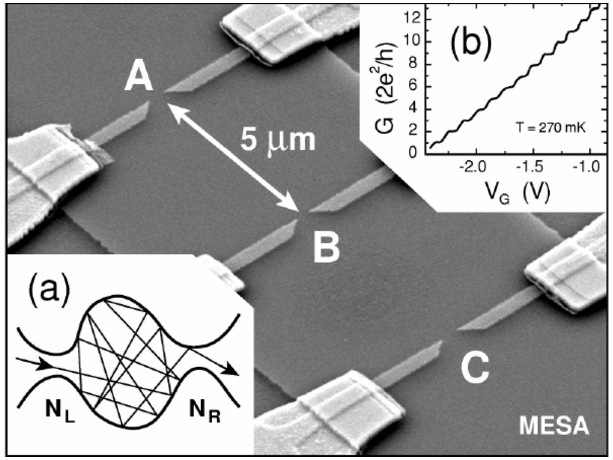
$$\overline{(\delta Q)^2} = 2\,q^2 T(1 - T)\,N$$

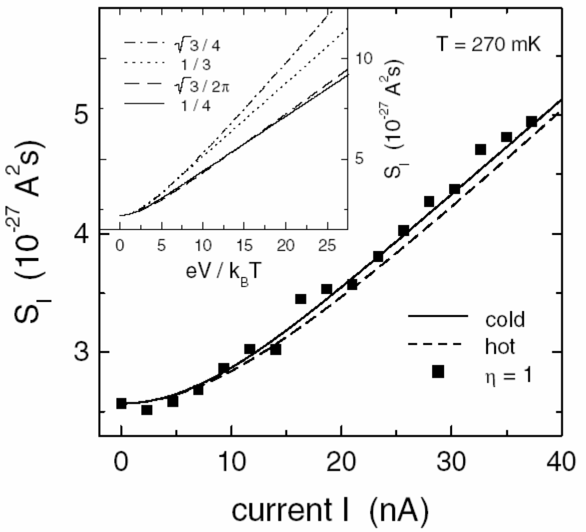
$$I = \frac{2q^2V}{h}\sum_n T_n \qquad\qquad S = \frac{4q^3V}{h}\sum_n T_n(1-T_n)$$

T_n's : mesoscopic « pin code »

Shot noise of chaotic cavity

(Oberholzer et al., PRL 2001; Nature 2002)

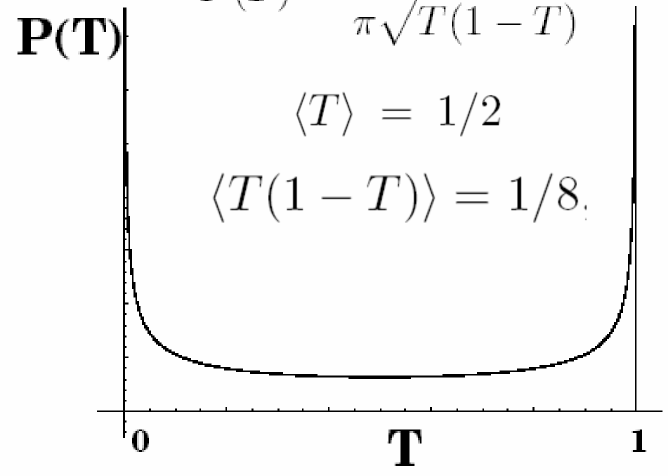




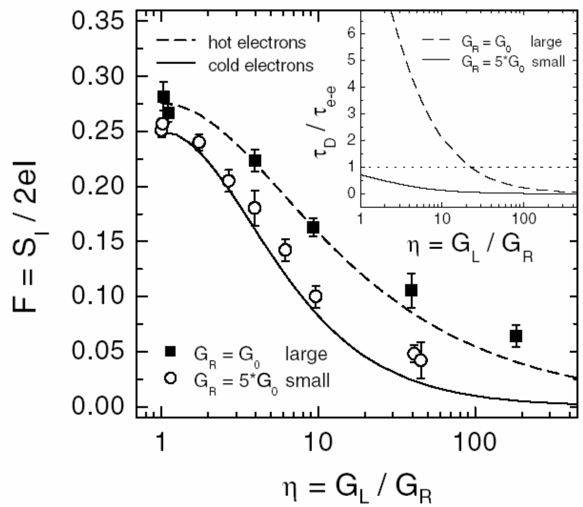
$$P(T) = \frac{1}{\pi\sqrt{T(1-T)}}$$

$$\langle T \rangle = 1/2$$

$$\langle T(1-T) \rangle = 1/8.$$

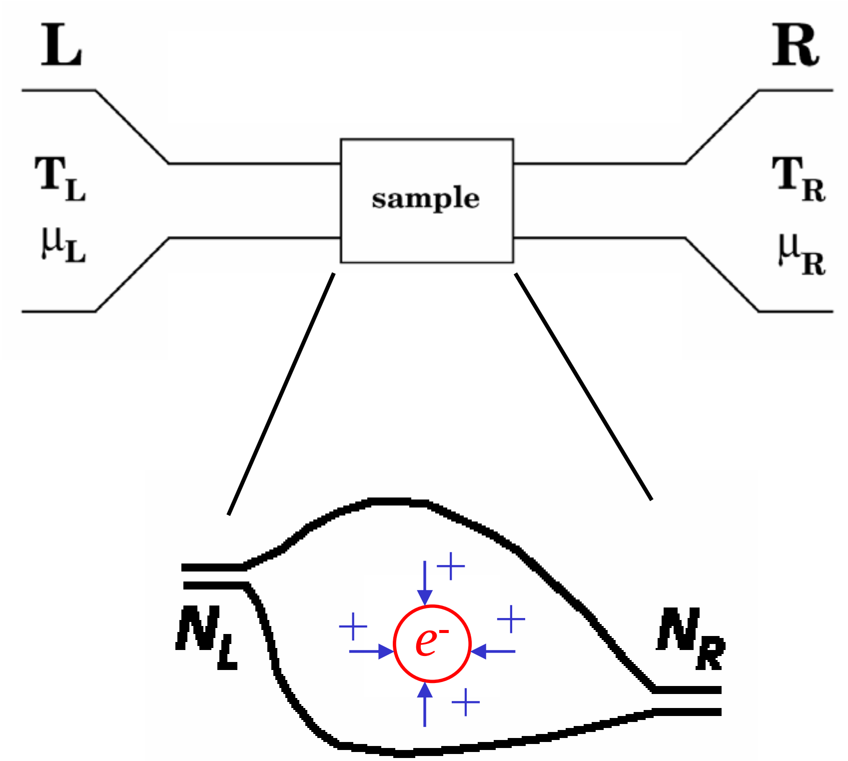


$$F = \frac{N_L N_R}{(N_L + N_R)^2}$$



"universal behavior!"

Finite frequency noise: various time scales



Time scales associated with reservoirs:

Bias voltage: $\hbar/eV$

Temperature: $\hbar/k_BT$

Time scales associated with conductor:

a. Time spent by electron:

Dwell time: τ_D

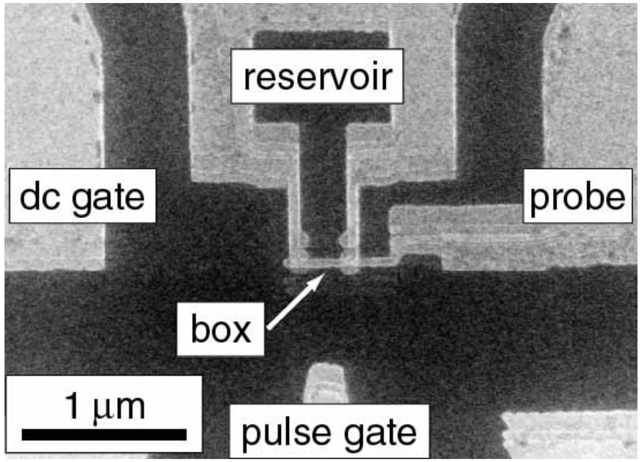
Thouless energy: $\hbar/E_{Th}$

b. Time scale associated with interactions:

Formation of screening cloud: τ_C

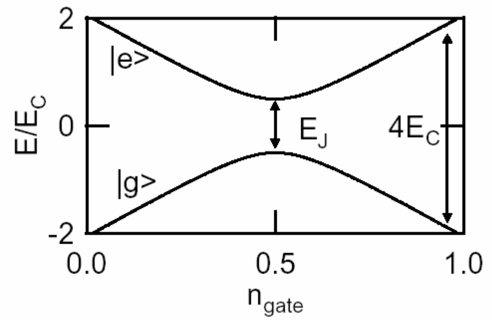
Noise spectroscopy with a mesoscopic device: superconducting qubit

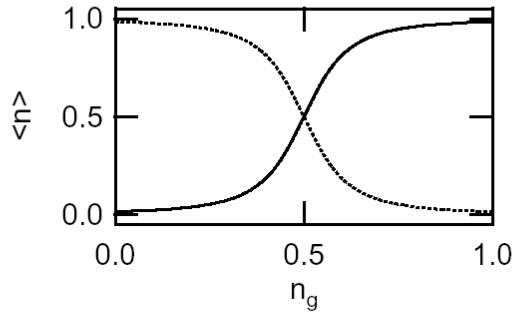
(Schoelkopf et al, 2003; Astafiev et al. PRL 2004)



(Nakamura et al, Nature 1999)

Qubit without a noise source: $H_0 = -\frac{\hbar\omega_{01}}{2}\sigma_z$



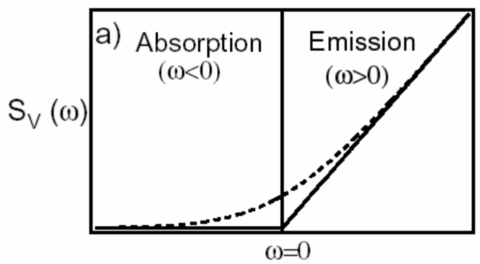


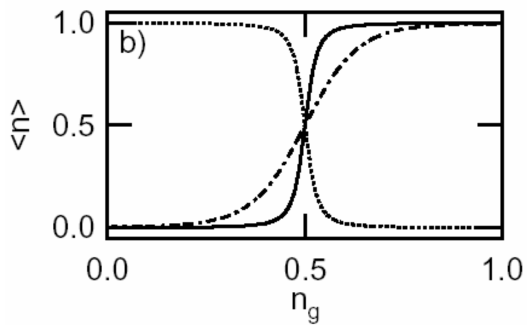
Coupling to a noise source: $V = Af(t)\sigma_x$

$$\Gamma_\uparrow = \frac{A^2}{\hbar^2} S_f(-\omega_{01})$$ (absorption)

$$\Gamma_\downarrow = \frac{A^2}{\hbar^2} S_f(+\omega_{01})$$ (emission)

Equilibrium noise:





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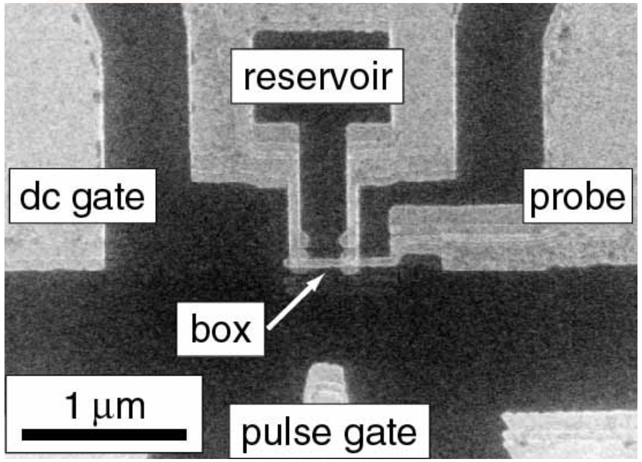

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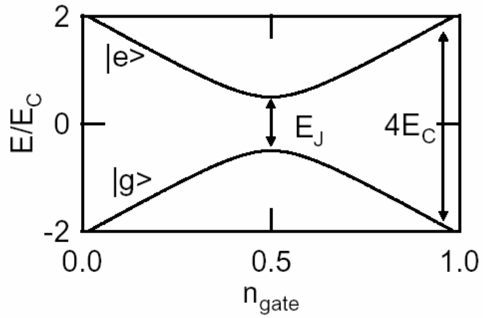

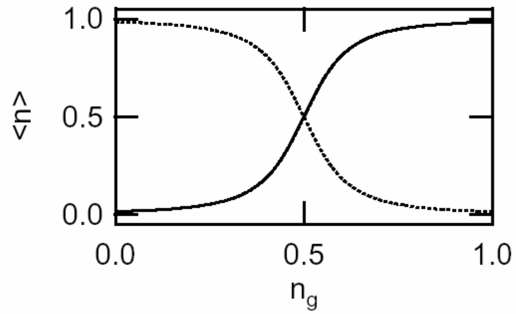

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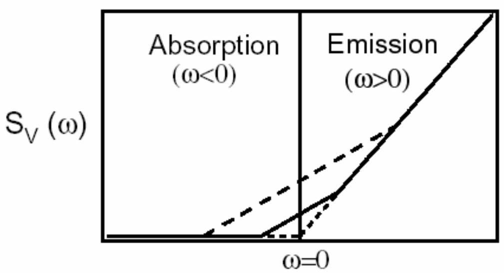
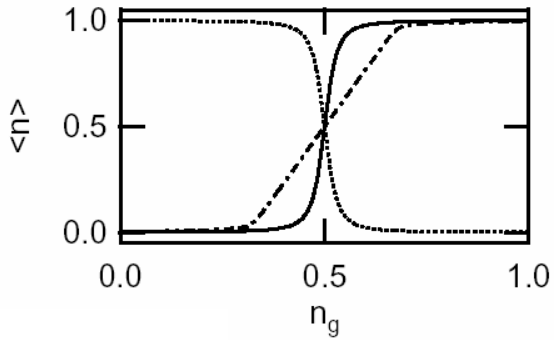
Shot noise





High frequency noise of a mesoscopic conductor

Related theoretical work:

-Mesoscopic capacitors *(Blanter & Büttiker, 2000)*

-Coulomb blockade systems

(Johansson et al. 2002, Käck et al.2003, Galaktionov et al. 2003)

- Quantum noise for energy-independent scattering

(Aguado and Kouwenhoven 2000, Zaikin et al. 2004)

- Quantum noise in Luttinger liquids

(Trauzettel et al. 2004, Dolcini et al. 2005, Lebedev et al. 2005)

- Chaotic cavity: stochastic path integral approach *(Nagaev et al. 2004)*
- Quantum noise, but without taking electron-electron interactions into account; i.e. non current-conserving *(various authors)*

This work: finite frequency quantum noise of chaotic cavity from scattering theory

- Take energy-dependence of scattering into account (random matrix theory)
- Take interaction effects (screening) into account (simple self-consistent RPA)

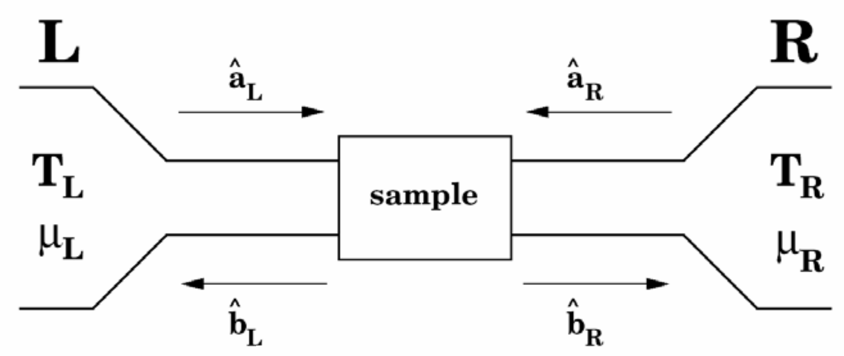
→ *Approach is current-conserving, no charge pile-up*

→ *Qualitatively similar behavior expected for other mesoscopic conductors*

→ *Easy to extend to calculate higher order correlation functions in the quantum limit*

Scattering theory of quantum transport (1): formalism

(Büttiker, PRB 1992; Blanter & Büttiker, Phys. Rep. 2000)



$$\begin{pmatrix} \hat{b}_{L1} \\ \cdots \\ \hat{b}_{LN_L} \\ \hat{b}_{R1} \\ \cdots \\ \hat{b}_{RN_R} \end{pmatrix} = s \begin{pmatrix} \hat{a}_{L1} \\ \cdots \\ \hat{a}_{LN_L} \\ \hat{a}_{R1} \\ \cdots \\ \hat{a}_{RN_R} \end{pmatrix}$$

$$s = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

$$\hat{I}_L(t) = \hat{I}_{L,in}(t) - \hat{I}_{L,out}(t)$$

$$\hat{I}_{L,in}(t) = \frac{e}{2\pi\hbar} \sum_n \int dE dE' e^{i(E-E')t/\hbar} \hat{a}^\dagger_{Ln}(E) \hat{a}_{Ln}(E')$$

$$\hat{I}_{L,out}(t) = \frac{e}{2\pi\hbar} \sum_{\alpha,\beta} \sum_{mnk} \int dE dE' e^{i(E-E')t/\hbar} \hat{a}^\dagger_{\alpha m}(E) s^\dagger_{L\alpha;mk}(E) s_{L\beta;kn}(E') \hat{a}_{\beta n}(E')$$

Scattering theory of quantum transport (2): current and noise

Average current: time-independent

$$\langle \hat{I}_L(t)\rangle = \frac{e}{2\pi\hbar}\int dE\,\mathrm{Tr}[t^\dagger(E)t(E)][f_L(E)-f_R(E)]$$

Current noise $\delta\hat{I} \equiv \hat{I} - \langle\hat{I}\rangle$: time-dependent

$$\begin{aligned}\langle\delta\hat{I}_L(t)\delta\hat{I}_L(0)\rangle &= \langle\delta\hat{I}_{L,in}(t)\delta\hat{I}_{L,in}(0)\rangle - \langle\delta\hat{I}_{L,in}(t)\delta\hat{I}_{L,out}(0)\rangle \\ &-\langle\delta\hat{I}_{L,out}(t)\delta\hat{I}_{L,in}(0)\rangle + \langle\delta\hat{I}_{L,out}(t)\delta\hat{I}_{L,out}(0)\rangle\end{aligned}$$

Frequency dependence: scattering matrix + Fermi functions

$$\langle\delta\hat{I}_{L,out}(t)\delta\hat{I}_{L,out}(0)\rangle =$$

$$= \frac{e^2}{h}\sum_{\alpha\beta}\int\frac{d\omega}{2\pi}e^{-i\omega t}\int dE\,\mathrm{Tr}[s^\dagger_{L\alpha}(E)s_{L\beta}(E+\hbar\omega)s^\dagger_{L\beta}(E+\hbar\omega)s_{L\alpha}(E)]f_\alpha(E)[1-f_\beta(E+\hbar\omega)].$$

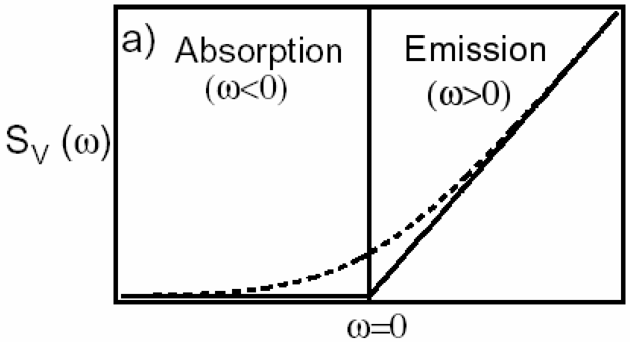
Scattering theory of quantum transport (3): current and noise for energy-independent scattering

Average current: "Landauer formula"

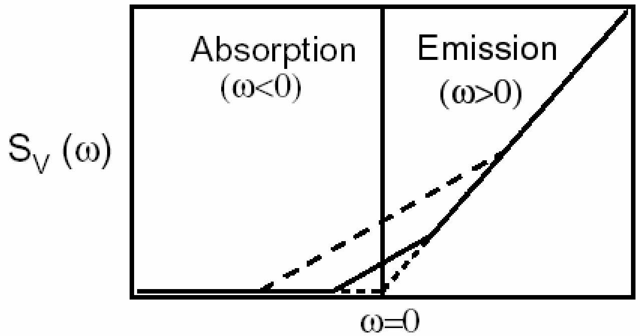
$$\langle \hat{I}_L(t) \rangle = \frac{e^2 V}{2\pi\hbar} \sum_n T_n = GV, \qquad G = (e^2/h) \sum_n T_n$$

Frequency-dependent current noise:

$$S_{\delta I \delta I}(\omega) = \frac{e^2}{h} \sum_n \left\{ T_n^2 \frac{2\hbar\omega}{1 - e^{-\beta\hbar\omega}} + T_n(1 - T_n) \left[\frac{\hbar\omega + eV}{1 - e^{-\beta(\hbar\omega + eV)}} + \frac{\hbar\omega - eV}{1 - e^{-\beta(\hbar\omega - eV)}} \right] \right\}$$



Equilibrium noise (V=0)



Non-equilibrium noise (T=0)

Chaotic cavity: characteristic time scales

Ehrenfest time

$$t_E = \frac{1}{\lambda}\ln\left(\frac{a}{\lambda_F}\right)$$

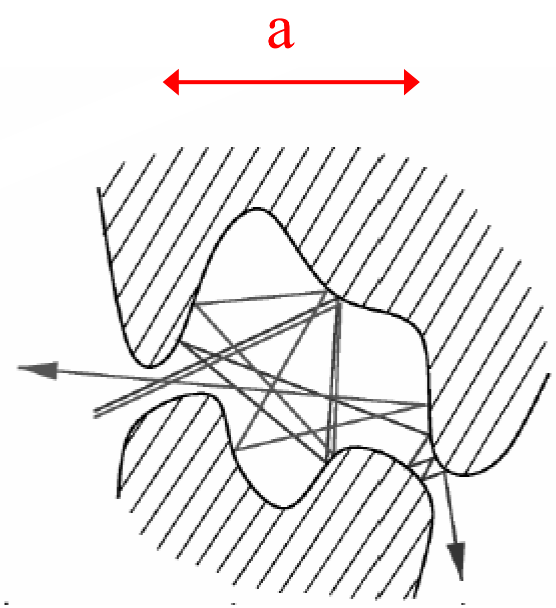
- *time for 2 classical trajectories to diverge*
- *time for wave packet to spread*

Dwell time

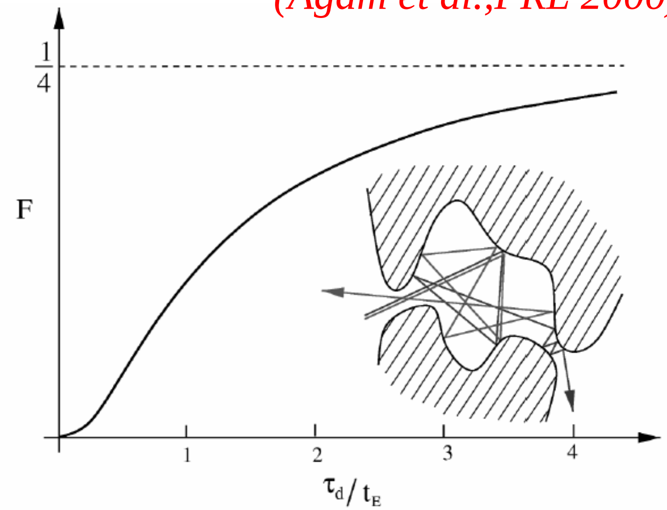
$$\tau_D = hN_F/N = R_Q e^2 N_F$$

Role of leads: $N_{L,R}$ = # channels in lead L,R

$N = N_L + N_R$: *L and R in parallel!*

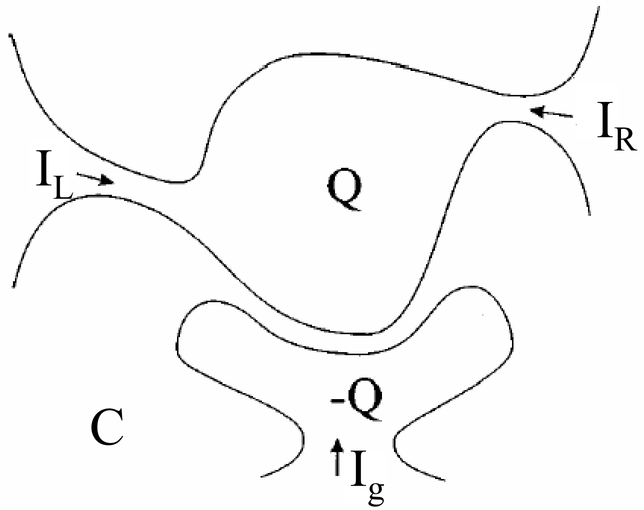


(Agam et al.,PRL 2000)



RMT applies for $\tau_D > t_E$
-universality ($N_{L,R}$ not too large)
-zero-dimensionality

Representative example of a mesoscopic scatterer: interacting chaotic cavity



Average current:

$$\langle \hat{I}_L(t)\rangle = \frac{e}{2\pi\hbar}\int dE\,\mathrm{Tr}[t^\dagger(E)t(E)][f_L(E)-f_R(E)]$$

Energy-dependent scattering: RMT

(Polianski & Brouwer, J. Phys. A 2003)

$$\langle \mathrm{Tr}[s^\dagger_{\alpha\beta}(E_1)s_{\alpha\beta}(E_2)]\rangle = \frac{N_\alpha N_\beta}{N}\frac{1}{1-i(E_2-E_1)\tau_D/\hbar}$$

- *Leads: # channels in lead L,R large:* N_L , $N_R >> 1$

-*Leading contribution in N, we ignore weak localization (but* $\tau_D > t_E$*)*

→ *treat interactions within self-consistent charging model*

(Brouwer et al, PRL & PRB 2005)

Landauer conductance: $I_L = GV$ $\qquad G = (e^2/h)\underbrace{N_L N_R/N}$ → $\sum_n \langle T_n\rangle$

High frequency noise in a non-interacting cavity

(Polianski & Brouwer, J. Phys. A 2003)

$$\langle \mathrm{Tr}[s^\dagger_{\beta\alpha}(E_1)s_{\beta\gamma}(E_2)s^\dagger_{\delta\gamma}(E_3)s_{\delta\alpha}(E_4)]\rangle =$$

$$\frac{N_\alpha N_\beta N_\delta}{N^2}\frac{\delta_{\alpha\gamma}}{[1-i(E_2-E_1)\tau_D/\hbar][1-i(E_4-E_3)\tau_D/\hbar]}$$

$$+\frac{N_\alpha N_\beta N_\gamma}{N^2}\frac{\delta_{\beta\delta}}{[1-i(E_4-E_1)\tau_D/\hbar][1-i(E_2-E_3)\tau_D/\hbar]}$$

$$-\frac{N_\alpha N_\beta N_\gamma N_\delta}{N^3}\frac{[1-i(E_2+E_4-E_1-E_3)\tau_D/\hbar]}{[1-i(E_2-E_1)\tau_D/\hbar][1-i(E_2-E_3)\tau_D/\hbar][1-i(E_4-E_1)\tau_D/\hbar][1-i(E_4-E_3)\tau_D/\hbar]}$$

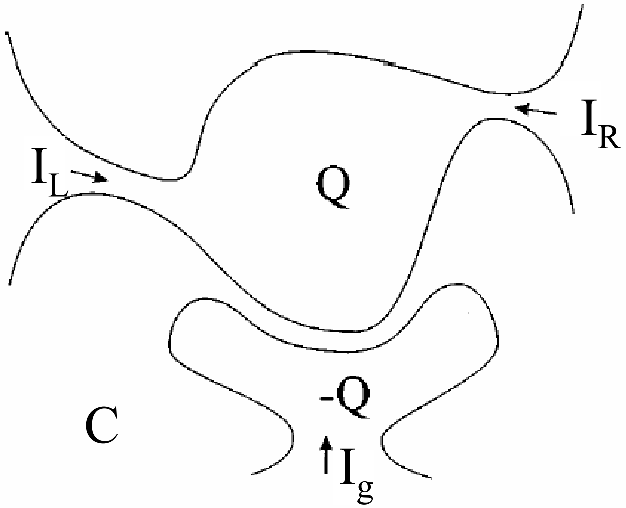
$$\overbrace{\frac{N_L N_R}{N}\left[1-\frac{N_L N_R}{N^2}\right]}^{\sum_n \langle T_n^2\rangle}$$

$$S^{cav,0}_{\delta I\delta I}(\omega) = \frac{e^2}{h}\frac{1+\omega^2\tau_D^2 N/N_R}{1+\omega^2\tau_D^2}\left\{\frac{N_L N_R}{N}\left[1-\frac{N_L N_R}{N^2}\right]\frac{2\hbar\omega}{1-e^{-\beta\hbar\omega}}+\right.$$
$$\left.\frac{N_L^2 N_R^2}{N^3}\left[\frac{\hbar\omega+eV}{1-e^{-\beta(\hbar\omega+eV)}}+\frac{\hbar\omega-eV}{1-e^{-\beta(\hbar\omega-eV)}}\right]\right\}.$$

$$\underbrace{\frac{N_L^2 N_R^2}{N^3}}_{\sum_n \langle T_n(1-T_n)\rangle}$$

Effect of Coulomb interactions: screening

(Pedersen et al. PRB 1998)



$$\hat{Q} = C\hat{U} = e\hat{\mathcal{N}} - e^2 N\hat{U}.$$

bare charge

$$\hat{I}_L(\omega) = e\int dE \sum_{\beta\gamma}\sum_{nm} \hat{a}^{\dagger}_{\beta m}(E) A^0_{\beta\gamma;mn}(L,E,E+\hbar\omega)\hat{a}_{\gamma n}(E+\hbar\omega).$$

$$A^0_{\delta\gamma}(\alpha,E,E') = \underbrace{\delta_{\alpha\delta}\delta_{\alpha\gamma}1_\alpha - s^{\dagger}_{\alpha\delta}(E)s_{\alpha\gamma}(E)}_{\text{zero-frequency part}} - \underbrace{2\pi i(E'-E)\mathcal{N}_{\delta\gamma}(\alpha,E,E')}_{\text{finite-frequency part}}$$

Density of states matrix:

$$\mathcal{N}_{\beta\gamma}(\alpha,E,E') = \frac{i}{2\pi}\frac{s^{\dagger}_{\alpha\beta}(E)\left(s_{\alpha\gamma}(E) - s_{\alpha\gamma}(E')\right)}{E'-E}$$

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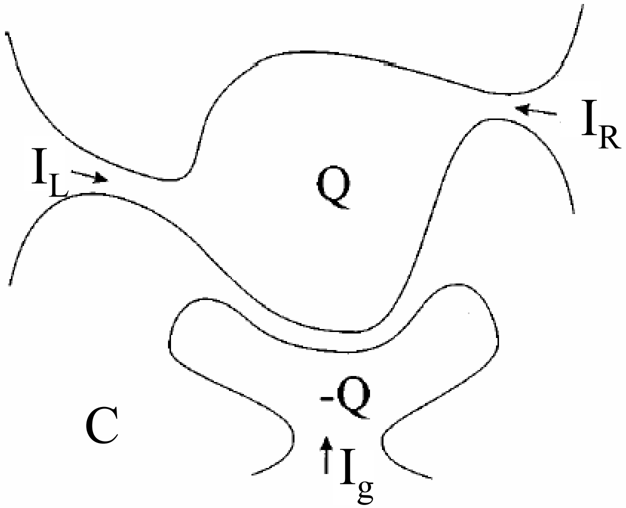

$$\hat{Q} = C\hat{U} = e\hat{\mathcal{N}} - e^2 N\hat{U}.$$

induced charge (RPA)

$$\hat{U} = Ge\hat{\mathcal{N}}, \qquad G(\omega) = (C + e^2 N(\omega))^{-1}.$$

Total current (bare +screening): $$\hat{I}_\alpha(\omega) = \hat{I}^0_\alpha(\omega) - i\omega e^2 \underline{N}_\alpha(\omega)\hat{U}(\omega)$$

Effective current matrix:

$$A_{\delta\gamma}(\alpha, E, E+\hbar\omega) = A^0_{\delta\gamma}(\alpha, E, E+\hbar\omega) + i2\pi\hbar\omega e^2 \underline{N}_\alpha G \mathcal{N}_{\delta\gamma}(E, E+\hbar\omega)$$

Density of states matrix: $$\mathcal{N}_{\beta\gamma}(\alpha, E, E') = \frac{i}{2\pi}\frac{s^\dagger_{\alpha\beta}(E)\left(s_{\alpha\gamma}(E) - s_{\alpha\gamma}(E')\right)}{E' - E}$$

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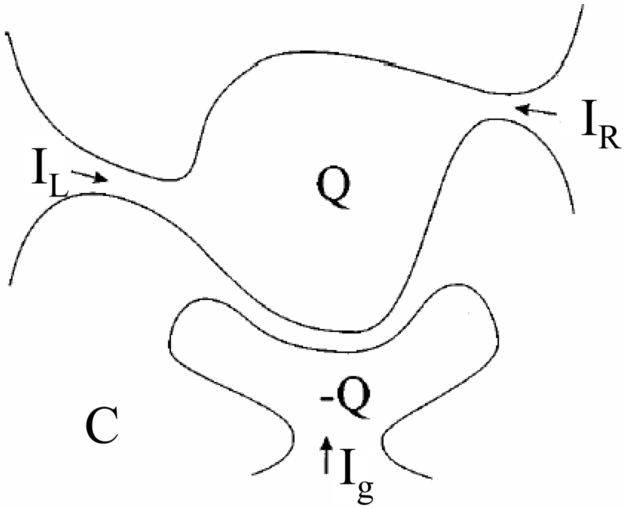

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Current conservation: $\sum_\nu A_{\delta\gamma}(\nu, E, E+\hbar\omega) = 0.$

Gate current:

$$\hat{I}_g(\omega) = i\omega C\hat{U}(\omega) \qquad A_{\delta\gamma}(0, E, E+\hbar\omega) = -i2\pi\hbar\omega C G \mathcal{N}_{\delta\gamma}(E, E+\hbar\omega)$$

High frequency quantum noise of an interacting cavity

$$S^{cav}_{\delta I \delta I}(\omega) = \frac{e^2}{h}\frac{1+\omega^2\tau^2 N/N_R}{1+\omega^2\tau^2}\left\{\frac{N_L N_R}{N}\left[1-\frac{N_L N_R}{N^2}\right]\frac{2\hbar\omega}{1-e^{-\beta\hbar\omega}}+ \frac{N_L^2 N_R^2}{N^3}\left[\frac{\hbar\omega+eV}{1-e^{-\beta(\hbar\omega+eV)}}+\frac{\hbar\omega-eV}{1-e^{-\beta(\hbar\omega-eV)}}\right]\right\}.$$

Non-interacting result with replacement $\tau_D \longrightarrow \tau$

$$\frac{1}{\tau} = \frac{1}{\tau_D} + \frac{Ne^2}{hC} = \frac{1}{R_Q C_\mu}$$

$\frac{1}{\tau_D}$ → dwell time

$\frac{Ne^2}{hC}$ → RC-time

C_μ → electrochemical capacitance

$$\frac{1}{C_\mu} = \frac{1}{e^2 N_F} + \frac{1}{C}$$

R_Q → charge relaxation resistance

$$R_Q = h/Ne^2$$

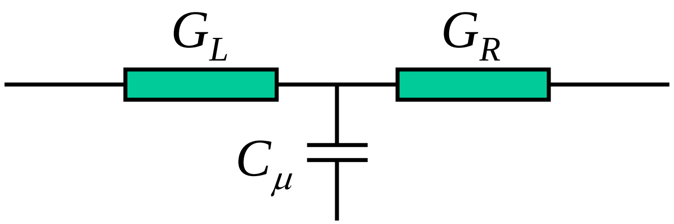
Symmetrized equilibrium noise of an interacting cavity: fluctuation dissipation theorem

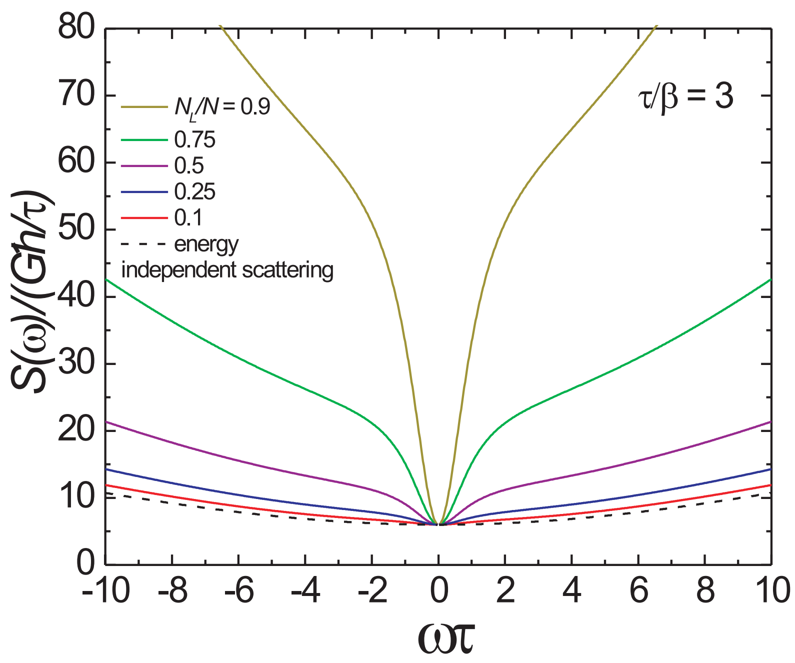
$$S^{cav}_{sym}(\omega) = \frac{e^2}{h}\frac{N_L N_R}{N}\hbar\omega \coth\left(\beta\hbar\omega/2\right)\frac{1+\omega^2\tau^2 N/N_R}{1+\omega^2\tau^2}$$

Fluctuation-dissipation theorem: $G_{LR}(\omega) = \frac{N_L N_R}{N}\frac{e^2}{h}\frac{1}{1-i\omega\tau}$

(Brouwer & Büttiker, EPL 1997)

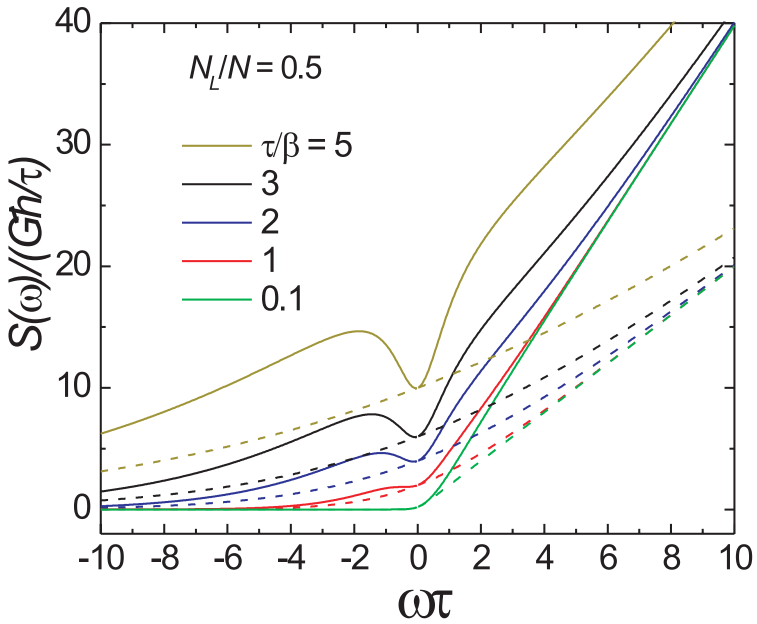
Effective circuit:

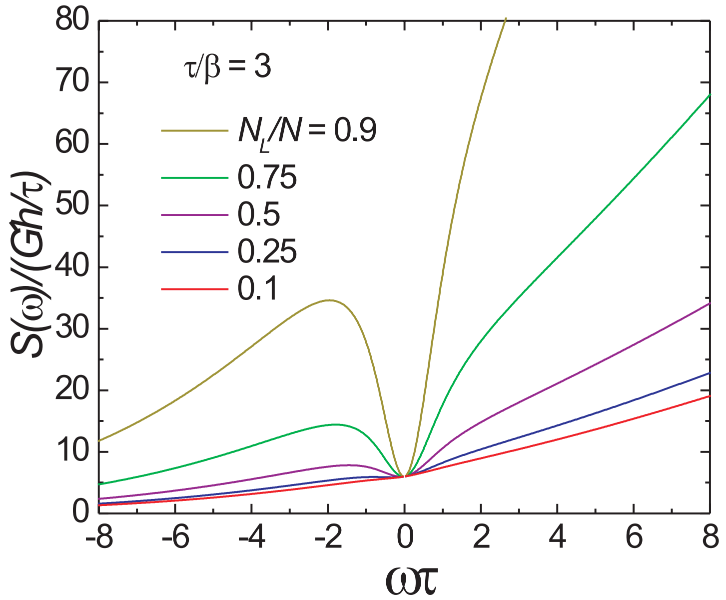




Equilibrium noise of an interacting cavity; absorption and emission

$$S^{cav}_{\delta I \delta I}(\omega) = \frac{e^2}{h} \frac{N_L N_R}{N} \frac{2\hbar\omega}{1 - e^{-\beta\hbar\omega}} \frac{1 + \omega^2\tau^2 N/N_R}{1 + \omega^2\tau^2}.$$





absorption emission

absorption emission

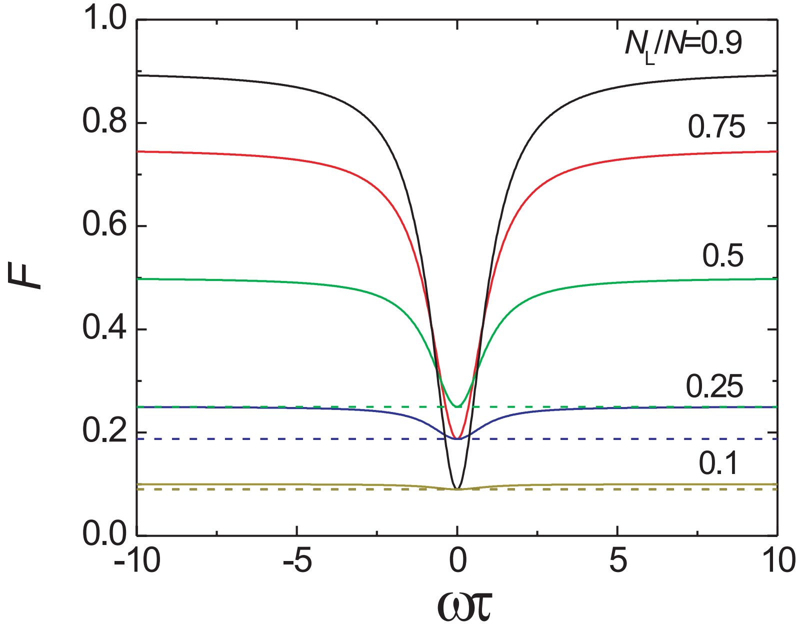
Non-equilibrium noise of an interacting cavity: shot noise and second cumulant of FCS

$$S^{cav}_{\delta I\delta I}(\omega) = \frac{e^2}{h}\frac{N_L^2 N_R^2}{N^3} eV \frac{1+\omega^2\tau^2 N/N_R}{1+\omega^2\tau^2},$$

Zero-frequency noise:

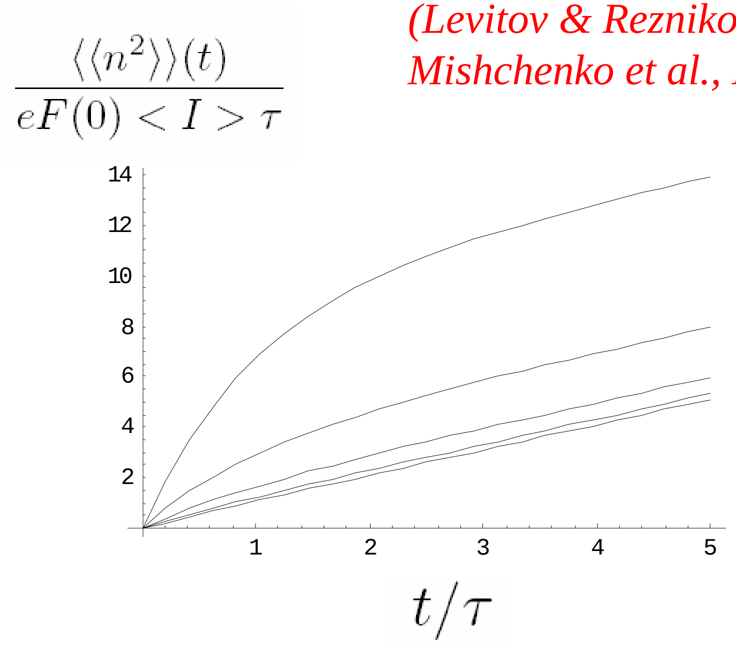
$$S^{cav}_{\delta I\delta I}(0) = \frac{N_L N_R}{N^2} e\langle I\rangle, \qquad \langle I\rangle = (e^2/h)(N_L N_R/N)V.$$

Frequency-dependent Fano factor:



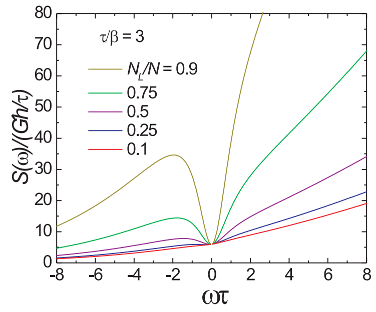
Time-dependent second cumulant:

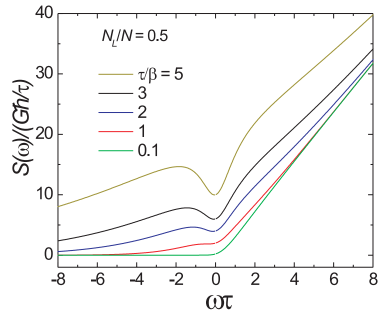
(Levitov & Reznikov, PRB 2004
Mishchenko et al., EPJB 2001)



Conclusions

- Interest in finite-frequency noise of mesoscopic conductors
- Absorption and emission phenomena
- Noise phenomena governed by charge relaxation time $R_Q C_\mu$ which contains both dwell time and RC time





Outlook

- Relevant frequencies in experimentally accessible range
- Method directly applicable to find frequency-dependent third cumulant
- Particular interest due to dependence on two frequencies: τ^{-1}, τ_D^{-1}