Dynamic and Stochastic Resource Capacity Allocation Problems and Adaptive Greedy Rules Based on Dynamic Prices

Peter Jacko
Basque Center for Applied Mathematics, Bilbao, Spain
e-mail: jacko@bcamath.org

Abstract

We will present a novel dynamic and stochastic model of resource allocation that generalizes a variety of problems addressed in the literature and we outline a unified methodology for designing adaptive greedy rules based on dynamic prices. Such rules are important in practice, since they may provide an easy-to-interpret and easy-to-implement solution to problems that are intractable for optimal solution due to the curse of dimensionality, or they embody an elegant optimal solution in some problems with simpler structure. We bridge the methodological gap between static/deterministic optimization and dynamic/stochastic optimization by stressing the connection between the classic knapsack problem and a group of related problems in management and stochastic scheduling unified by our model.

Paper Outline

Consider a collection of competitors with resource capacity demands that in aggregate are, at least at some time instants, beyond the available resource capacity. Suppose that to each competitor we can assign a price, independent of other competitors, that measures the efficiency of attaining a joint goal if a particular amount of resource capacity is allocated to her at a given moment. We are interested in designing and evaluating rules based on these prices and the resource capacity demands which accomplish the goal of good resource capacity allocation, whatever "good" may mean.

In this work we allow for dynamically and stochastically evolving competitors. Therefore, we shift our focus to greedy rules that are *adaptive*. Several questions must be addressed in such a setting:

1) [Economic question] For a given joint goal, is it possible to define dynamic quantities for each competitor that can be interpreted as prices? And if yes,
2) [Algorithmic question] How to calculate such prices quickly?
3) [Mathematical question] Under what conditions is there a greedy rule that achieves optimal resource capacity allocation?
4) [Experimental question] If greedy rules are not optimal, how close to optimality do they come? And how do they compare to alternative rules?

When the competitors are dynamic (even if they are non-stochastic), such a problem is PSPACE-hard. This *curse of dimensionality* justifies the interest in greedy rules, since optimal solutions for high-dimensional problems appearing in the real world are unlikely to be obtained. In addition, this result implies that we can expect that optimality of greedy rules will occur only in problems with largely restricted dynamics.

We present a Markov decision process formulation of the *Dynamic and Stochastic Resource Capacity Allocation Problem*. This novel general model builds on several special cases of increasing complexity that have been addressed in the literature since 1950's, such as the job sequencing problem, the multi-armed bandit problem, the multi-armed restless bandit problem, and a large number of their special cases and extensions.

We show how the Whittle relaxation followed by the Lagrangian relaxation of the problem allows its decomposition into single-competitor subproblems that can be efficiently solved under certain natural conditions in terms of dynamic prices. Such a solution is known in the bandit problem literature also as an *index policy*. We further present well-known cases in which such dynamic prices indeed lead to an optimal greedy rule for the original problem, and illustrate its nearly-optimal performance in more complex (intractable) problems arising in communications networks, sensor management, marketing, and other fields.

References

[1] Jacko, P. (2009). Adaptive Greedy Rules for Dynamic and Stochastic Resource Capacity Allocation Problems, *Medium for Econometric Applications* 17(4):10–16, Available online at http://www.met-online.nl .