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THE CONTEMPORARY ARTS

Exploring variations through computational analysis

Alan Marsden, Lancaster University, UK

Possibilities of using computation

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Using computers changes or even challenges the practices of music analysis.

- **Musical data can be analysed with greater precision.**
- **Greater quantities of music can be analysed.**
- **Using computers changes the questions asked in analysis.**

Alan Marsden, '"What was the question?": Music analysis and the computer', in Tim Crawford & Lorna Gibson, *Modern Methods for Musicology: Prospects, Proposals and Realities* (Ashgate, 2009).

Necessities of using computation

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Some music theory and analysis makes general claims about music.

- **General claims require evidence and arguments of general validity.**
- **Empirically verifiable claims should be empirically verified.**
- **Cook & Clarke call for musicology to become a 'data rich' discipline (*Empirical Musicology* (OUP, 2004)).**

Validity requires

- **No bias (objectivity)**
- **Sufficient evidence**
- **Precision of argument**

Computers, suitably programmed and with suitable databases, deliver these. They are difficult to obtain by purely human means.

How is a variation related to a theme?

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A general question about a kind of music.

Needs first to be framed more precisely:

- **What properties does a variation share (or share more) with the theme of which it is a variation, but not share (or share less) with a different theme?**

No bias

- **Selection of material on objective criteria**

Sufficient evidence?

- **10 themes, 76 variations, but only four bars of each**

Precise argument

- **Computational comparison**
- **Mathematical analysis of results**

Automatic Schenkerian reduction

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Previous work (Kassler, etc.) has shown the theoretical possibility of Schenkerian reduction by computer, but implementation is a complex problem.

AHRC-sponsored project to investigate Schenkerian reduction by computer.

- **System capable of deriving a reduction from small extracts of keyboard music (c. 4-8 bars).**
- **For short themes with Ursatz, matches human-produced reductions moderately well.**

Essential problem is that there are a vast number of possible reductions of even short extracts. Identifying 'the best' reduction is difficult.

Example (hand-made) reduction

Formalisation of theory

Structure

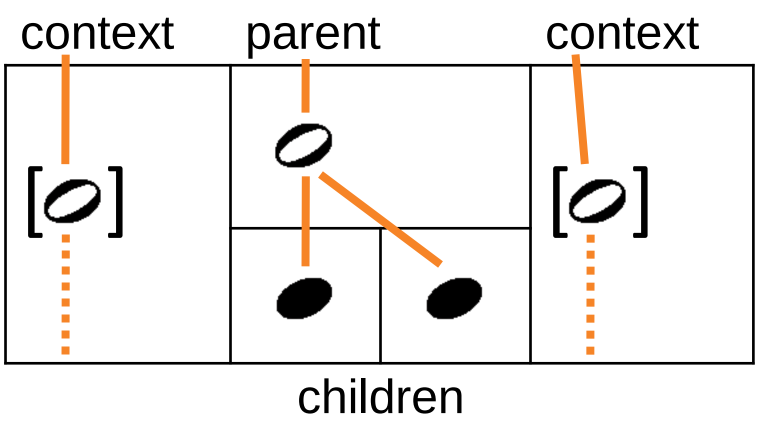
- **binary trees**
  - **parallel congruent trees for different voices**
  - **can share parts of their structure**
  - **so properly a directed acyclic graph (DAG, digraph)**

Nodes = notes and rests

- **pitch, duration & tie**
- **harmony (global key & metre)**
- **no explicit voices**

Arcs = **'atomic reductions'**

- **one parent; two children**
- **constraints on immediately preceding and following context**
- **harmonic constraints**
- **inheritance of harmony**



Example atomic reductions

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Appoggiatura

- **First child: no tie**
- **Second child: no tie; pitch one step above or below first child**
- **Parent: no tie; pitch equal to second child; harmony equal to second child's; pitch of second child consonant**
- **Required pre-context: none**
- **Required post-context: none**

Neighbour Note

- **First child: [no constraint]**
- **Second child: no tie**
- **Parent: tied if first child tied; pitch equal to first child; harmony equal to first child's; pitch of first child consonant**
- **Required pre-context: none**
- **Required post-context: note one step above or below second child**

Atomic reductions

No context constraints:

- **hold (tied)**
- **repetition**
- **shortening (followed by rest)**
- **delay (preceded by rest)**
- **appoggiatura**
- **consonant skip 1 (first pitch = parent)**
- **consonant skip 2 (second pitch = parent)**
- **interruption (I-V)**

Constraint on following context:

- **anticipation**
- **neighbour note (incomplete; resolves to following context note)**

Constraint on preceding context:

- **suspension**

Other reductions can be constructed from combinations of these

Discussion and detail of formalisation in

Alan Marsden, 'Generative Structural Representation of Tonal Music', *Journal of New Music Research*, 34 (2005), 409-428

Computational process

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Basic process:

1. **Divide the score into a sequence of 'segments'.**
   - **each segment covers a span where no note begins or ends**
2. **For each pair of segments, compute the possible reductions, deriving new segments.**
   - **do this recursively for pairs involving derived segments also**
3. **Select only analyses which contain an Ursatz.**
4. **Select the best alternative.**

The number of alternatives is far too great for a naive process.

- **number of possibilities related to $n!$**
  - **(n = number of segments in the piece)**
  - **$n! = n \times (n - 1) \times (n - 2) \times \ldots \times 1$**

Chart parser; CYK algorithm

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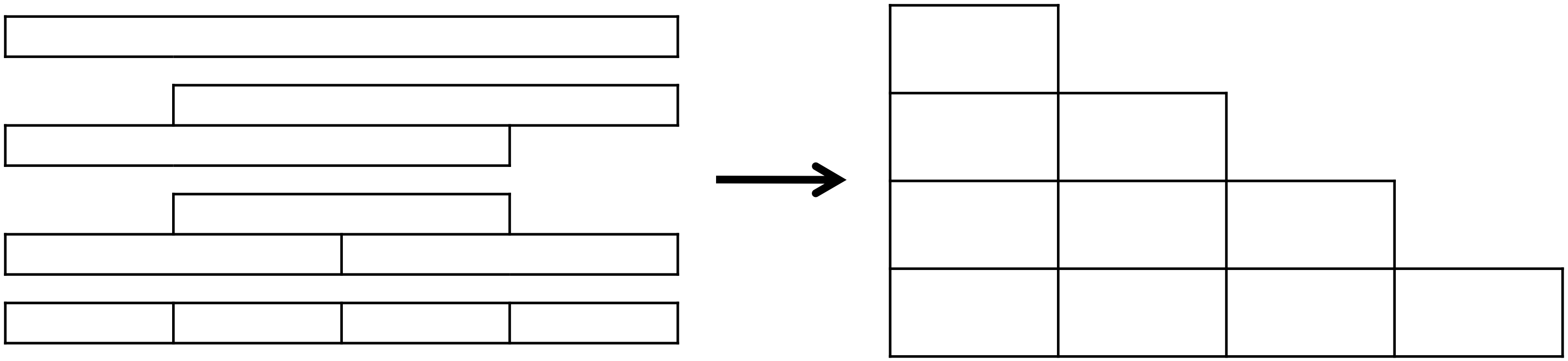
Instead of making a set of analyses, make a chart of possible reductions at each point, from which complete analyses can be extracted.

- **Triangular matrix of cells**
  - **bottom row contains segments of the 'surface'**
  - **higher rows contain derived segments spanning 2, 3 … surface segments**
  - **top row has a single cell spanning the entire piece**



- **Lower computational complexity**
  - **in principle, cubic (n^3) instead of factorial**

Up-Down Process

'Up':

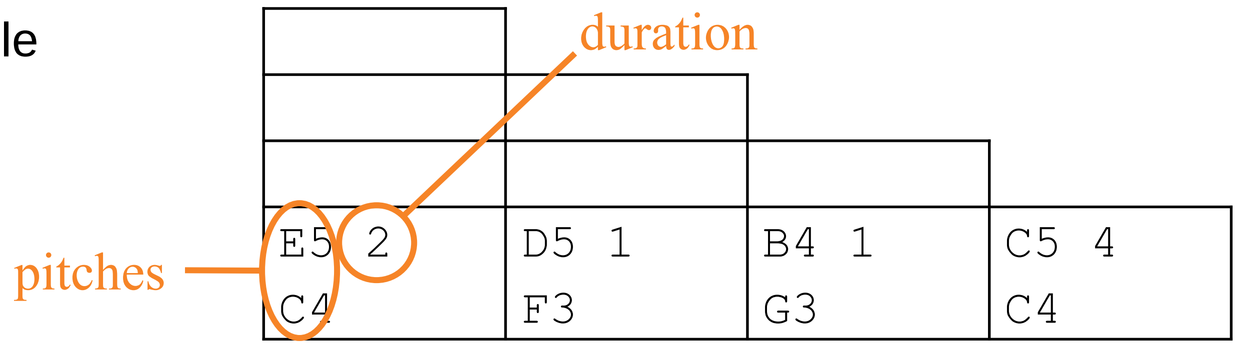
- **Derive segments**
- **Record best score for each possibility**
- **Record possible Ursatz membership for each segment**

'Down':

- **Prune segments which have no parent**
- **Prune segments which cannot be part of an Ursatz or be reduced to a member of an Ursatz**
- **Select best-scoring analysis**
  - **best-first search**

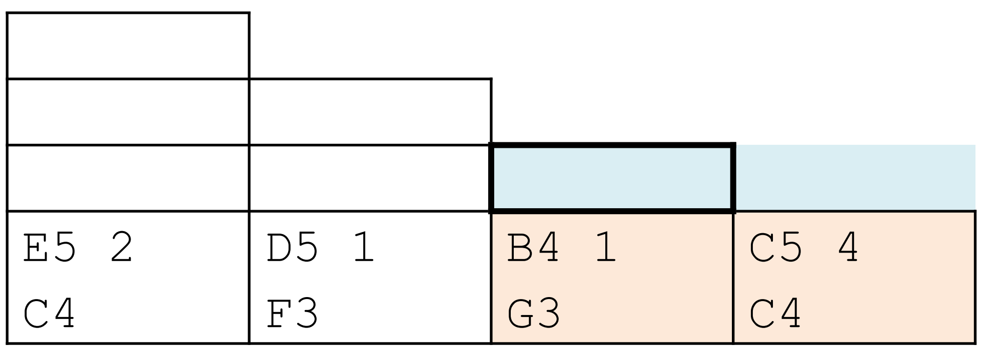
Reduction Process 1

Initial table



Step 1

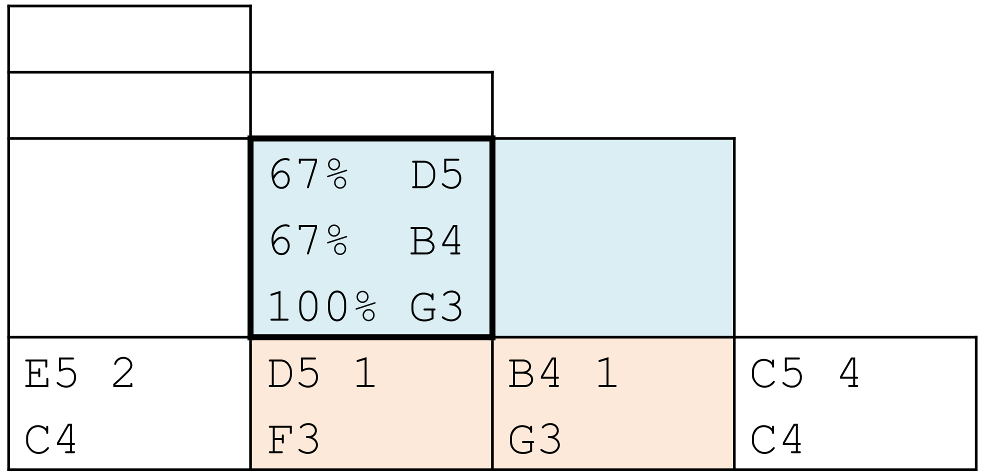
No new segments



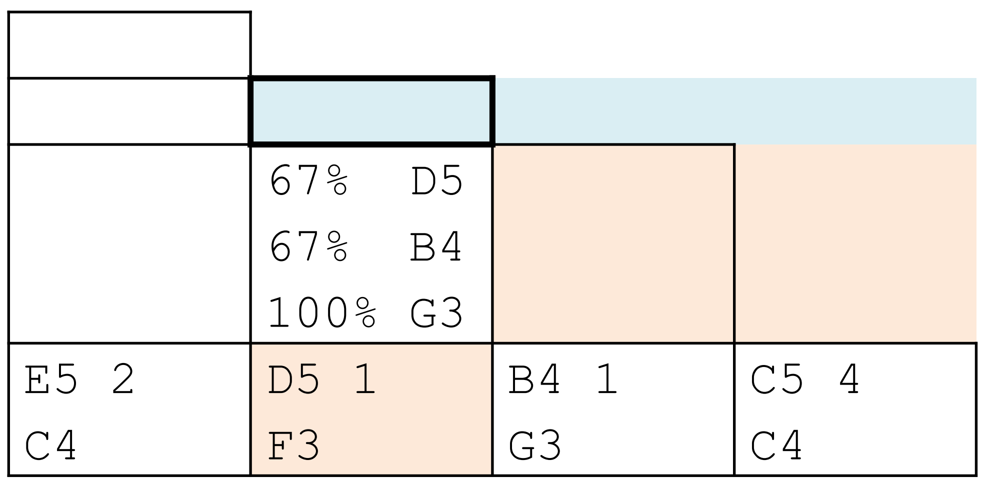
Reduction Process 2

Step 2

3 new segments:
G3 B4, G3 D5,
G3 B4 D5



Step 3a
No new segments



Reduction Process 3

Step 3b

2 new segments:

C4 C5, G3 C4 C5

	100% C5 100% C4 50% G3		
	67% D5 67% B4 100% G3		
E5 2 C4	D5 1 F3	B4 1 G3	C5 4 C4

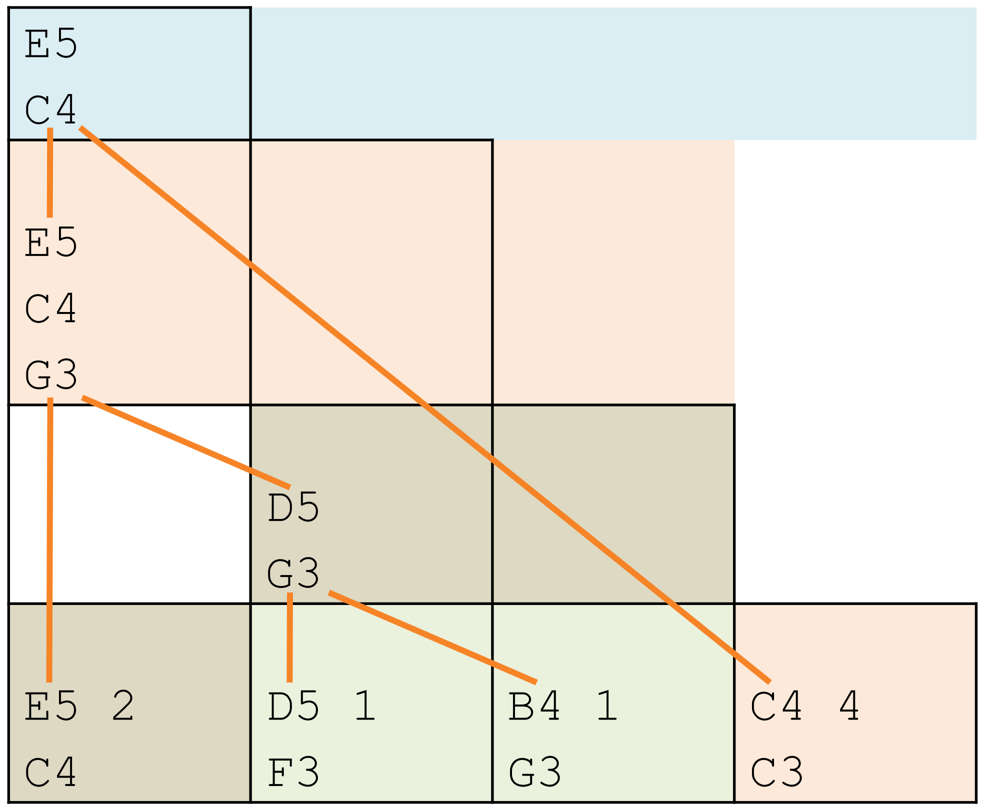
Reduction Process 4

Full table

63% E5 75% C5 75% C4 63% G3			
100% E5 100% C3 50% G3	100% C5 100% C4 50% G3		
	67% D5 67% B4 100% G3		
E5 2 C4	D5 1 F3	B4 1 G3	C5 4 C4

Selection of Best Analysis

Prune and select best scoring



Weights

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To find a good analysis

1. Select higher level pitches which are more often present in the surface.
2. Avoid splitting and joining of voices.
3. Select reductions with small intervals between notes reduced together.
4. Reduce segments of approximately equal duration together.
5. Avoid reductions which create syncopations at higher levels.
6. Avoid reducing a shorter segment with a following longer segment.
7. Prefer reductions with more tonic and dominant harmony.
8. Avoid reductions where a note is followed by a rest.
9. Prefer reductions where higher level harmonies are more often consonant with the surface.

Automatically derived best-scoring analysis

Exploring variations

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Hypothesis: Variations and themes share a common structure.

- **The reduction of a variation will match the reduction of the theme, at least at higher levels.**
- **The match will be greater than a match based on the surface alone.**

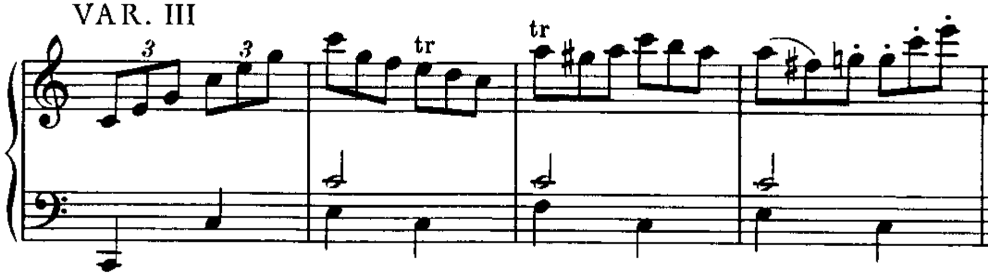
Method: Compare how much variations match their theme with how much they match unrelated themes.

- **Take corresponding extracts of variations and themes.**
  - **First four bars of all Mozart piano variations in simple triple and duple metres, avoiding variations in a different key or metre, and two juvenile pieces.**
- **Match each variation with each theme.**
  - **match surface with surface and best reduction of theme with reduction matrix of variation**
- **Test for a greater degree of match with the correct theme.**

Examples of materials

Theme

Variations



Matching methods

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All combinations of

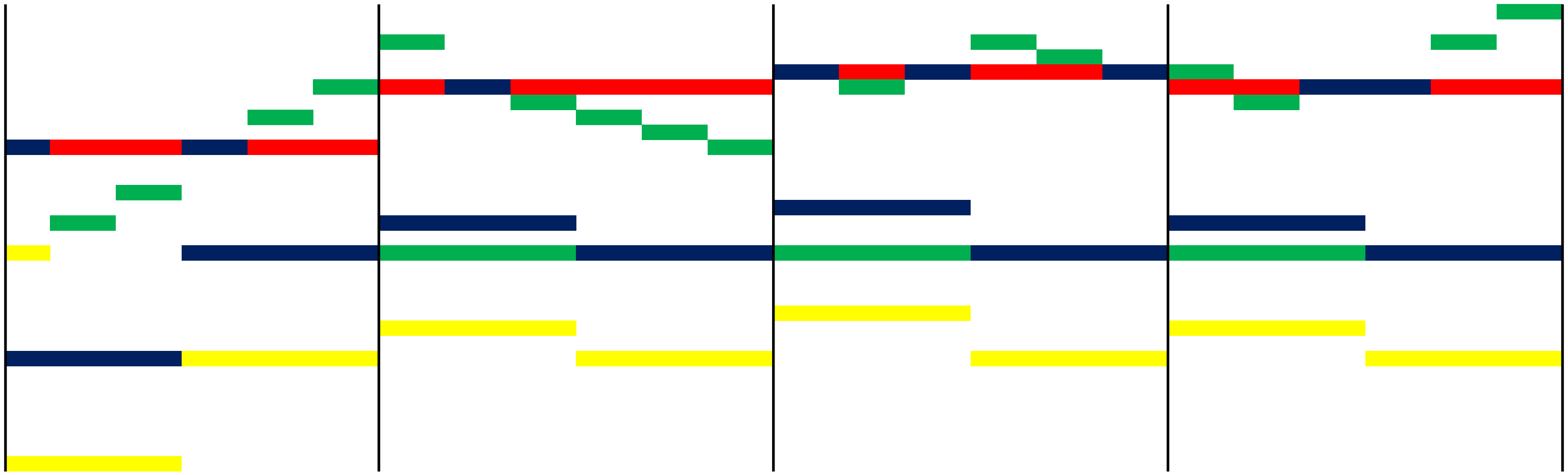
- **Pitch matching: pitches/pitch classes**
- **Pitches from: full texture/melody+bass/melody/bass**
- **Voice must match (melody, middle or bass): yes/no**
- **Match tied notes: yes/no**
- **Weight by metre/reduction level: yes/no**
- **Limit by parent match (reduction only): yes/no**
- **Value recorded:**
  - **surface: proportion of span/present in span/present in bar**
  - **reduction (from multiple possible segment): maximum/simple average/score-weighted average**

384 different combinations for surface matches.

1024 different combinations for reduction matches.

Surface-matching example

K.265 theme with K.265 variation 3



Blue: portions of theme notes matched with variation notes

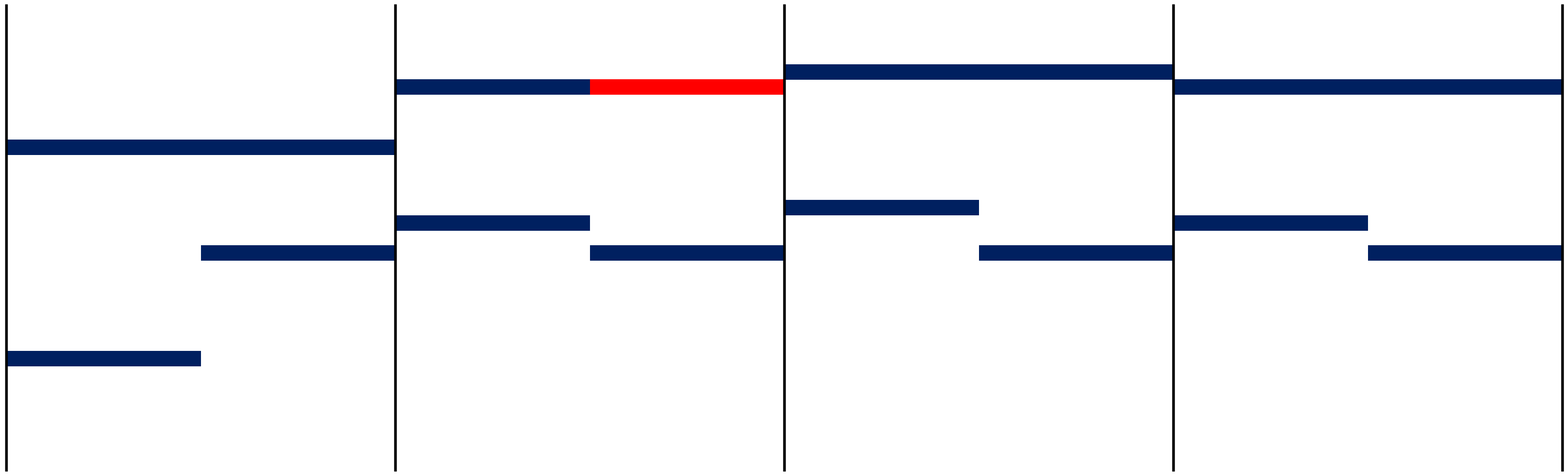
Red: portions of theme notes not matched with variation notes

Yellow: variation notes matched with theme notes

Green: variation notes not matched with theme notes

Reduction-matching example

K.265 theme and K.265 variation 3



Blue: portions of theme notes matched with variation notes in *some* corresponding segment

Red: portions of theme notes not matched with variation notes

- **Matches are also made at higher levels of reduction. In this case higher levels match perfectly.**

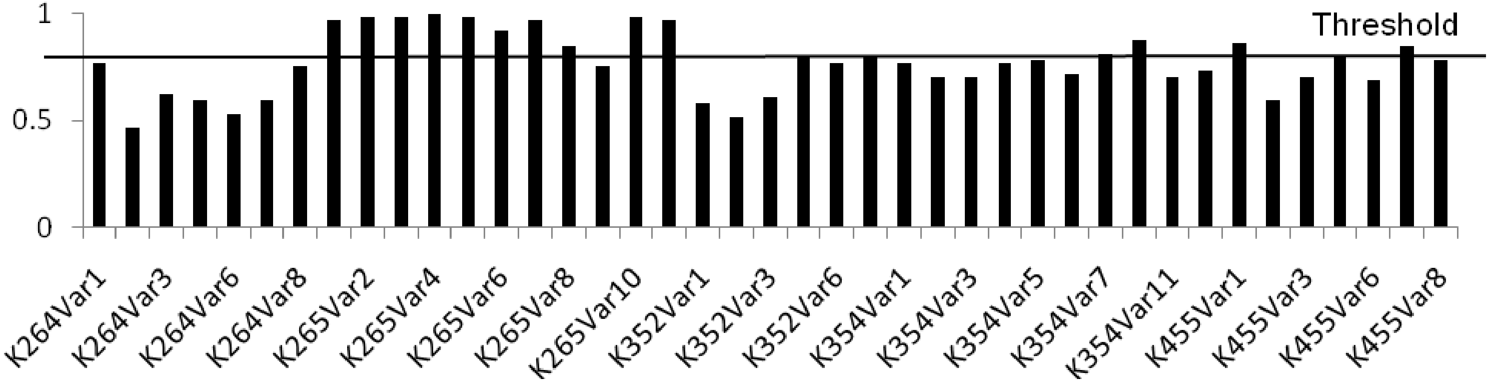
Method

1. All themes and variations were transposed to F major.
2. Every theme was compared to every variation in the same metre with each method and the degree of match recorded.
3. For each theme and each method, a maximum possible F-measure was calculated.
   - Select a threshold of match.
   - Count how many variations of this theme have a degree of match to the theme greater than this threshold (tp), and how many less (fn).
   - Count how many variations of other themes have a degree of match to the theme greater than this threshold (fp).
   - F-measure is 2 * tp/(2 * tp + fn + fp).
   - Test for all possible thresholds.
4. High F-measure indicates a method which tests what a theme and its variations have in common

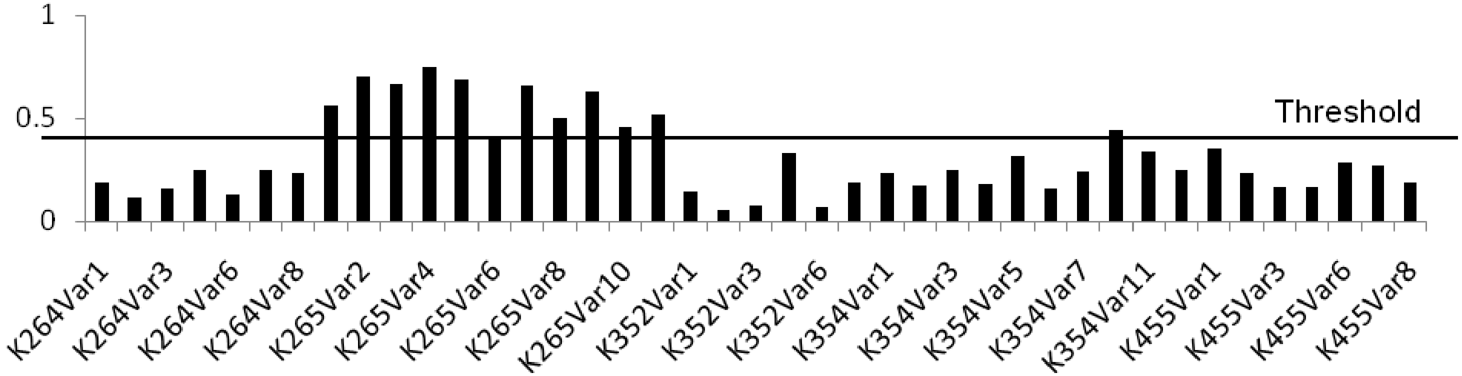
Example results

For theme of K.265

Reduction-based result



Surface-based result



Overall results

Surface methods	Average F-measure	Reduction methods	Average F-measure
Best	0.867	Best	0.842
Average	0.776	Average	0.748
Worst	0.540	Worst	0.671

1. **Contrary to the hypothesis, variations and themes do not appear to be more similar in their reductions than at the surface.**
2. **Best surface-based method matches pitch classes rather than pitches, matches notes in their respective voices, includes tied notes, weights by duration, and measures the proportion of span which matches.**
3. **Best reduction-based method matches pitch classes in melody and bass in their voices, ignores tied notes, weights by duration, and measures the maximum match in alternative segments.**

Going about computational analysis

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- Write your own software.
  - requires expertise
  - very time-consuming
- Use an existing package
  - Sonic Visualiser for analysis from audio
  - Humdrum for score-based analysis
  - not many packages
  - still require some expertise
- Use general-purpose software
  - Excel or similar
  - Matlab or similar
- Get someone else to write the software for you
  - computer-science student as project
  - collaborate with a computer scientist
  - software service such as centre shortly to be established at QMUL Centre for Digital Music

The computational approach

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- Precise definition of data
- Unambiguous and tractable analysis processes
  - what is to be found out
  - how to find it out
- Rigorous assessment of results
  - mathematical analysis
  - tests for significance

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