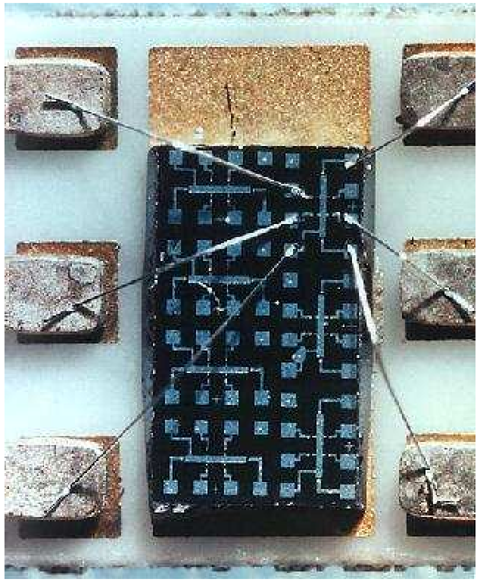
Non-equilibrium entanglement and noise in coupled qubits

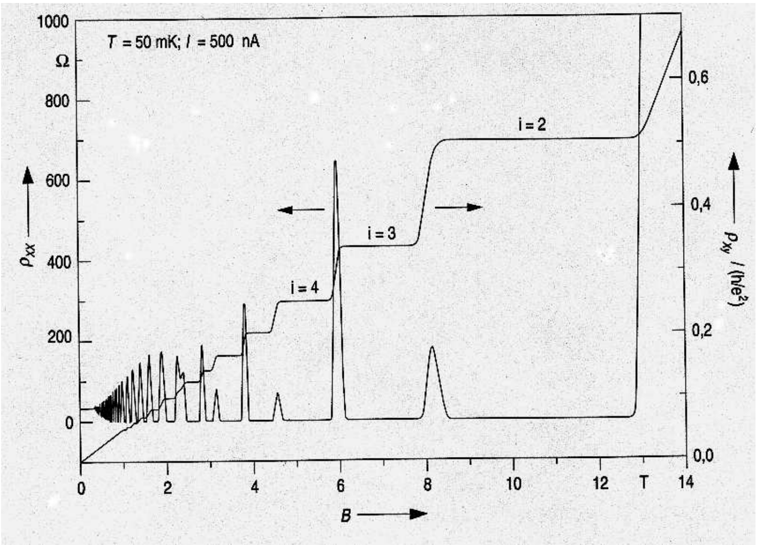
T. Brandes

- Quantum Mechanical Transport
- Quantum Noise
- Transport, noise, entanglement

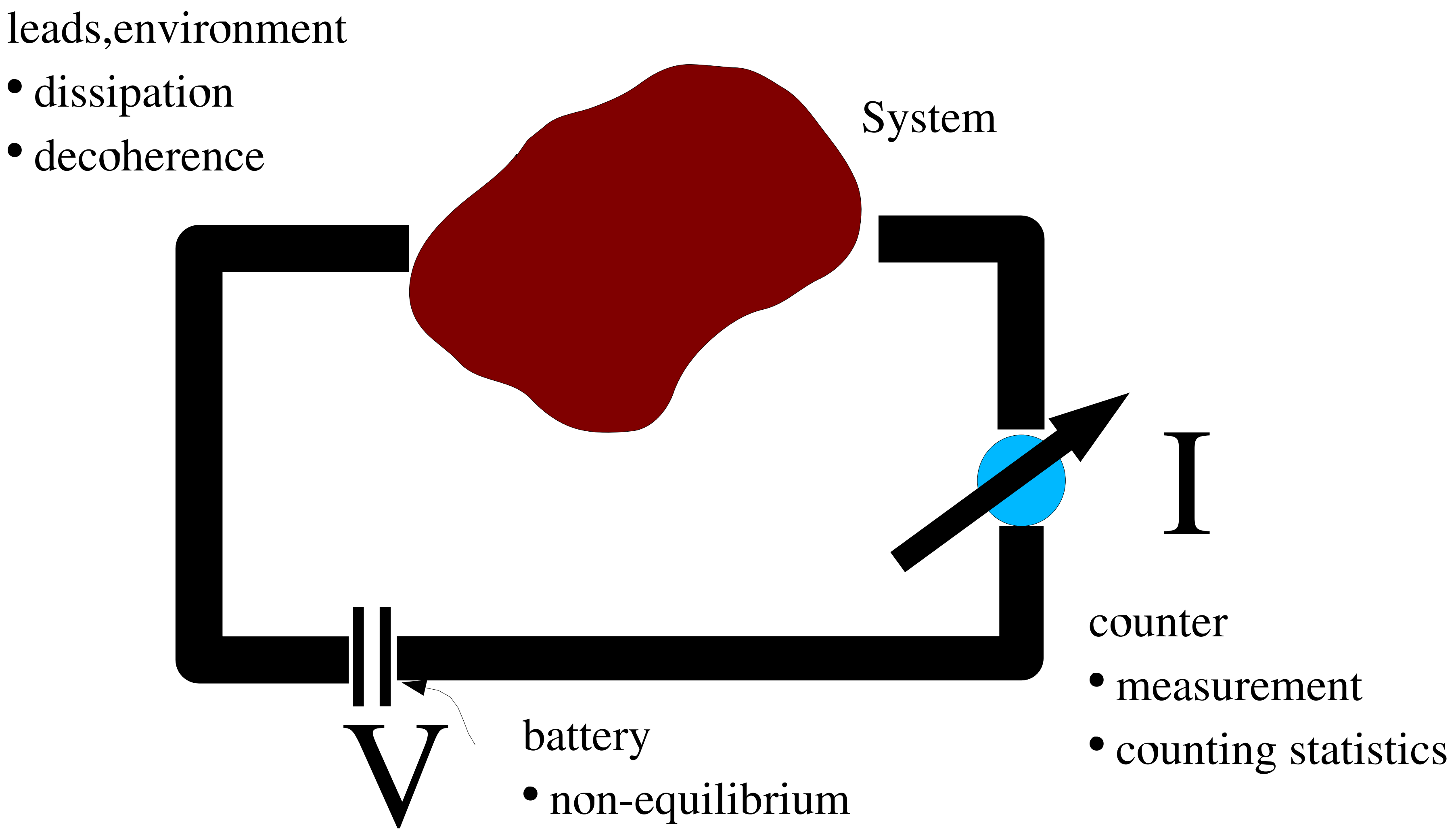
Co-workers: R. Aguado (Madrid), N. Lambert (Tokyo),

Electronic Transport





$$R_K = \frac{h}{e^2} = 25812,807\Omega.$$



TRANSPORT = system + non-equilibrium + external world

Electronic Transport

Things are difficult. Start from something simple?

SMALL STUFF:

- Dimension 2 (2DEG), 1 (wires), 0 (few-level quantum systems).
- Single Electron Transistor.
- charge/flux/spin qubits (controllable two-level systems)

- tunneling $\rightsquigarrow$ quantum superpositions
- interactions $\rightsquigarrow$ entanglement
- environment $\rightsquigarrow$ decoherence

$\rightsquigarrow$ arena of *Mesoscopic Physics*.

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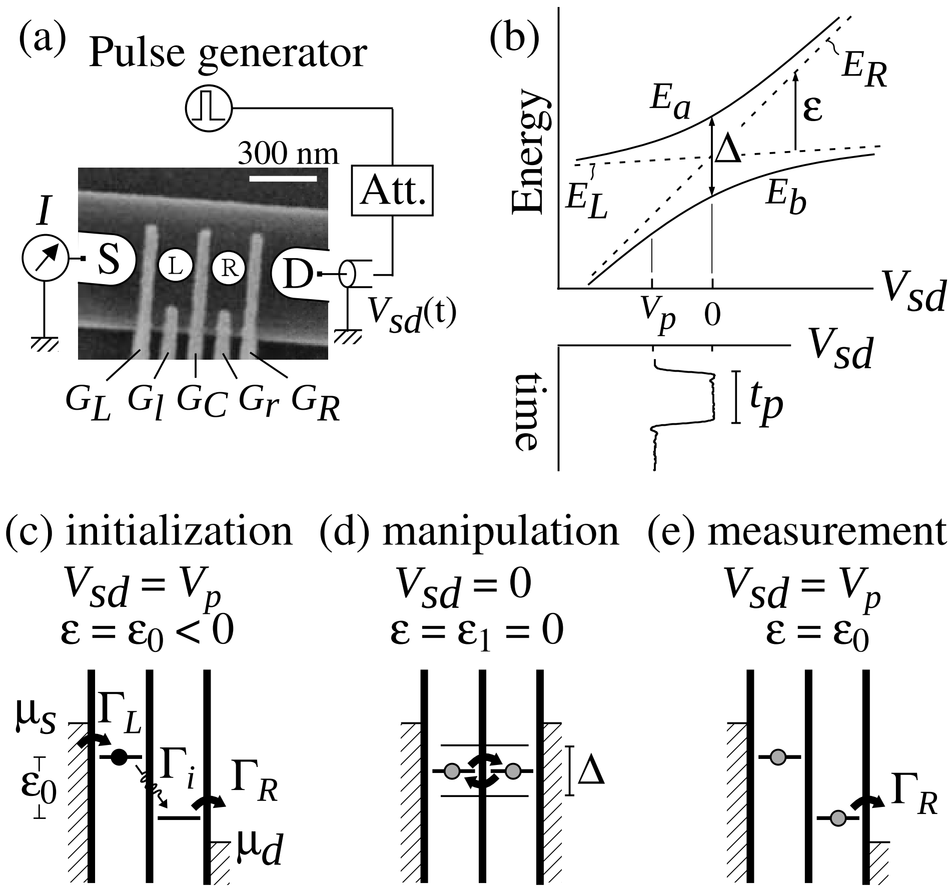
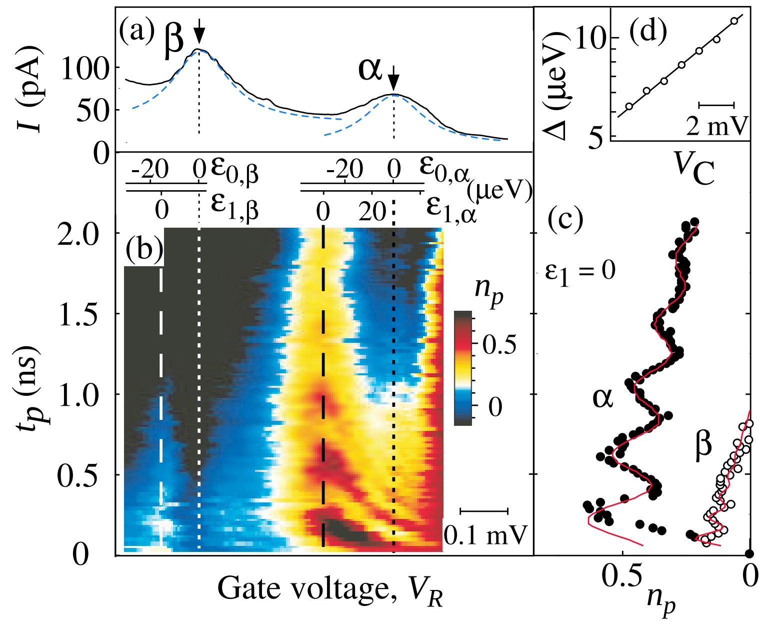
Coherent Manipulation of Electronic States in a Double Quantum Dot

T. Hayashi,[1] T. Fujisawa,[1] H. D. Cheong,[2] Y. H. Jeong,[3] and Y. Hirayama[1,4]
[1]*NTT Basic Research Laboratories, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, 243-0198, Japan*



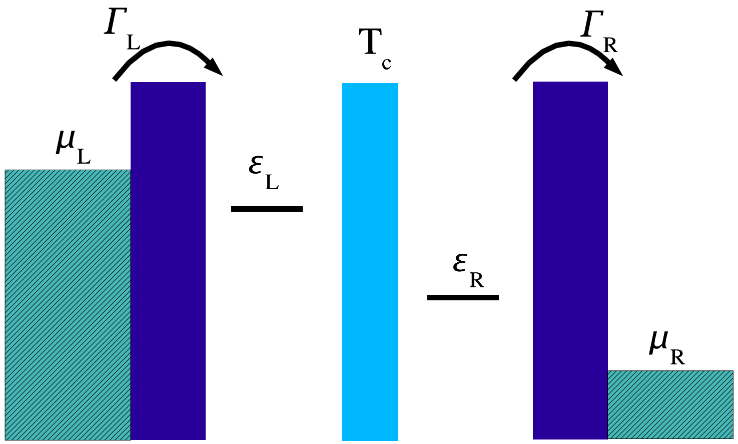


cf. TB, T. Vorrath, PRB **66**, 075341 (2003); U. Hartmann, F. K. Wilhelm, PRB **67**, 161307 (2003), M. Thorwart, J. Eckel, E.R. Mucciolo cond-mat/0505621 (2005).

These are also useful in order to understand transport ‘from scratch’.

Three-State Transport Model

- Transport model for the smallest quantum system: $SU(2)$ plus one empty state.
- $|L\rangle = |N_L + 1, N_R\rangle$ 'left', $|R\rangle = |N_L, N_R + 1\rangle$ 'right', $|0\rangle = |N_L, N_R\rangle$ 'empty'.



- internal bias $\varepsilon = \varepsilon_L - \varepsilon_R$, tunnel coupling T_c.

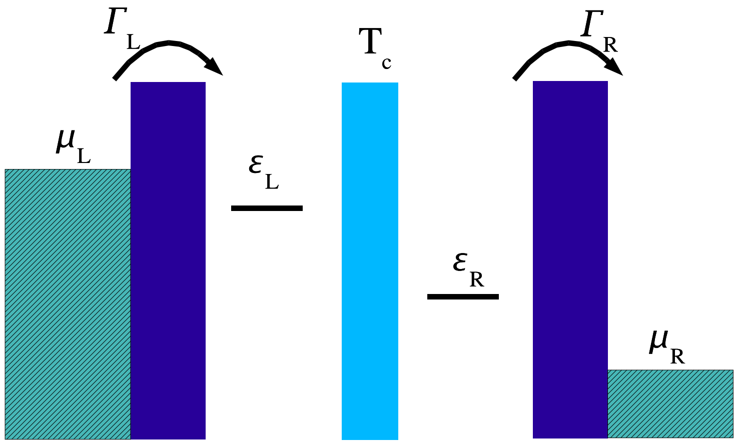
$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_{res} + \mathcal{H}_T, \quad \mathcal{H}_S = \frac{\varepsilon}{2}\hat{\sigma}_z + T_c\hat{\sigma}_x$$

$$\mathcal{H}_T = \sum_{k_i} (V_k^i c_{k_i}^\dagger |0\rangle\langle i| + H.c.), i = L, R.$$

One goal: calculate density operator ρ for $t \to \infty$. ρ has 4 (not 3) real parameters,

$$\rho = \begin{pmatrix} \rho_{00} & 0 & 0 \\ 0 & \rho_{LL} & \rho_{LR} \\ 0 & \rho_{RL} & \rho_{RR} \end{pmatrix}, \quad \rho_{00} = 1 - \rho_{LL} - \rho_{RR}.$$

- Double quantum dots, strong Coulomb blockade $U \to \infty$.

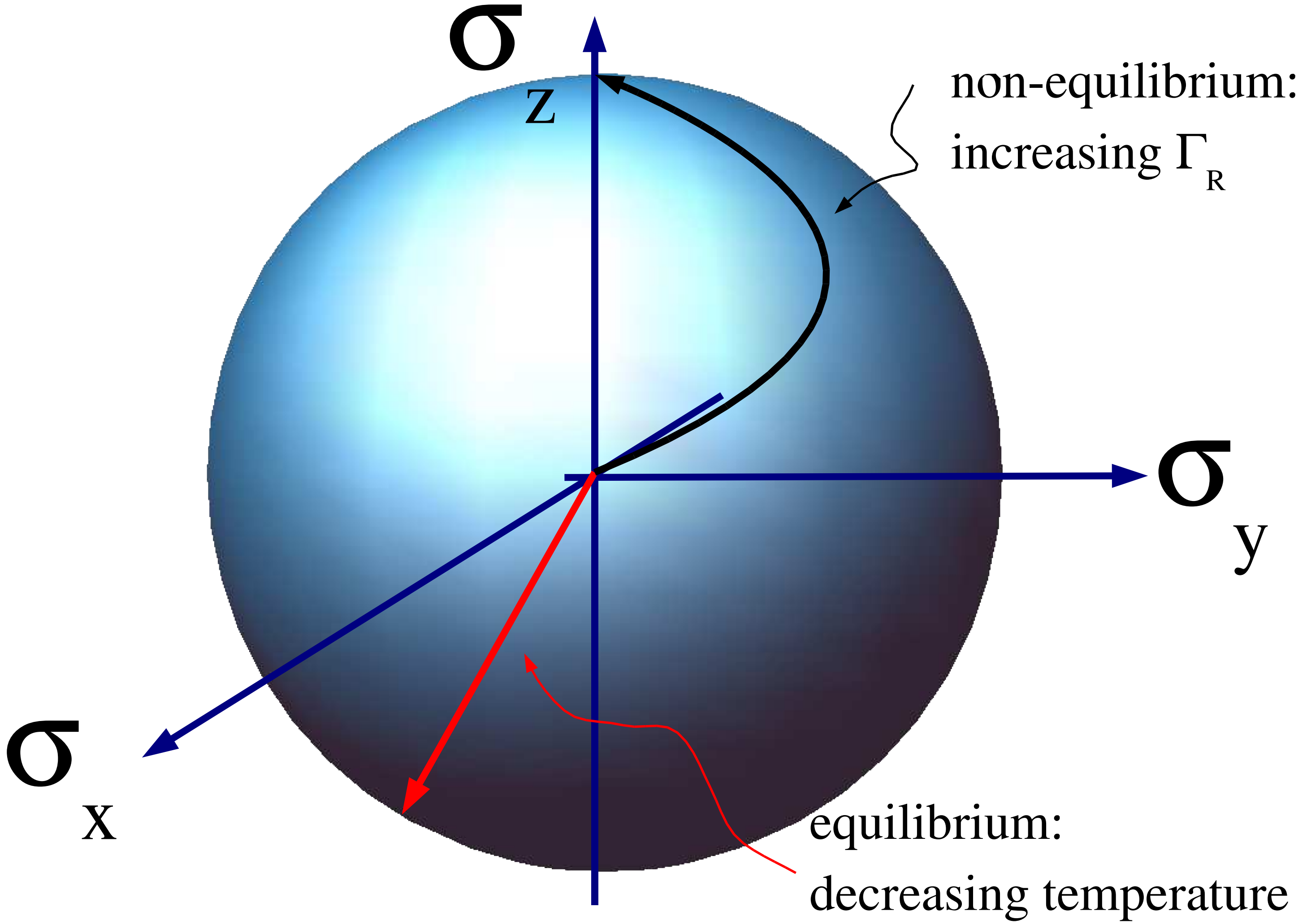


- Complicated problem for any bias $|\mu_L - \mu_R| < \infty$.
- Only $\mu_L - \mu_R \to \infty$ relatively easy. Then, exact (?) solution in Markovian limit (flat tunneling DOS, no memory).
- External tunnel rates; $\Gamma_i(\varepsilon) = 2\pi \sum_{k_i} |V_k^i|^2 \delta(\varepsilon - \varepsilon_{k_i})$.

- Solve Liouville-von-Neumann eq. $\rightsquigarrow$ stationary current (Stoof-Nazarov 1996, Gurvitz 1996)

$$\langle \hat{I} \rangle_{t\to\infty}^{\mathrm{SN}} = -e \frac{T_c^2 \Gamma_R}{\Gamma_R^2/4 + \varepsilon^2 + T_c^2(2 + \Gamma_R/\Gamma_L)},$$

- Just Breit-Wigner. Nothing on spectrum, $\pm\frac{1}{2}\sqrt{\varepsilon^2 + 4T_c^2}$.
- Pure state for $\Gamma_R \to \infty$ (no current): quantum Zeno effect (continuous measurement version): right lead as detector with ∞ bandwidth.

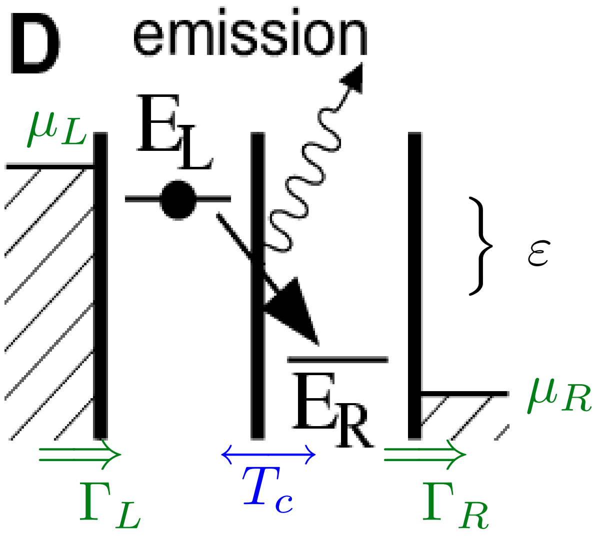


Single DQD Bloch-sphere ($\Gamma_L \rightarrow \infty$) in L-R basis.

Double Quantum Dots

3 states $|L\rangle$, $|R\rangle$, $|0\rangle$

$\hat{\sigma}_z \equiv |L\rangle\langle L| - |R\rangle\langle R|, \quad \hat{\sigma}_x \equiv |L\rangle\langle R| + |R\rangle\langle L|.$



$$\mathcal{H} = \mathcal{H}_{SB} + \mathcal{H}_{res} + \mathcal{H}_T$$

$$\mathcal{H}_T = \sum_{k_\alpha} (V_k^\alpha c_{k_\alpha}^\dagger |0\rangle\langle\alpha| + H.c.), \alpha = L, R$$

$$\mathcal{H}_{SB} = \left[\frac{\varepsilon}{2} + \sum_{\mathbf{Q}} \frac{g_Q}{2}\left(a_{-\mathbf{Q}} + a_{\mathbf{Q}}^\dagger\right)\right]\hat{\sigma}_z + T_c\hat{\sigma}_x + \mathcal{H}_B.$$

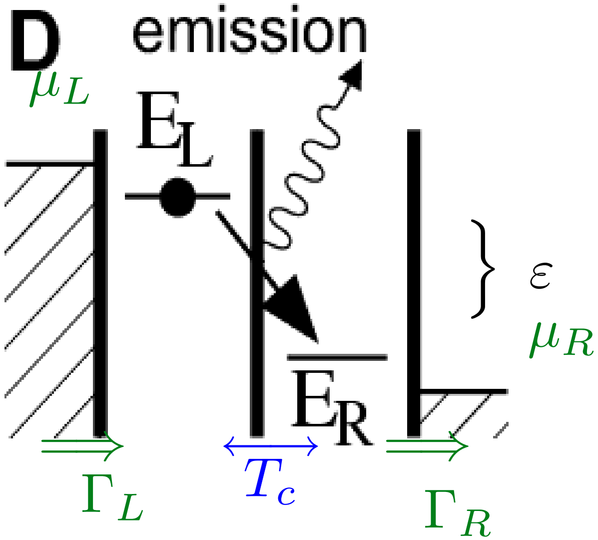
<u>*Loc-Deloc Transition at $\alpha = 1$*</u>, *Leggett et al 87*

- ‘Internal’ Parameter ε, T_c;

$$J(\omega) \equiv \sum_{\mathbf{Q}} |g_Q|^2 \delta(\omega - \omega_Q) = \begin{cases} 2\alpha\omega_{\mathrm{ph}}^{1-s}\omega^s e^{-\frac{\omega}{\omega_c}} \\ \text{microscopic model: } \textit{Phonons...} \end{cases}$$

- ‘External’ parameters $\mu_L, \mu_R, \Gamma_\alpha(\varepsilon) = 2\pi \sum_{k_\alpha} |V_k^\alpha|^2 \delta(\varepsilon - \varepsilon_{k_\alpha}), \alpha = L/R$.

Formulation



- EOM for reduced density operator

$$\langle \mathbf{A}(t)\rangle = \langle \mathbf{A}(0)\rangle + \int_0^t dt' \left\{ M(t,t') \langle \mathbf{A}(t')\rangle + \Gamma_L \mathbf{e}_1 \right\}.$$

- $\mu_L - \mu_R \to \infty$ (Gurvitz, Prager 1996, Stoof, Nazarov 1996, Gurvitz 1998.)

- 'Memory Kernel'

$$z\hat{M}(z) = \begin{bmatrix} -\hat{G} & \hat{T}_c \\ \hat{D}_z & \hat{\Sigma}_z \end{bmatrix}, \quad \hat{G} \equiv \begin{pmatrix} \Gamma_L & \Gamma_L \\ 0 & \Gamma_R \end{pmatrix}, \quad \hat{T}_c \equiv iT_c(\sigma_x - 1)$$

- Blocks $\hat{D}_z$, $\hat{\Sigma}_z$: Dephasing, Relaxation $\left\{ \begin{array}{l} \text{PER} \\ \text{POL} \end{array} \right.$

Driven nonequilibrium: transport Floquet theory

$\varepsilon + \Delta \cos \Omega t$

$$\mathcal{H}(t) = \left[\frac{\varepsilon(t)}{2} + \sum_{\mathbf{Q}} \frac{g_Q}{2}\left(a_{-\mathbf{Q}} + a^\dagger_{\mathbf{Q}}\right)\right]\hat{\sigma}_z + T_c\hat{\sigma}_x + \mathcal{H}_B + \mathcal{H}_{res} + \mathcal{H}_T$$

- Limiting cases
  - $\mathcal{H}_B = 0$, perturbation theory in T_c: P. K. Tien, J. R. Gordon 1963
  - $\mathcal{H}_{res} + \mathcal{H}_T \equiv 0$ M. Grifoni, P. Hänggi 1998,...
- Method: Fourier components of $\rho(t) \longrightarrow$

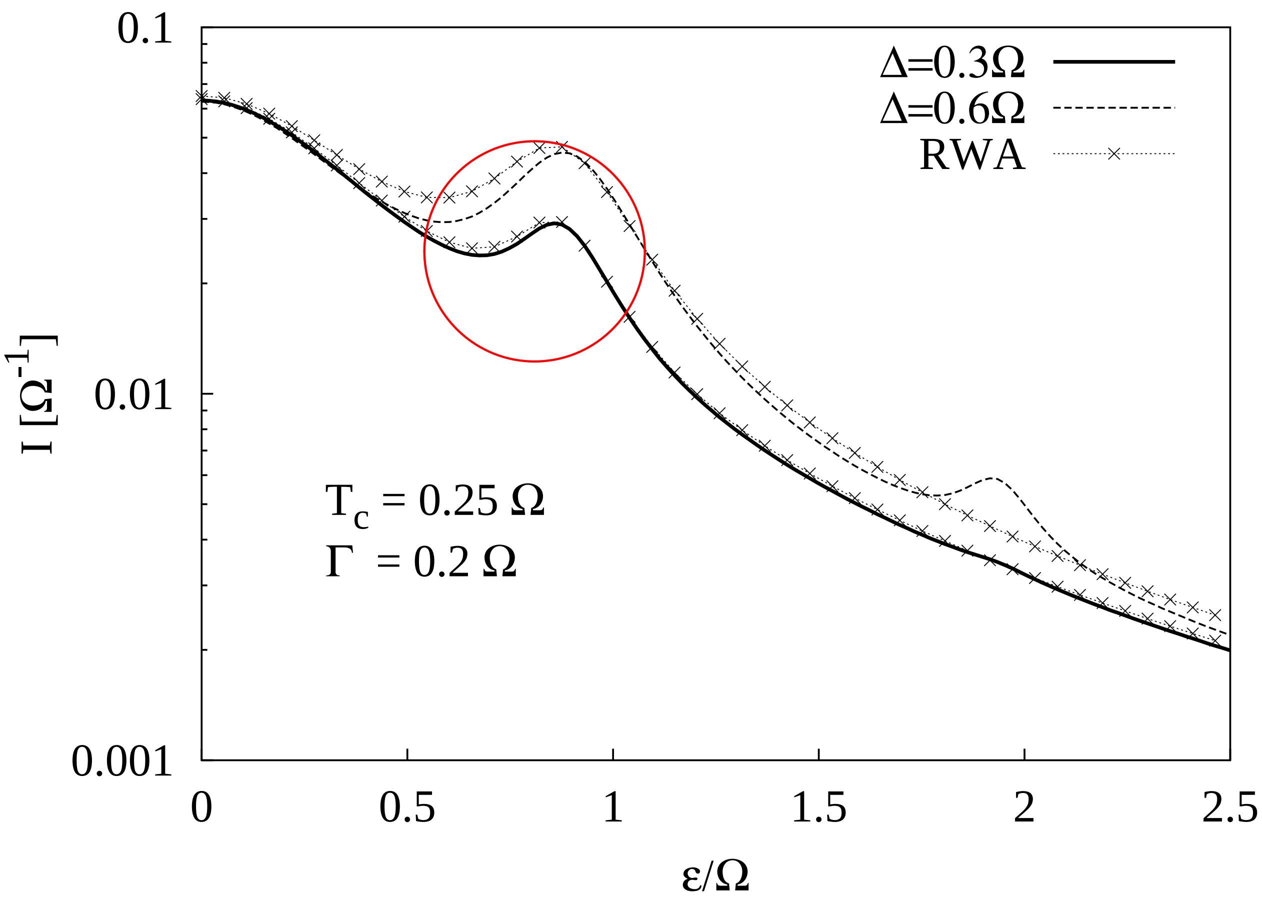
$$K_m(-im'\Omega) = i^{-m}T_c^2 \sum_n \left[J_n\left(\frac{\Delta}{\Omega}\right) J_{n-m}\left(\frac{\Delta}{\Omega}\right)\hat{D}_{\varepsilon+(m'-n)\Omega} + J_n\left(\frac{\Delta}{\Omega}\right) J_{n+m}\left(\frac{\Delta}{\Omega}\right)\hat{D}^*_{\varepsilon-(m'+n)\Omega}\right]$$

$$G_m(-im'\Omega) = i^{-m}T_c^2 \sum_n \left[J_n\left(\frac{\Delta}{\Omega}\right) J_{n-m}\left(\frac{\Delta}{\Omega}\right)\hat{E}_{\varepsilon+(m'-n)\Omega} + J_n\left(\frac{\Delta}{\Omega}\right) J_{n+m}\left(\frac{\Delta}{\Omega}\right)\hat{E}^*_{\varepsilon-(m'+n)\Omega}\right].$$

$\uparrow$ *dynamical localisation*

$$\hat{D}_\varepsilon(z) \equiv \frac{1}{\hat{C}_\varepsilon(z)^{-1} + \Gamma_R/2}$$

$\uparrow$ *dissipation*



RWA result

T.H. Stoof, Yu. V. Nazarov, PRB **53**, 1050 (1996).

Bloch-Siegert shift for large ac amplitudes Δ.

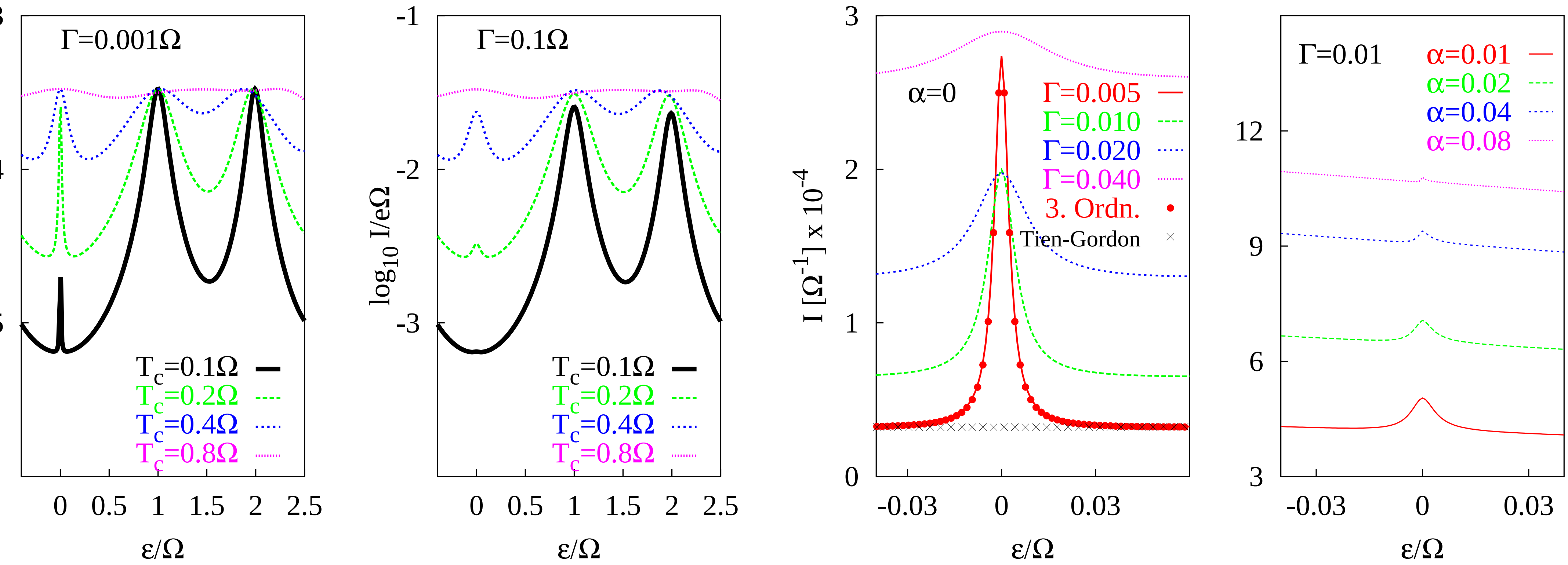
TB, R. Aguado, G.Platero, PRB **69**, 205326 (2004).

Resonance at $\varepsilon = \sqrt{\Omega^2 - 4T_c^2}$: quantum coherence.

EXP: R. H. Blick, D. W. van der Weide, R. J. Haug, K. Eberl; PRL **81**, 689 (1998); T. H. Oosterkamp, T. Fujisawa, W. G. van der Wiel, K. Ishibashi, R. V. Hijman, S. Tarucha, L. P. Kouwenhoven, (1998).

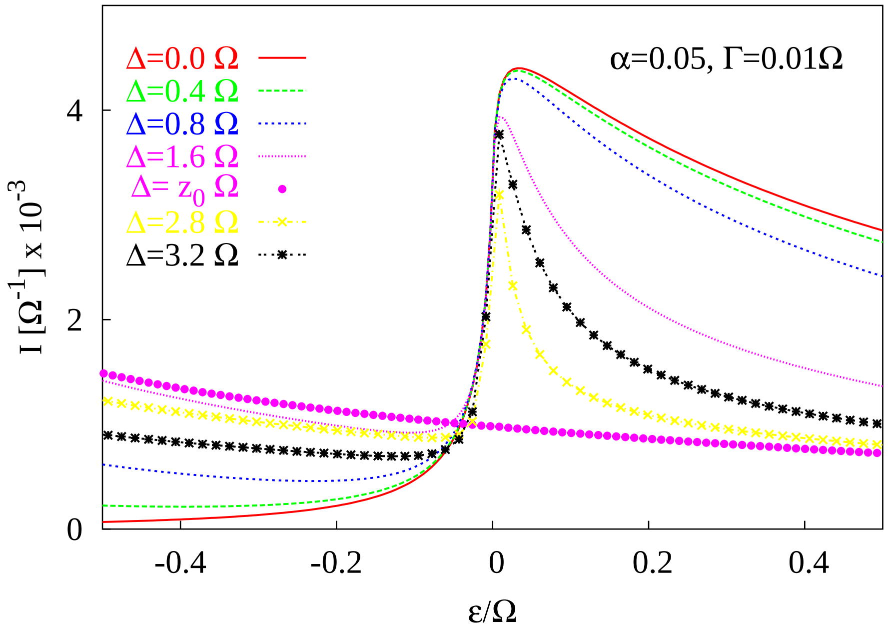
Dynamical localisation: $T_c \to T_c J_0(\Delta/\Omega)$ (Tien-Gordon)...

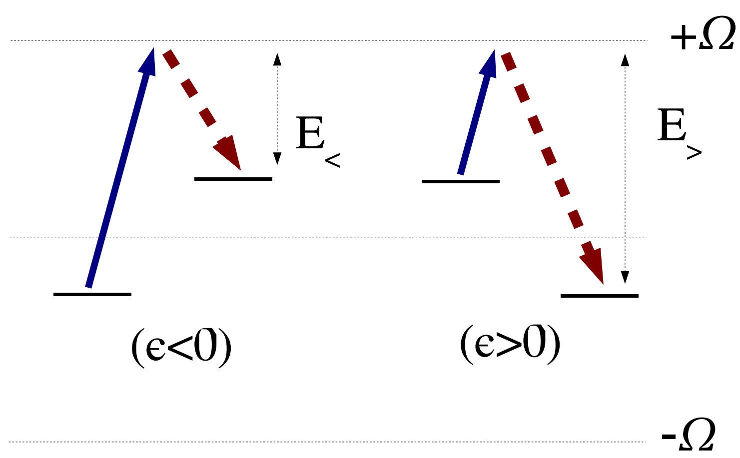
- Tien-Gordon *wrong for large* T_c: 6. order T_c Barata, Wreszinski, PRL **84**, 2112 (2000).



'Coherent lifting' of dynamical localisation ... vanishes for *stronger dissipation*

Dissipation and ac fields





- Dynamical localisation destroys asymmetry between spontaneous emission and absorption.
- $P(E) \propto E^{2\alpha-1} e^{-E/\omega_c} \rightsquigarrow P(E_<) > P(E_>)$.

- Quantum Mechanical Transport
- Quantum Noise
- Entanglement

Here: noise in presence of Coulomb blockade, quantum coherence and dissipation.

Condensed-matter physics

The noise is the signal

Rolf Landauer

Noise is not only a hindrance to signal detection. Advances in measurement techniques mean that it can now be regulate and reduce the fluctuations.

The new investigations[1–4] were prompted by precise measurements of noise at quan-

Nature 392, 658 - 659 (16 April 1998)

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Quantum Noise: p_n-technology...

- Jump-resolved Master equations (Cook 1982, resonance fluorescence).

$$
\begin{aligned}
\dot\rho(t) &= (\mathcal{L}_0 + s\mathcal{L}_1)\,\rho(t), \quad s=1 \\
\dot\rho^{(n)}(t) &= \mathcal{L}_0\rho^{(n)}(t) + s\mathcal{L}_1\rho^{(n-1)}(t) \\
G(s,t) &\equiv \sum_n s^n \rho^{(n)}(t) \quad \text{generating function, } G(s=1,t)=\rho(t).
\end{aligned}
$$

- Yields $p_n(t, t+T)$: probability for n electrons tunneling out to the right in time interval $[t, t+T)$.

- $p_n(t, t+T)$ also related to noise correlation functions.

- Long-time behaviour from lowest eigenvalue of $\mathcal{L}_0 + s\mathcal{L}_1$.

Quantum Noise: $S(\omega)$-technology...

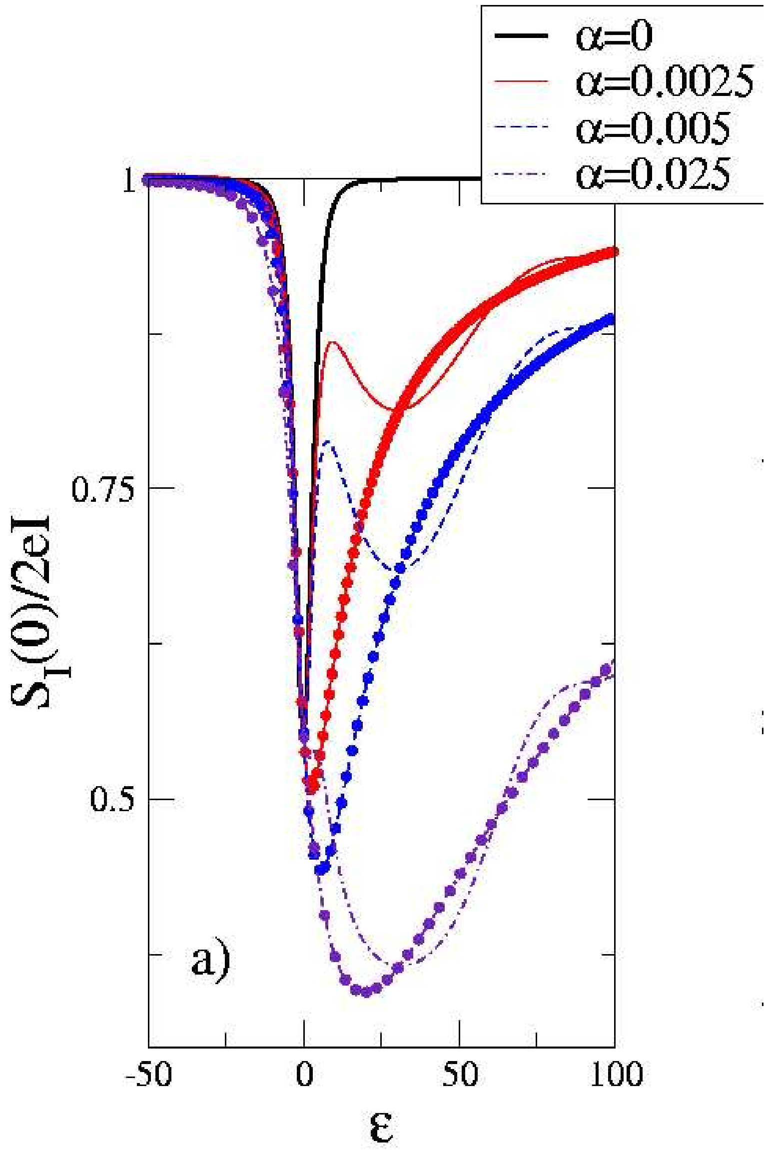
- Noise-Spectrum with current conservation $I_L - I_R = \dot{Q}$, $I = aI_L + bI_R$,

$$\mathcal{S}_{\mathcal{I}\mathcal{I}}(\omega) \equiv \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle \{\Delta\hat{I}(\tau), \Delta\hat{I}(0)\}\rangle = \underline{aS_{I_L I_L}(\omega) + bS_{I_R I_R}(\omega)} - ab\omega^2 \underline{S_Q(\omega)}.$$

- Version for many leads α, β (Cottet, Belzig, Bruder 2004, cf. Flindt, Novotny, Jauho 2004)

$$\mathcal{S}_{\mathcal{I}_\alpha \mathcal{I}_\beta}(\omega) = -2e^2 \mathrm{Tr}\left(\mathcal{L}_\alpha [i\omega + \mathcal{L}]^{-1} \mathcal{L}_\beta \rho_{\mathrm{stat}} + (\omega \leftrightarrow -\omega) + (\alpha \leftrightarrow \beta)\right) + 2e^2 \delta_{\alpha\beta} \mathrm{Tr} \mathcal{L}_\alpha \rho_{\mathrm{stat}}.$$

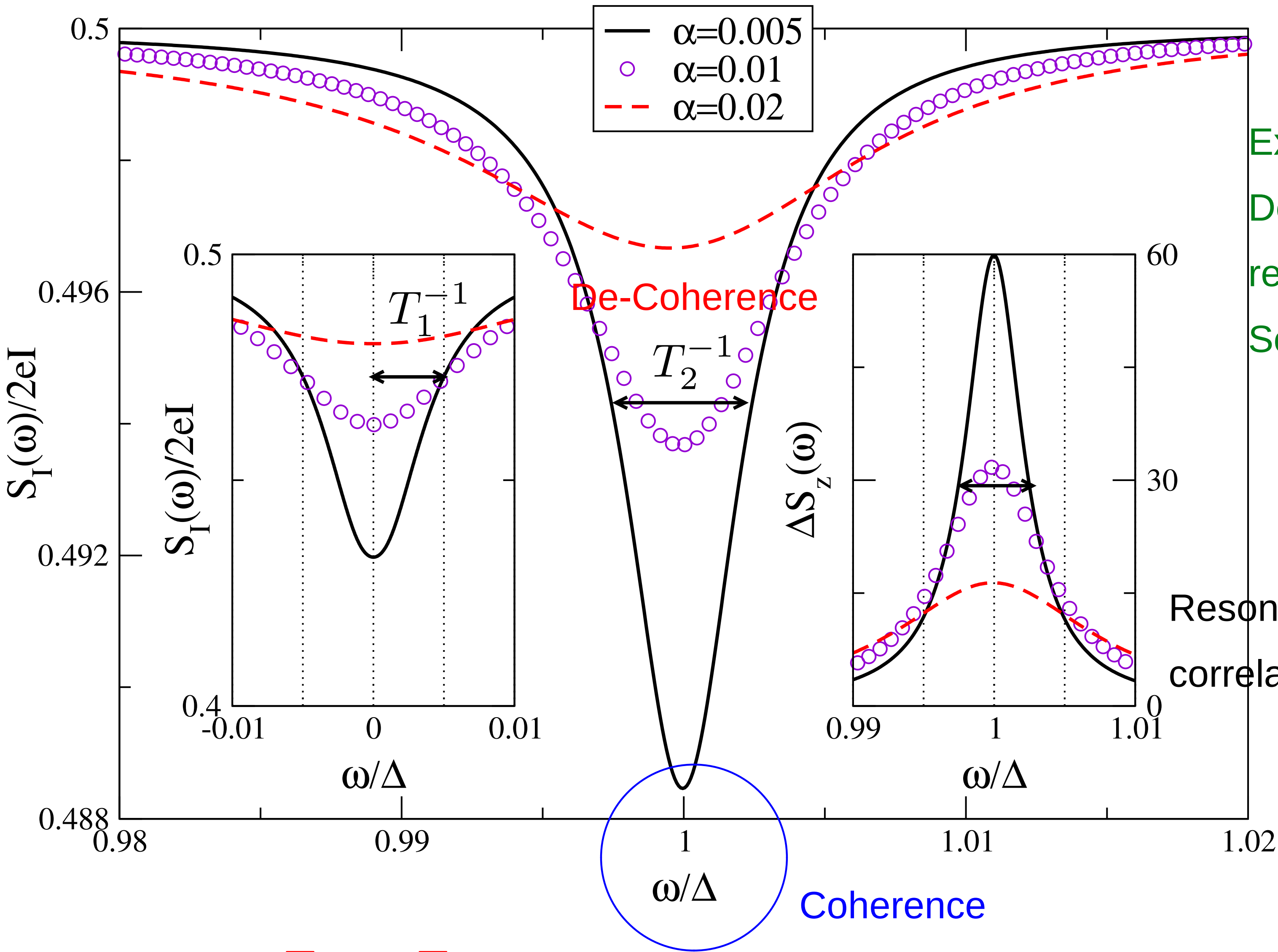
Fano Factor



- Interaction $(U \to \infty) \rightsquigarrow$ no Khlus-Lesovik form '$T(1-T)$'.
- ($\alpha = 0$) coherence suppresses noise: minimum at $\varepsilon = 0$.
- ($\alpha = 0$) large $|\varepsilon|$ 'localises' charge.
- ($\alpha \neq 0$) for $\varepsilon > 0$: dissipation suppresses noise.
- Maximal for $\gamma_p = \Gamma_R$.

MANCHESTER 1824 The University of Manchester frequency dependent noise spectrum

R. Aguado, TB, Phys. Rev. Lett. **92**, 206601 (2004), Eur. Phys. J. B **40**, 357 (2004).



Exp. Cooper-Pair Box: R. Deblock, E. Onac, L. Gurevich, L. P. Kouwenhoven, Science **301**, 203 (2003)

Resonance as Pseudo-Spin-correlation function

Coherence

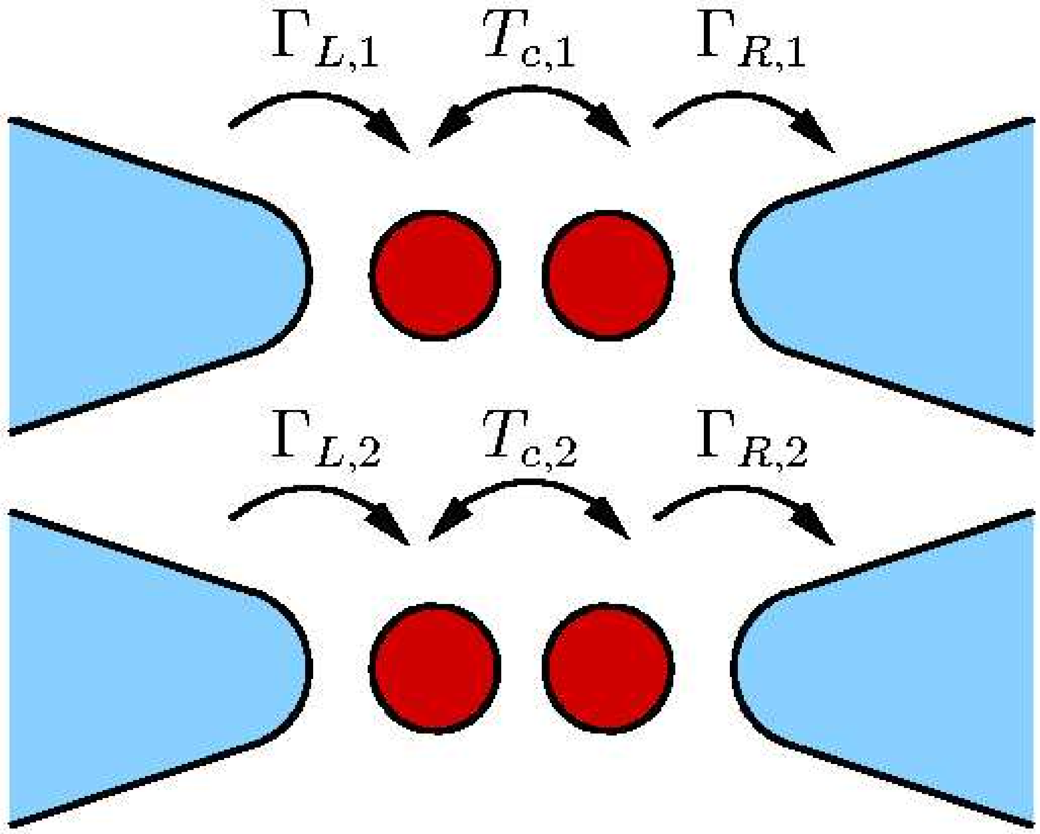
contains T_1 und T_2 (PER) !

- Quantum Mechanical Transport
- Quantum Noise
- Entanglement

- Entanglement in non-equilibrium.

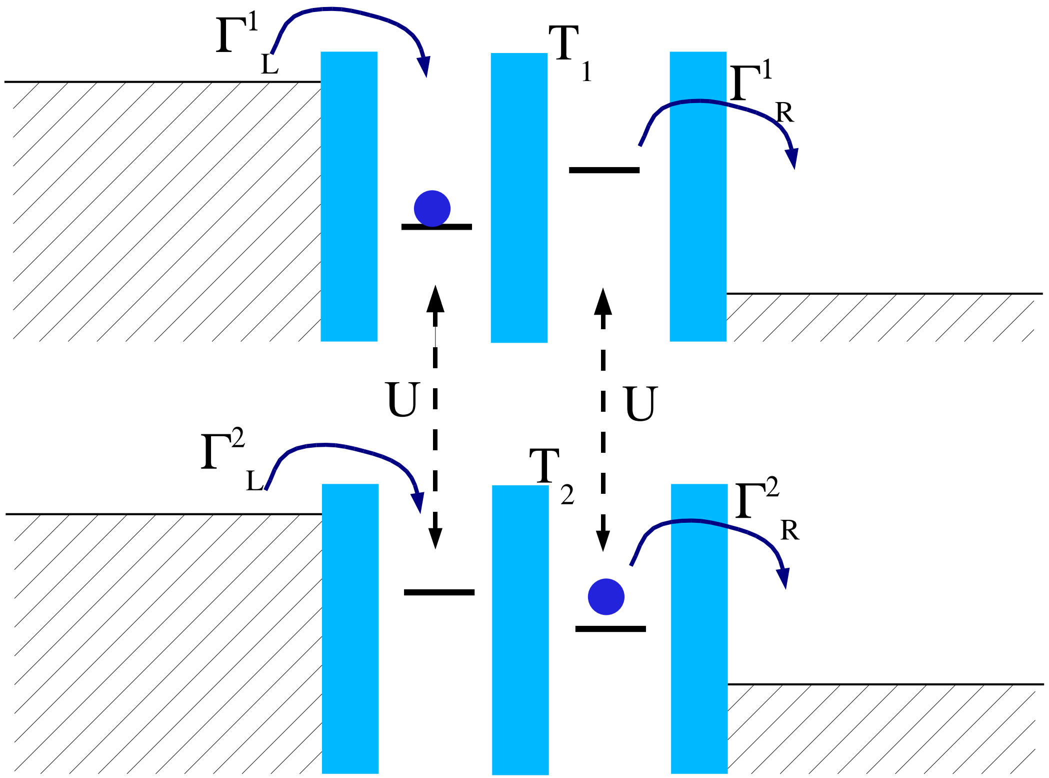
Current balance: Ampere, Biot-Savart etc.

Transport through coupled 2-Qubits



- *Phonon* coupling: effective interaction, Dicke effect T. Vorrath, TB, PRB 2003.

Transport through coupled 2-Qubits



- *Coulomb* coupling: two-site Hubbard with (pseudo) spin N. Lambert, R. Aguado, TB 2005.

Two Double Quantum Dots: Coulomb-Coupling U

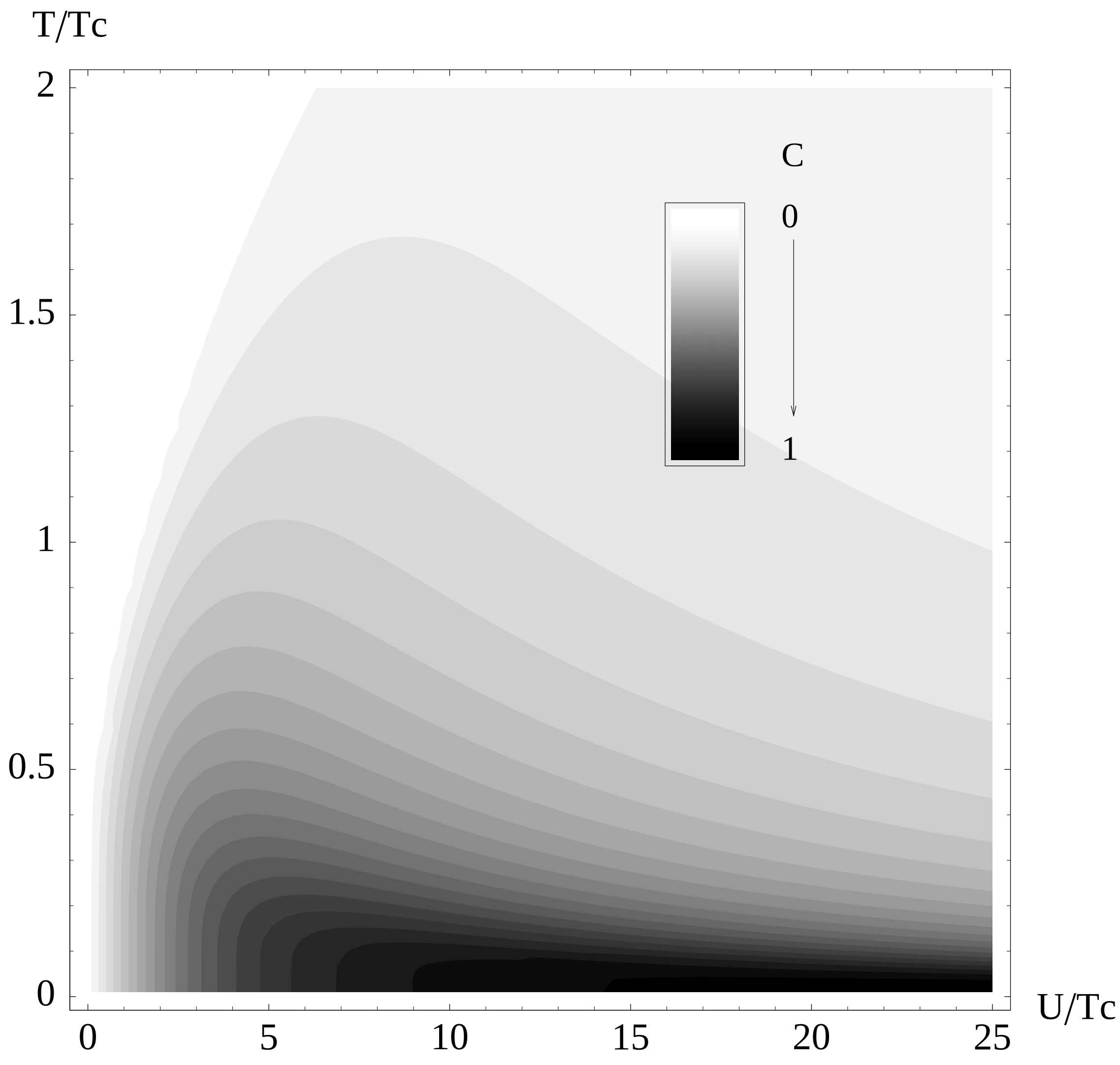
- Total Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_T + \mathcal{H}_{\rm res}$.
- Double qubit

$$\begin{aligned}\mathcal{H}_0 &= \sum_{i=1,2}\left(\varepsilon_i(\hat{n}_L^{(i)} - \hat{n}_R^{(i)}) + T_i(\hat{n}_{LR}^{(i)} + \hat{n}_{RL}^{(i)})\right) \\ &+ \frac{U}{2}\left(\hat{n}_L^{(1)}\hat{n}_L^{(2)} + \hat{n}_R^{(1)}\hat{n}_R^{(2)}\right).\end{aligned}$$

- Electron reservoir Hamiltonians $\mathcal{H}_{\rm res}$.
- Tunnel Hamiltonian

$$\mathcal{H}_T = \sum_k (V_k^{\alpha i} c^\dagger_{ki\alpha} s^i_\alpha + H.c.), \quad \hat{s}^i_\alpha = |0_i\rangle\langle\alpha_i|, \quad \alpha = L, R, \quad i = 1, 2.$$

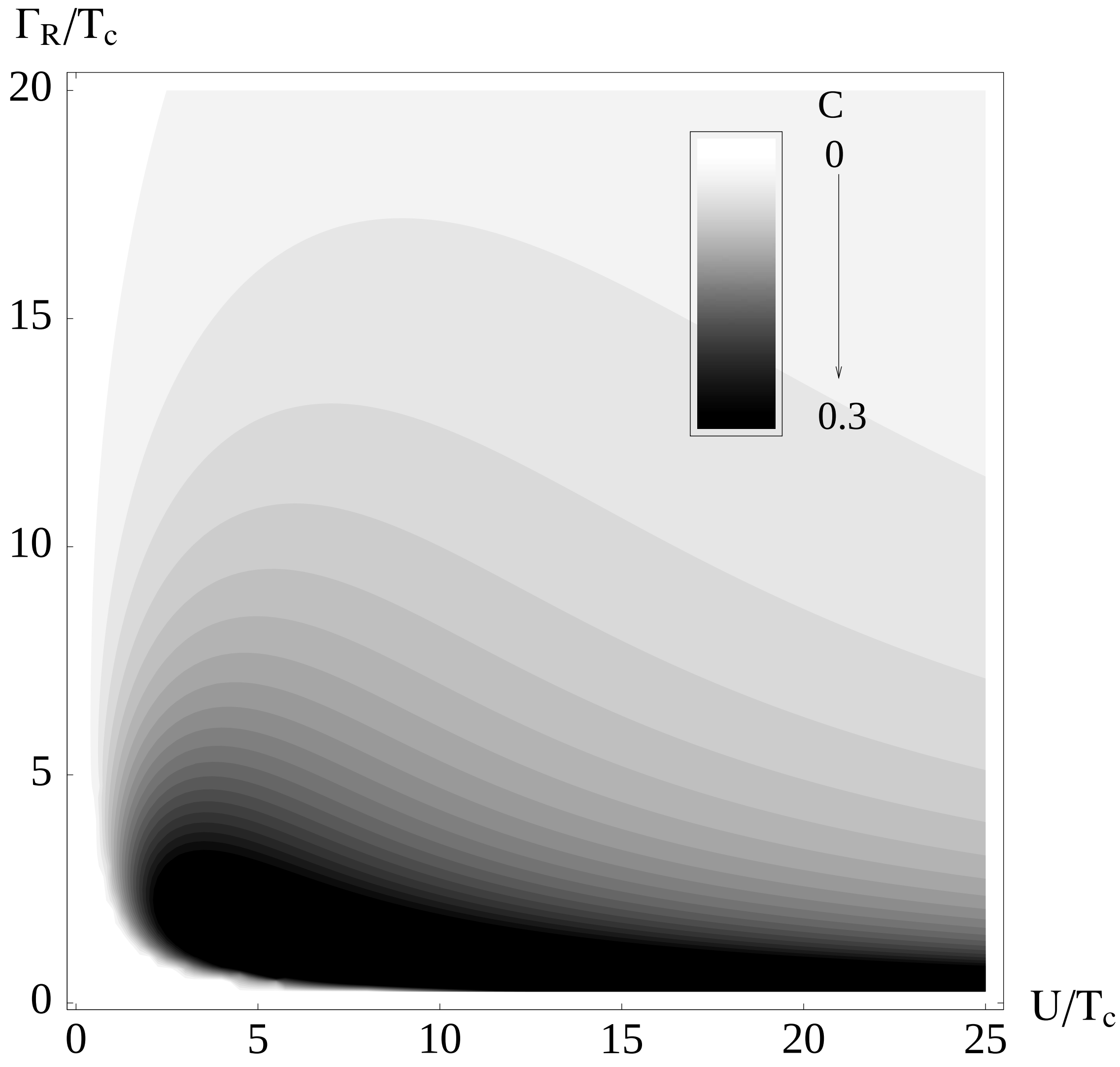
Equilibrium Entanglement ($\mathcal{H}_T = 0$) for $\rho(T) = e^{-\mathcal{H}_0/T}/Z$



- Concurrence from four eigenvalues of $\mathcal{H}_0$.
- $\rho(T)$ too mixed to be entangled below certain U-threshold (cf. Werner state).
- Entanglement maximum at optimal U-value.

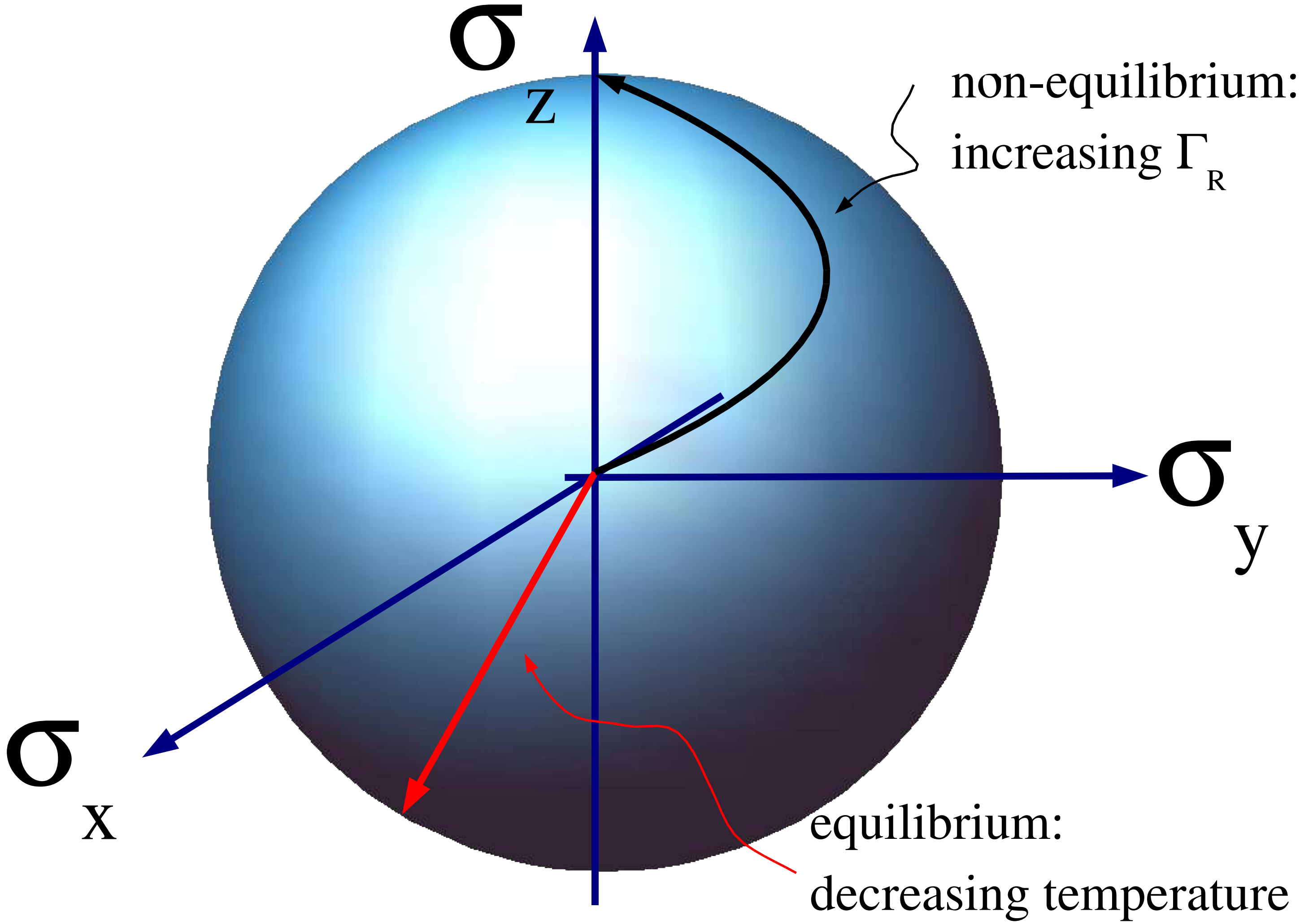
N. Lambert 2005.

Non-Equilibrium Entanglement ($\mathcal{H}_T \neq 0$).



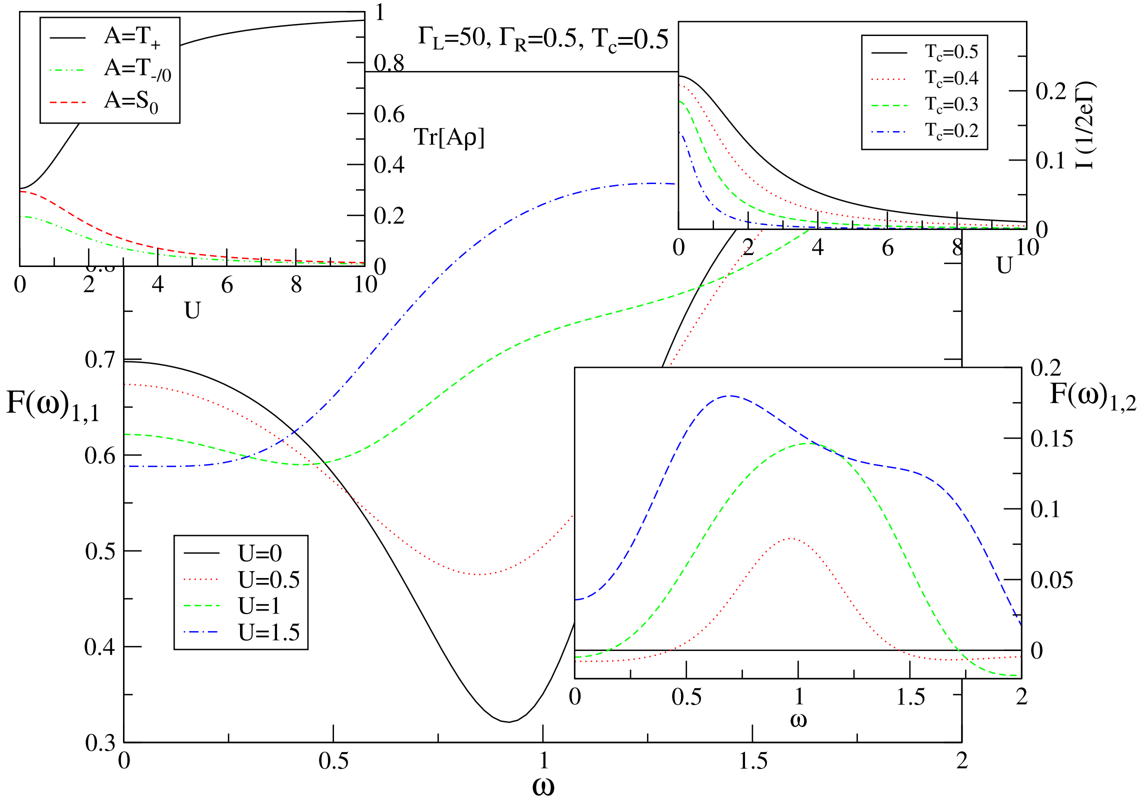
- Concurrence of two-electron projection $\hat{P}\rho_\infty$, good for $\Gamma_L \gg \Gamma_R$.
- Zero entanglement below $U \sim 2T_c^2/\Gamma_R$.
- State strongly mixed for $\Gamma_R \to 0$, continuous from $U = 0$.
- Zeno-trapped for $\Gamma_R \to \infty$: pure left and un-entangled.

N. Lambert 2005.



Single DQD Bloch-sphere ($\Gamma_L \to \infty$) in L-R basis.

Non-equilibrium noise spectrum $S_{ij}(\omega)$



- Diagonal noise reveals double qubit *spectrum* $\leftrightarrow$ stationary current.
- Resonances at Bohr frequencies $1/2(U \pm \sqrt{16T_c^2 + U^2})$.
- Cross-noise can become negative.

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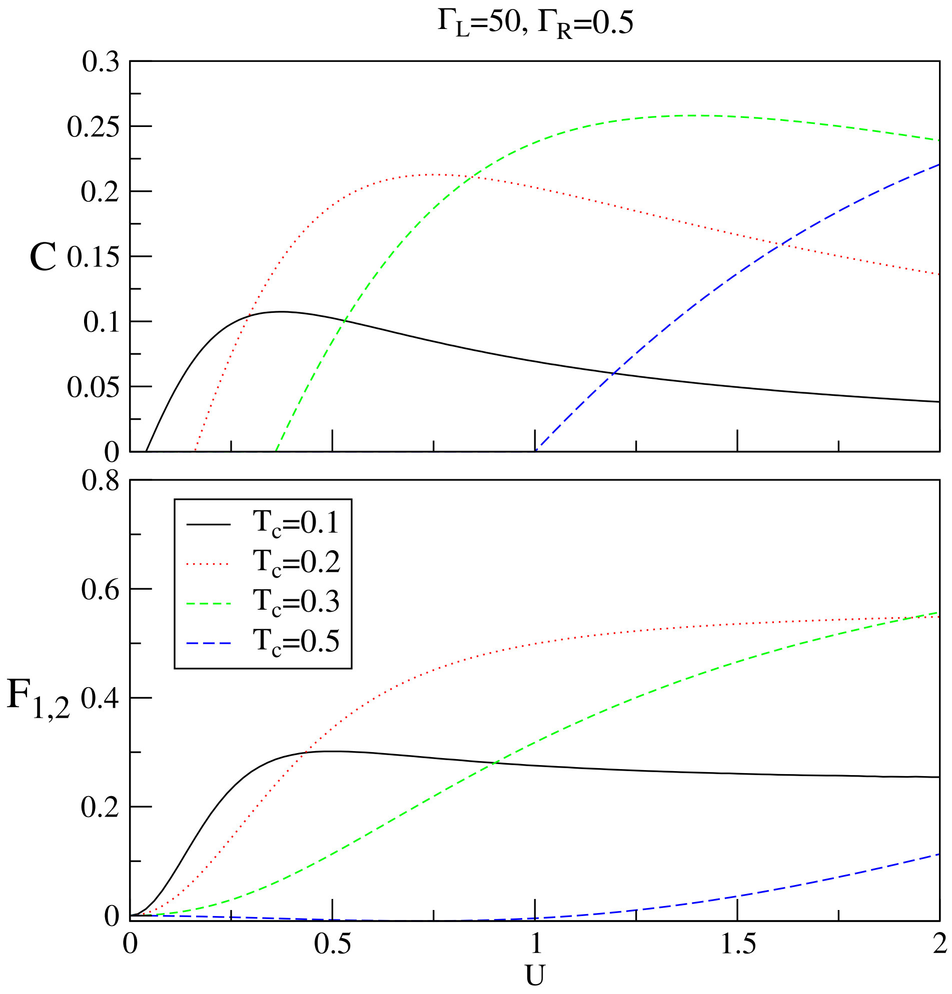
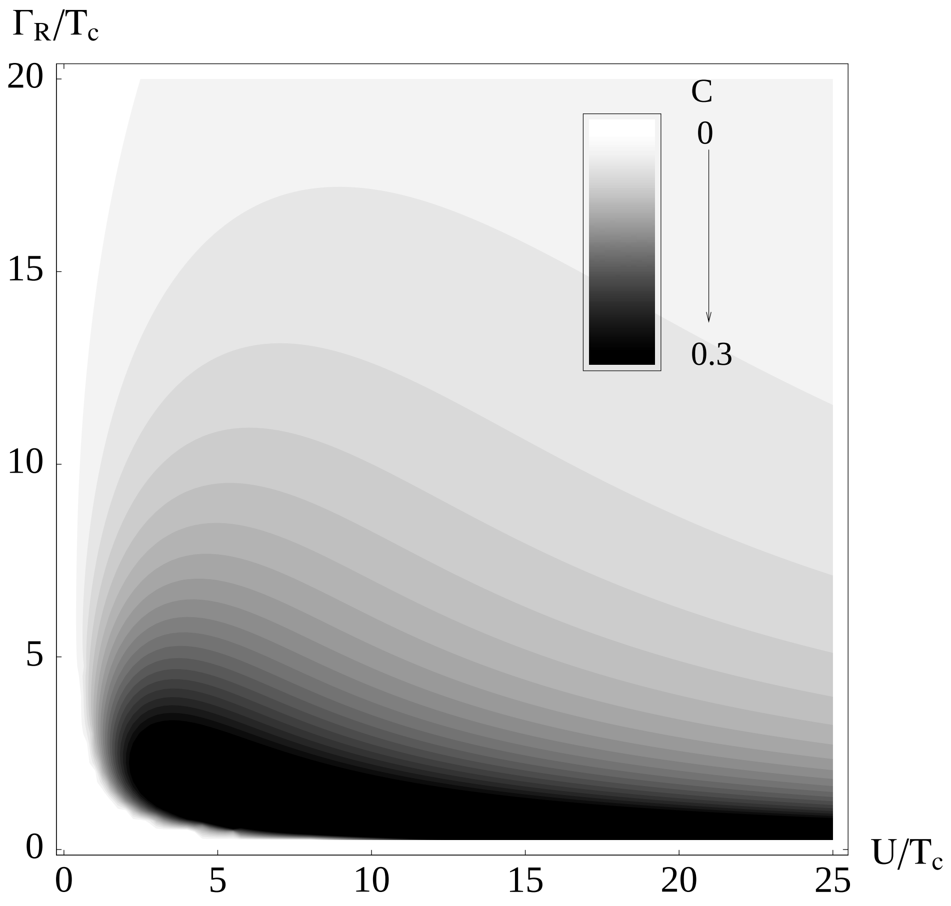
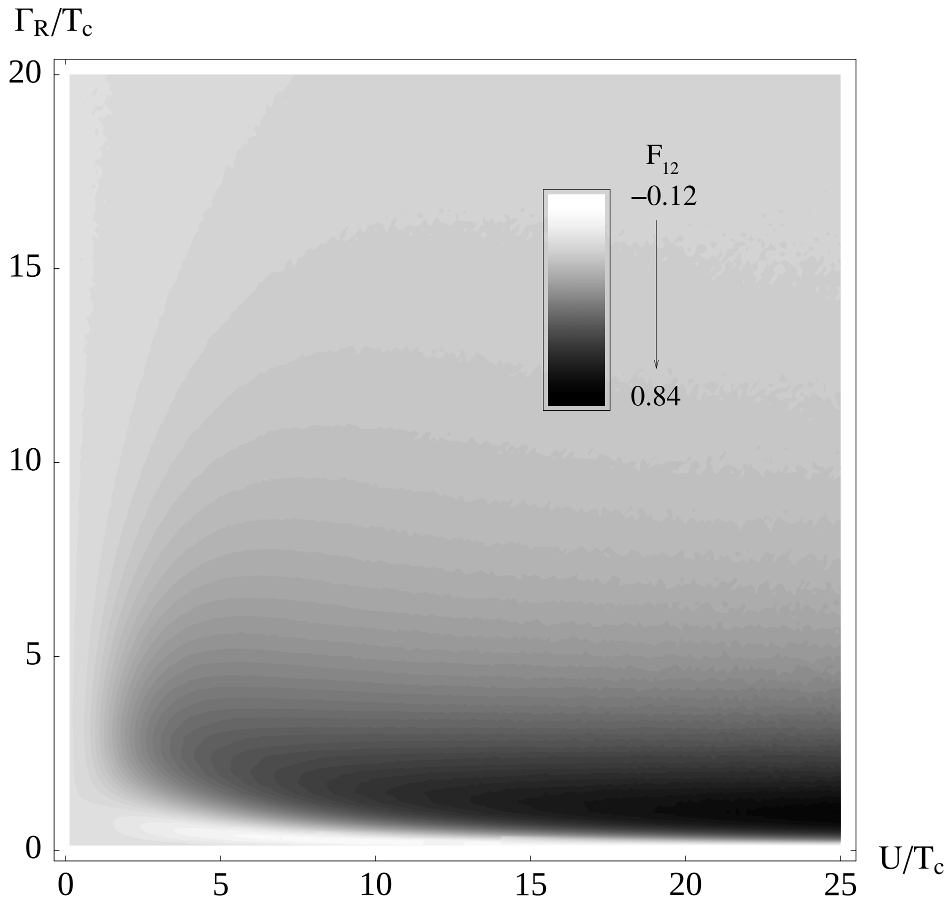
Entanglement and cross-noise Fano factor $F_{12} \equiv S_{12}(0)/2eI$



- Qualitative resemblence to concurrence C.
- Switching on in C corresponds to negative-positive re-emergence in F.

Concurrence and cross-noise Fano factor $F_{12} \equiv S_{12}(0)/2eI$





Summary

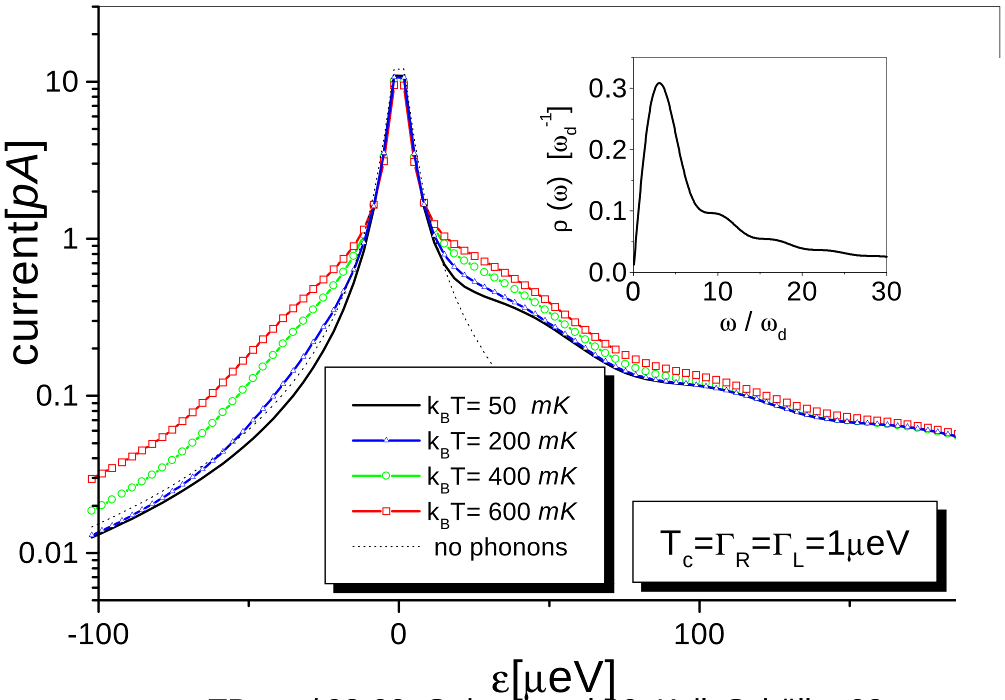
- $N = 1, 2$ 'Non-equilibrium qubits'
- '3 state transport pseudo-spin-boson' model: dissipation, quantum noise.
- QIP tasks, Q-Optics effects, NEMS stuff (single phonon).
- So far infinite bias limit. Finite bias: Co-tunneling, Kondo physics ...

TB, Phys. Rep. **408**, 315 (2005).

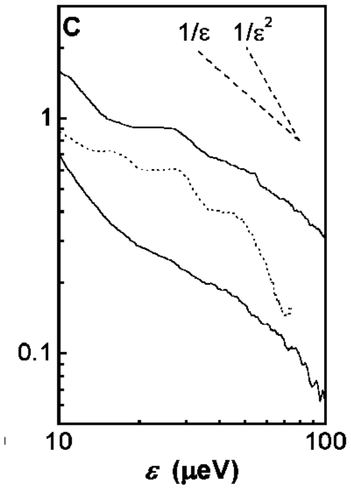
- Polaron-Transformation (POL) $\equiv$ NIBA (non-interacting blib approximation): calculate $\hat{D}_z$ and $\hat{\Sigma}_z$ using bosonic correlation function

$$C_\varepsilon^{[*]}(z) \equiv \int_0^\infty dt e^{-zt} e^{[-]i\varepsilon t} \exp(-\int_0^\infty d\omega \frac{J(\omega)}{\omega^2}\left[(1-\cos\omega t)\coth\left(\frac{\beta\omega}{2}\right) \pm i \sin\omega t\right]).$$

- Polaron tunneling $\rightsquigarrow$ 'boson shake-up' effect
- Re$[C_\varepsilon(z)]|_{z=\pm i\omega} = \pi P(\varepsilon \mp \omega)$: P(E)-Theory.



TB *et al* 98-00; Guinea et al 00; Keil, Schöller 02



T. Fujisawa, T. H. Oosterkamp, W. G. van der Wiel, B. W. Broer, R. Aguado, S. Tarucha, and L. P. Kouwenhoven, Science **282**, 932 (1998)

$\propto \varepsilon^{1+2\alpha}, \alpha \approx 0.1$