Optimal Dynamic Promotion and the Knapsack Problem for Perishable Items (Extended Abstract)

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Senior managers in retail industry make important decisions upon assortment planning, product pricing, and product promotion. While product assortment is a strategic decision taken over a long-term planning period (Kök et al., 2006), the latter two are both strategic and tactical: they can be used in day-to-day marketing decisions to dynamically adjust to demand variations. Within the food retail industry, the necessity, frequency, and complexity of pricing and promotion decisions are further magnified by *perishability* of food products.

There is a strong need by retail managers for a "soft" marketing tool, which would dynamically allow them to improve sales and revenues, yet not altering product prices. For, dynamic pricing models may prescribe to change prices too often or in an "unsystematic" fashion, which contradicts discrete-time decision making, implementation costs, and retail brand image strategy. In addition, the price reduction must usually be done over all units of the product, thus losing possible profit from customers willing to pay the original, higher price. Furthermore, dynamic pricing results in a trade-off between markdowns and stockouts, since markdowns may damage producers, while stockouts may damage retailers.

We design a revenue management model, in which demand is stimulated by moving a number of product units to a promotion space, rather than by price changes. Thus, we address the problem of filling a promotion location with limited space to maximize the expected revenue, which we have termed the *Knapsack Problem for Perishable Items* (KPPI). Examples of the promotion space include shelves close to the cash register, promotion kiosks, or a depot used for selling via the Internet.

We solve the KPPI using a problem decomposition into single product unit subproblems. A natural mathematical setting for the KPPI subproblems is the restless bandit model (cf. Whittle, 1988; Niño-Mora, 2002), a fundamental stochastic model for resolving a trade-off between exploration and exploitation in an optimal fashion. In our model the bandits (perishable items) are *restless*, because they can get sold regardless of being in the knapsack or not, the time horizon is *finite*, and we are to select *more than one* item for the knapsack, which is allowed to be filled partially, due to the *heterogeneity* of the items.

Each product unit is assigned a *promotion priority index*, which captures the opportunity cost of promotion, as a function of its price, lifetime, expected demand, and expected promotion power. These indices are then used for each item as objective-function value coefficients in a (classic) knapsack problem, whose solution yields a well-performing heuristic for the KPPI. We thus mix up two models: the restless bandit problem and the knapsack problem.

1 Optimal Dynamic Promotion under Time-Homogeneous Demand

Suppose that the item perishes in T time periods, implying a final cost $c > 0$ if not sold before. Let $1 - p$ be the probability that a promoted item is sold in one period, and $1 - q$ that of a non-promoted item ($q > p$). Future costs are discounted by the one-period discount factor β. The next proposition asserts that, optimally, an item with lower probabilities of being sold when not promoted is assigned a higher promotion priority index, and that the index is nondecreasing over time. Hence, once the item is optimally chosen for promotion, then it should remain promoted until it perishes.

Proposition 1. *The perishable item is indexable with its index*

$$\nu^* = \frac{c(\beta q - \beta p)(\beta p)^{T-1}}{1 - (\beta q - \beta p)\frac{1-(\beta p)^{T-1}}{1-\beta p}}. \tag{1}$$

The results can be extended to the long-run average criterion by taking the limit $\beta \to 1$. Further, if $q = 1$, then the index reduces to $\nu^* = c(1 - p)$, the well-known $c\mu$-rule. The details and extensions, including the case with non-homogeneous demand and product inventories, are given in the full-length paper (Jacko, 2007), where we prove that, under certain demand regularity conditions, the necessity of marketing actions increases with larger inventory and with shorter product lifetime.

2 Knapsack Problem for Perishable Items

Since the dynamic programming formulation of the KPPI is most likely to be intractable, we relax the problem, which allows us to decompose it into single-item cases solved above.

Whittle (1988) proposed for restless bandits what has become known as the *Whittle's relaxation*: replace the infinite set of resource constraints by one constraint requiring to use the full resource only *in expectation*, resulting in a sort of *perfect market assumption*. This is solved by the Lagrangian method: for a given penalizing parameter, the relaxation can be decomposed and analyzed separately for each item, so we can apply the results obtained above. The penalizing parameter can be interpreted as the competitive market price of space, the resource provided by the knapsack.

Since the indices measure the opportunity cost (the true economic value) of promoting an item, we propose the following heuristic construction for the (non-relaxed) KPPI: "Promote the items that are given by an optimal solution to a knapsack problem with indices as the objective function value coefficients and item volumes as the knapsack constraint weights". Our simulation studies suggest that the proposed heuristic outperforms the existing ones of Whittle (1988) and Niño-Mora (2002).

3 Conclusions

We have developed a dynamic and stochastic model for promotion of perishable products, and proposed a solution that has a natural economic interpretation and suggests itself to be easily implementable in practice. These advantages come at the cost of possible suboptimality of such a dynamic solution, which is, however, negligible and smaller than the cost of implementing naive marketing solutions or other existing proposals. The model has an appealing property of being extensible to a variety of ad-hoc requirements that managers or certain circumstances may impose.

More generally, KPPI offers a comprehensive modeling framework that may be used in other applications, since the items considered in knapsack problems are often perishable, either naturally or due to special restrictions. Apart from product promotion in other retail industries, applications arise transplantation medicine or task management.

References

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