A Framework for Investigation of Schenkerian Reduction by Computer

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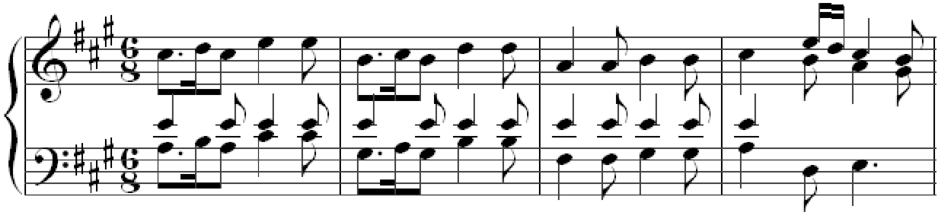
Goldsmiths, 10 Apr. 2008

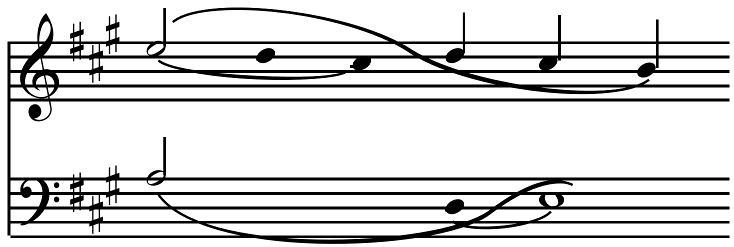
Schenkerian Analysis

Progressively reduces a score, removing less essential features, to reveal the ‘background’ structure.

Mozart:



Schenker:



Lerdahl & Jackendoff GTTM

F. Lerdahl & R. Jackendoff,
A Generative Theory of Tonal Music
(1983), MIT Press

Benefits

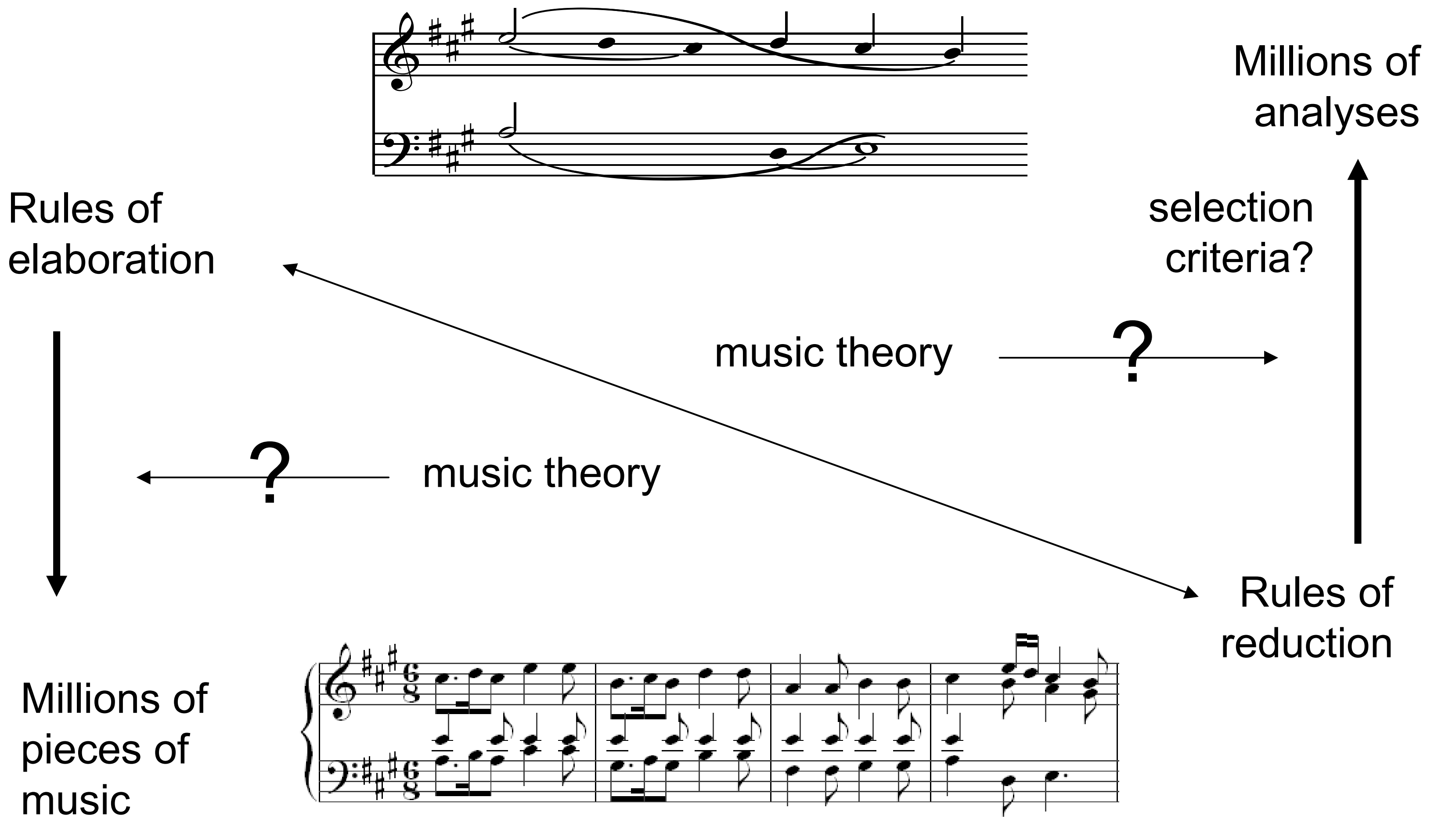
- The most influential and widely adopted theory and method of analysis for tonal music since the last quarter of the 20th c.
- Adumbrates many aspects of musical structure (key, harmony, segmentation, metre).
- Some evidence that it corresponds to perception and cognition of music.
- Based on two centuries of previous music theory.

BUT does remain controversial among musicians, and suffers from obscure arguments about detail.

Previous Work

- Kassler (1967, 1975, 1977, 1988)
  - program which successfully analyses three-voice middlegrounds
- Smoliar et al. (1976, 1978, 1980)
  - program capable of verifying an analysis
- Lerdahl & Jackendoff (1983, 2001)
  - rule-based system for quasi-Schenkerian reduction
  - not demonstrably computable
- Mavromatis & Brown (2004)
  - demonstration of theoretical possibility of Schenkerian analysis by context-free grammar
- Hamanaka, Hirata & Tojo (2005-7)
  - implementation of Lerdahl & Jackendoff reduction with adjustment of parameters (now moving towards automatic parameter-setting)
- Gilbert & Conklin (2007)
  - probabilistic grammar for melodic reduction

The Research Problem



A Framework for Empirical Research

1. Formalise rules of reduction.
2. Derive all possible reductions of a fragment of music.
3. Measure certain characteristics of a sample.
4. Measure the same characteristics in ‘correct’ analyses of the same fragments.
5. Compare the distribution of values from the sample to the values from the analyses.
6. Characteristics where the analyses are consistently distinguished in the sample distribution suggest possible selection criteria.

1. Formalisation of Rules of Reduction

- See Alan Marsden, ‘Generative Structural Representation of Tonal Music’, *Journal of New Music Research*, 34 (2005), 409-428

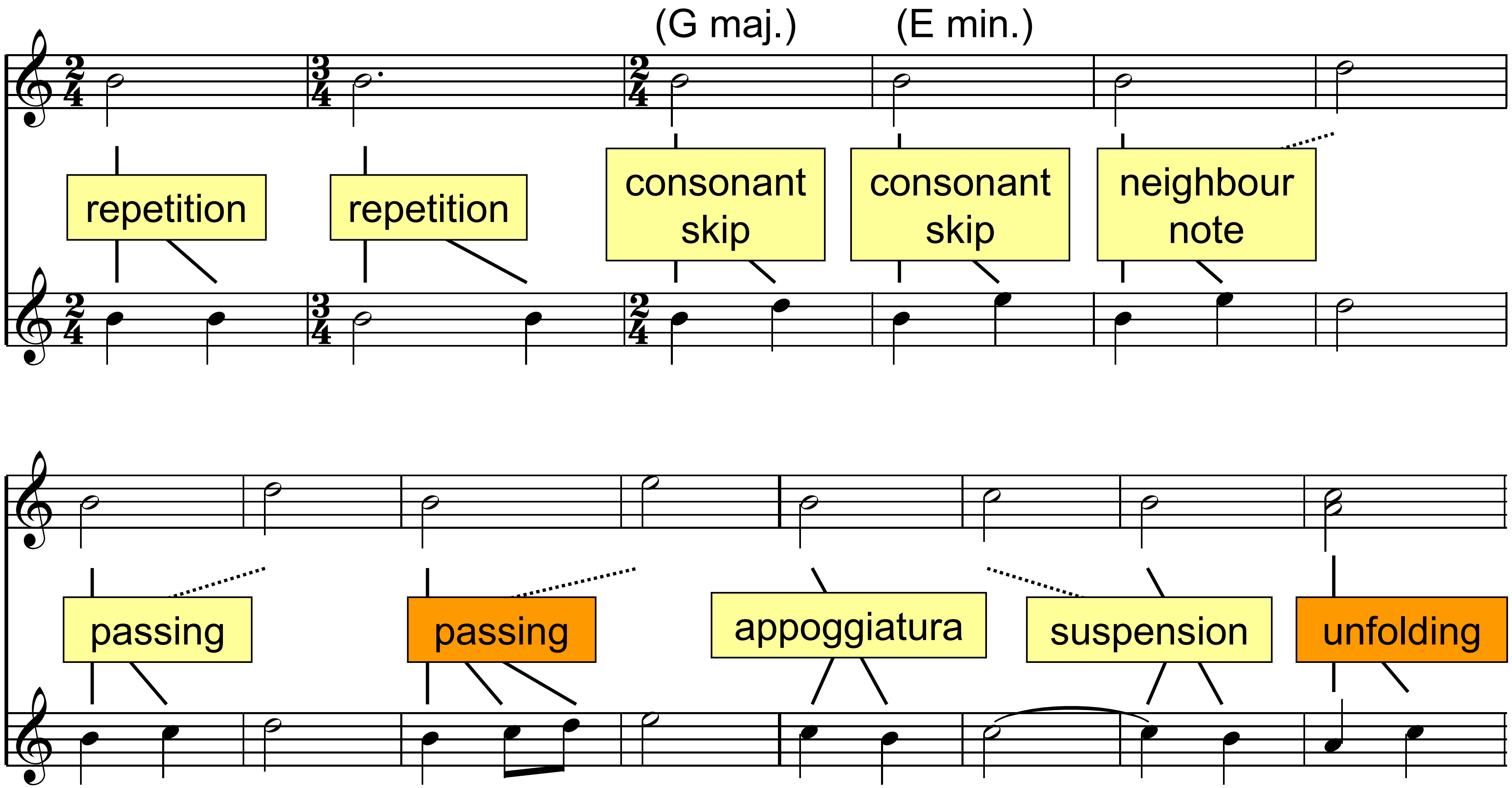
1. All elaborations are binary.
    - elaborations producing more than one new note accommodated by special intermediate ‘notes’
2. Elaborations generate new notes within the same time-span (cf. Lerdahl & Jackendoff, Komar).
3. Only certain kinds of elaborations are possible.
4. Elaborations have harmonic constraints.
5. Some elaborations require specific preceding or following context notes.

Formalisation (non contentious)

1) Notes are defined by pitch and time (start and duration).
2) All notes on the ‘surface’ of the piece derive by a process of iterative elaboration of a single chord (i.e., several notes all with the same start and duration).
3) Only certain kinds of elaboration are possible.
4) Elaborations can have an associated key and harmony.
5) Simultaneous elaborations (in different parts/voices) must be consistent in key and harmony.

A piece of music is a tree-like structure of elaborations, BUT it has simultaneous trees (for different voices) and these may intertwine (a note can belong to more than one tree).

Elaborations



Further detail in Marsden, *CHum* (2001) and *JNMR* (2005).

Formalisation (contentious)

6) All elaborations produce two ‘children’.
7) All elaborations have one ‘parent’ note.

   (So trees are binary. Special ‘note sequences’ are produced in extended passing elaborations. Unfoldings, which should have multiple parents, are represented by multiple elaborations.)

8) Elaborations may require a specific preceding or following ‘context note’.

   (So branches of trees are not independent of each other.)

Restrictions (Temporary?)

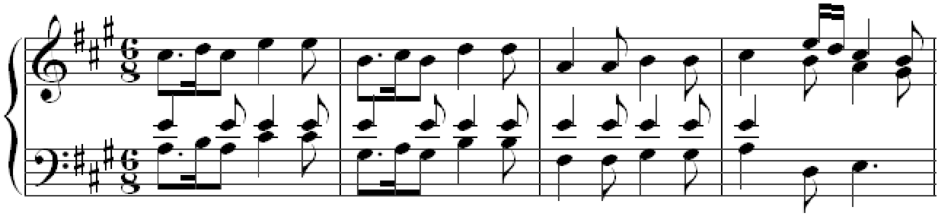
In order to allow a less inefficient analysis algorithm:

9) Simultaneous branching in trees must produce children with the same durations in each tree.
10) Preceding context notes must be present on the surface (e.g., in the case of the preparation of a suspension).
11) Voices cannot cross each other.

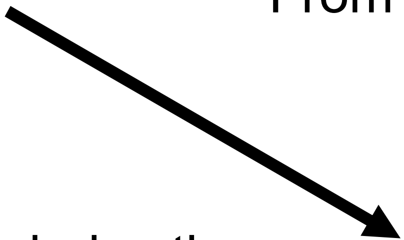
Plus some arbitrary restrictions to avoid crazy solutions:

12) Chords in reductions must not be larger than a certain small number of notes.
13) Pairs of notes reduced must have a moderately simple ratio of durations.

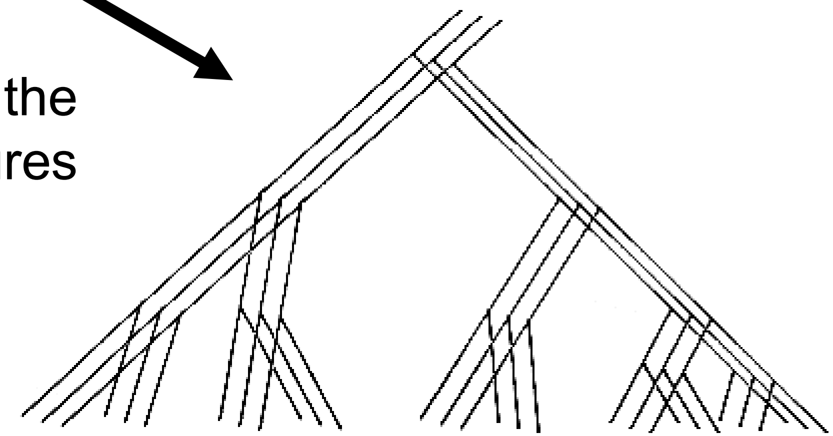
The Process



From the score …



… to derive the tree structures



Local Solution-Finding

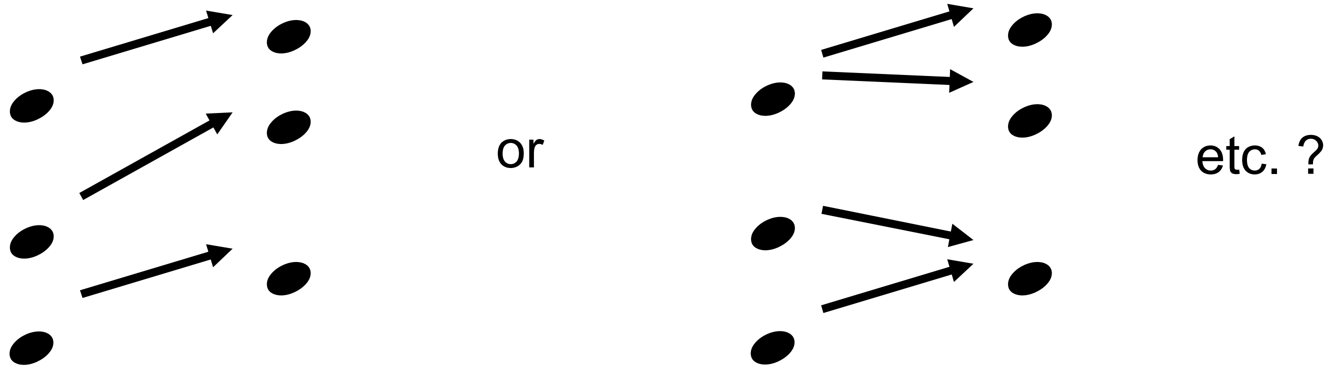
For any pair of notes, given knowledge of the preceding notes (on the surface) and possible and actual following notes (both on the surface and at higher levels), we can determine:

- which elaborations, if any, can produce these notes,
- what the parent note must be for each elaboration,
- what the requirements of key and harmony are for each elaboration.

So, given any pair of consecutive chords, knowledge of preceding and following chords, and rules of harmonic and tonal consistency, we can determine the possible parent chords of that sequence.

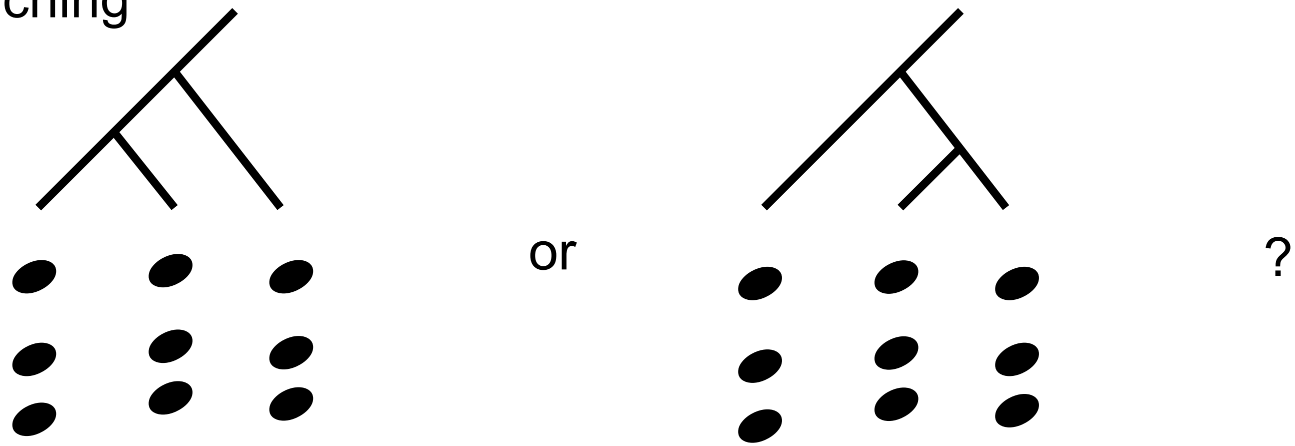
Combinatorial Problems

1. Voices



Increases exponentially with the size of a piece

2. Branching



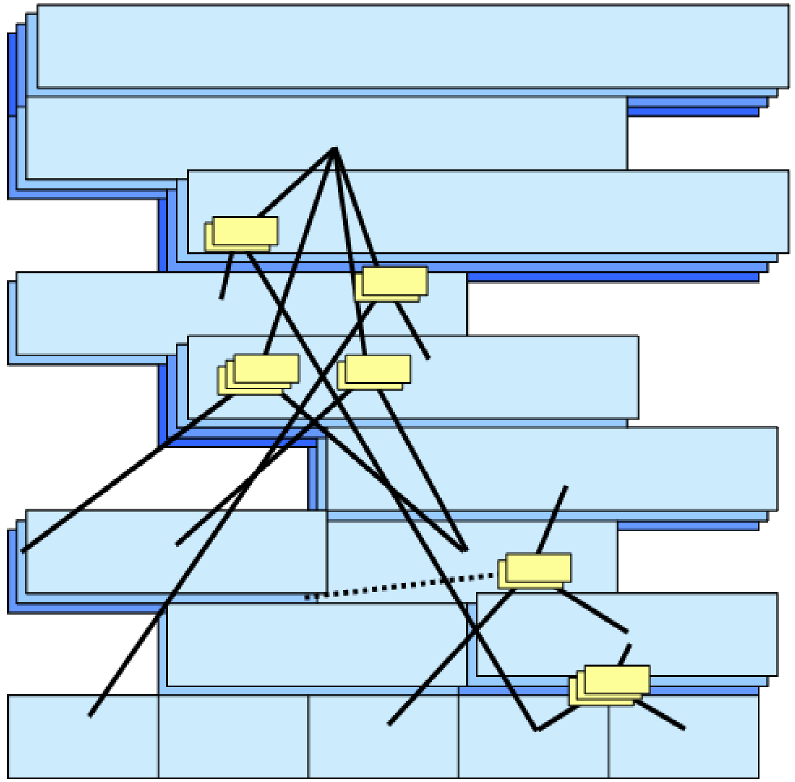
Increases factorially with the size of a piece

Attempted Solution

- Inspired by dynamic programming.
- Construct a 3D matrix of valid local solutions.
  - lowest level is all the ‘chords’ of the surface of the piece:
    1D, *n* cells
  - higher levels are all possible chords derived by reduction from all possible pairs of chords below:
    2D, (*n* – *l*) * *x* cells
    (*l* level of reduction, *x* unknown but limited number of possibilities)
- Any valid reduction tree can be derived from the matrix by selecting a top-level cell and then iteratively selecting pairs of possible children.

2. Derivation of All Possible Reductions

- Not possible explicitly, because of ‘combinatorial explosion’
  - number of possible reductions related to *n*! (where *n* is the length of the music)
- Derivation of a matrix of local solutions, from which all possible reductions may be derived
  - size theoretically related to n^3



Example of Reduction Matrix

```
Row 5
0-5 16
 67 E5
 67 C5
 75 C4
 50 A3
 25 G3
Row 4
0-4 8       1-5 14
 63 E5       67 _E5
 38 D5       67 C5
 25 C4       75 C4
 50 B3       50 A3
 25 A3       25 G3
 38 G3
Row 3
0-3 7       1-4 6       2-5 12
 67 E5       33 _E5     100 C5
 33 D5       33 D5       75 C4
 33 C4       67 B3       50 A3
 33 B3       22 A3       25 G3
 50 A3       44 G3
Row 2
0-2 6       1-3 5       2-4 4       3-5 10
100 E5       50 _E5      43 D5      100 C5
 50 C4       30 D5       57 B3      100 C4
 25 B3       40 pB3-G3   14 A3       50 G3
 50 A3       40 B3       57 G3
             40 A3
Row 1
0-1 4       1-2 4       2-3 3       3-4 2       4-5 9
100 E5       67 _E5      50 D5      100 D5      100 C5
 33 pC4-A3   50 pB3-G3   50 B3       67 B3      100 C4
 33 C4       17 B3       50 A3       67 G3       50 G3
 33 B3       67 A3
Row 0
0 2         1 2         2 2         3 1         4 1         5 8
100 E5      100 _E5     100 A3      100 D5      100 _D5     100 C5
100 C4      100 B3                  100 B3      100 G3      100 C4
```

Example of Selection

```
Row 5
0-5 16
100 E5
100 C4

Row 4
0-4 8         1-5 14
100 E5
100 C4

Row 3
0-3 7         1-4 6         2-5 12

Row 2
0-2 6         1-3 5         2-4 4         3-5 10
100 E5
100 C4

Row 1
0-1 4         1-2 4         2-3 3         3-4 2         4-5 9
              100 _E5                     100 D5
              100 pB3-G3                  100 G3

Row 0
0 2           1 2           2 2           3 1           4 1           5 8
100 E5        100 _E5       100 A3        100 D5        100 _D5       100 C5
100 C4        100 B3                      100 B3        100 G3        100 C4
```

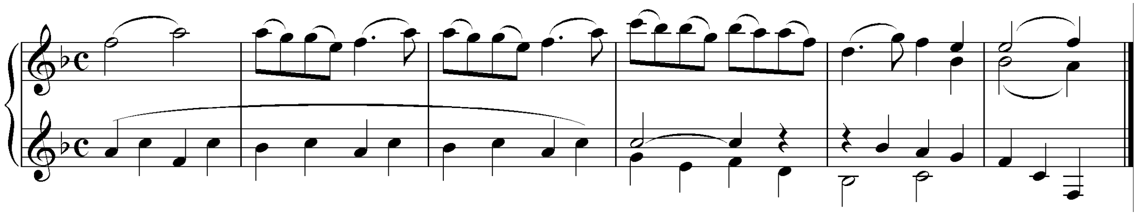
3. Selection and Measurement of a Sample

- Selecting a random sample is not trivial
  – selecting an option at one point in the matrix affects options at other points
  – currently selects top-down giving equal likelihood to each remaining option at each point
- Which measures to try?
  – guesses based on expertise
  – suggestions from Schenkerian literature (Plum, Schachter, teaching materials)
  – Lerdahl & Jackendoff preference rules

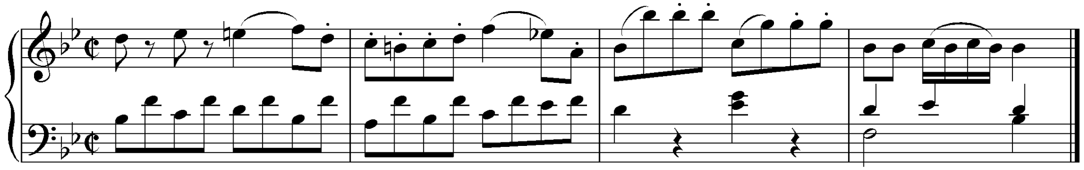
Sample Fragments

Rondo themes from Mozart piano sonatas

1a 1b



2a 2b



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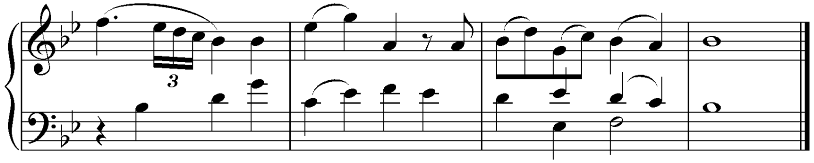


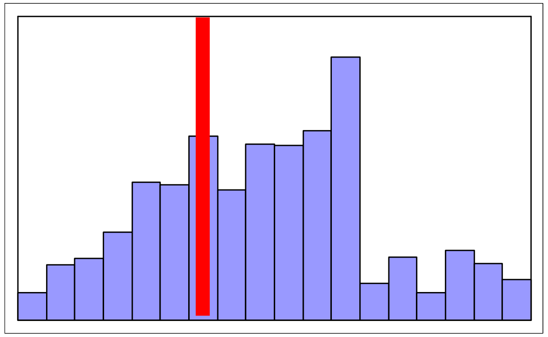
4

1 & 2 were analysed in two halves

4. Measurement of Characteristics

- ‘Correct’ analyses derived from teaching materials
  - selection of the closest match from the possibilities in the reduction matrix
- Characteristics measured
  1. number of notes
  2. consistency of voices
  3. ratio of durations
  4. order of durations
  5. syncopation
  6. harmonic support

Number of Notes



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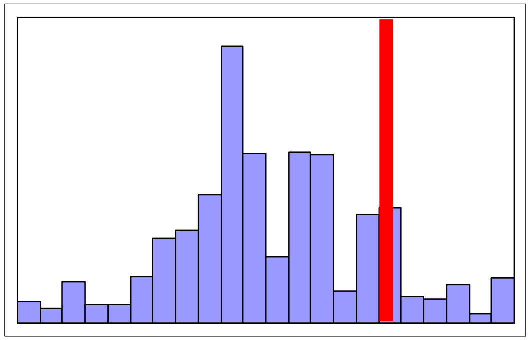

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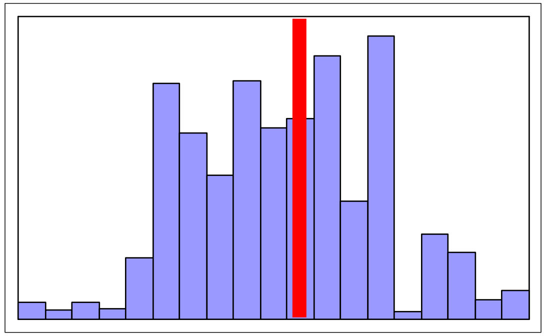

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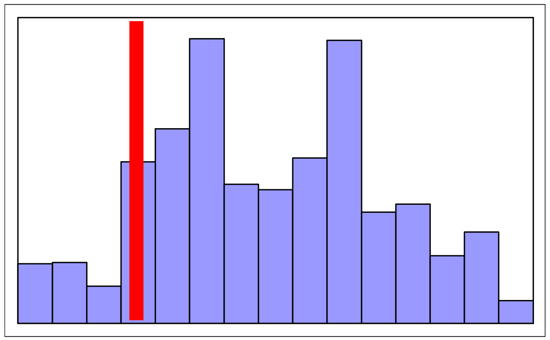

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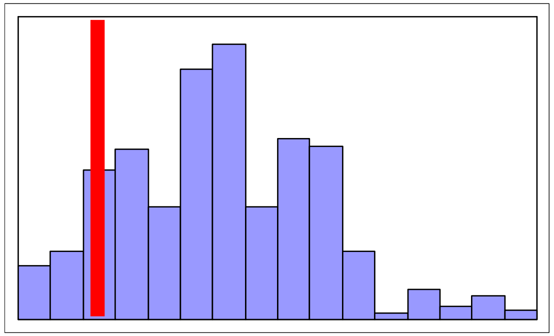

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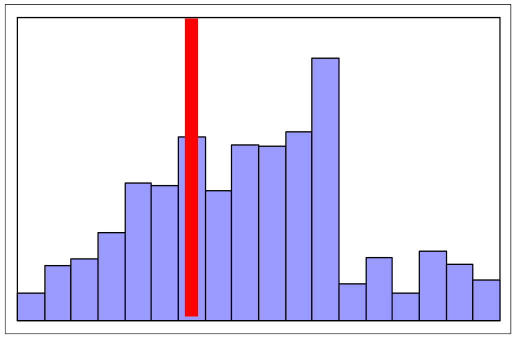

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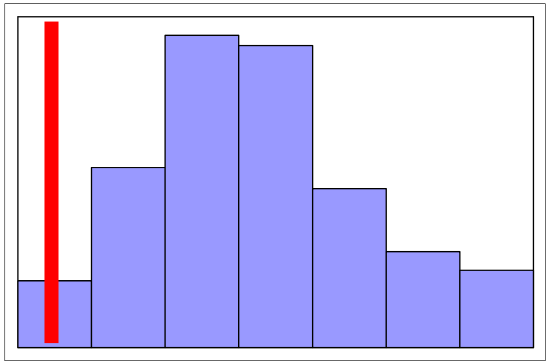
Number of Reductions with Fewer Voices



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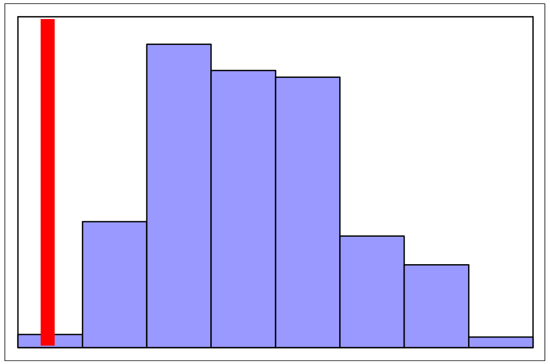

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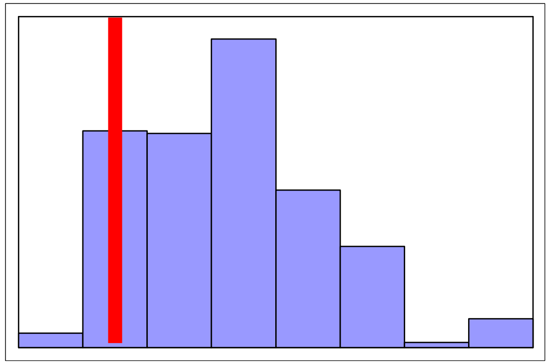

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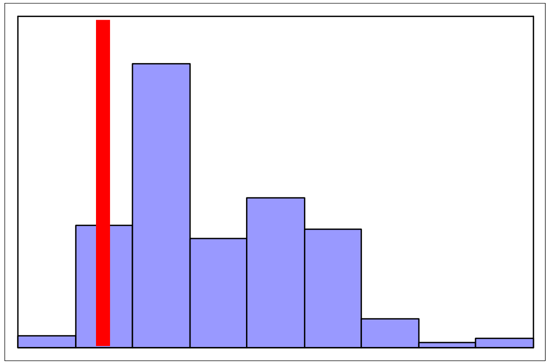

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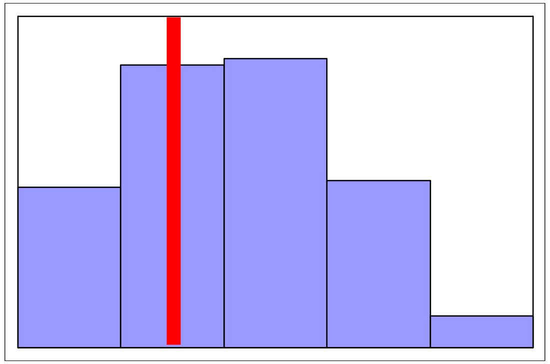

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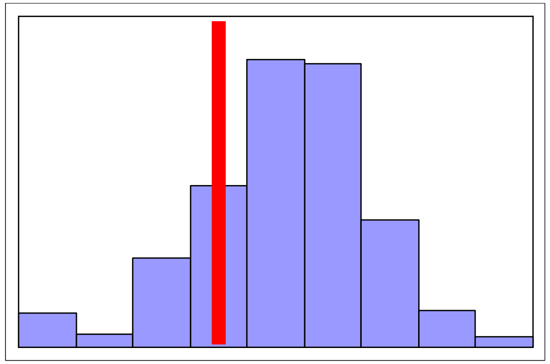
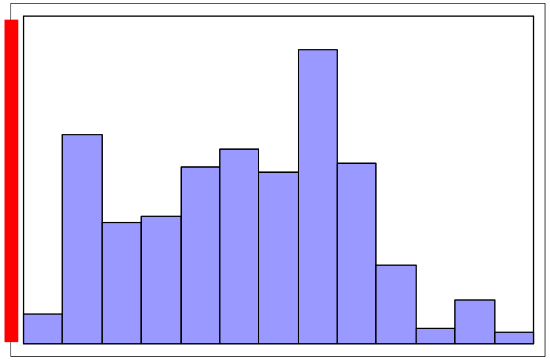


4

Ratio of Durations



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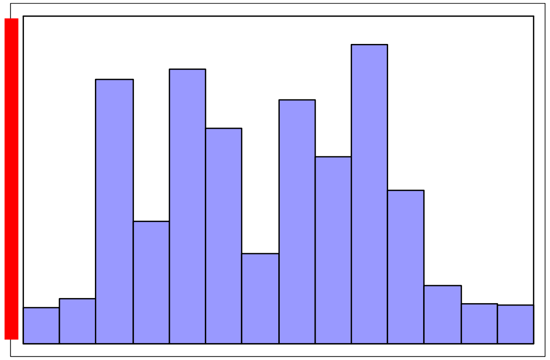

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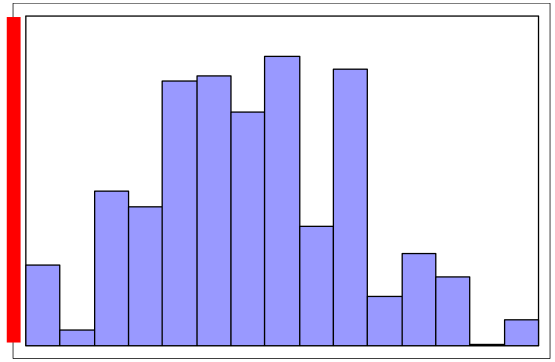

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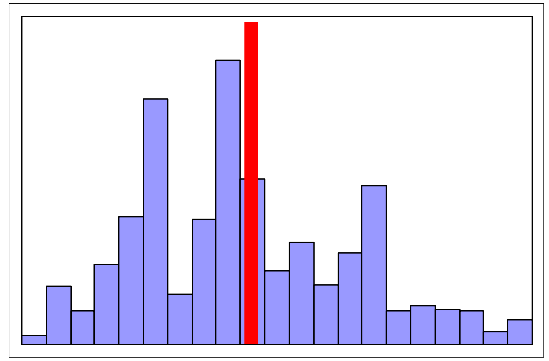

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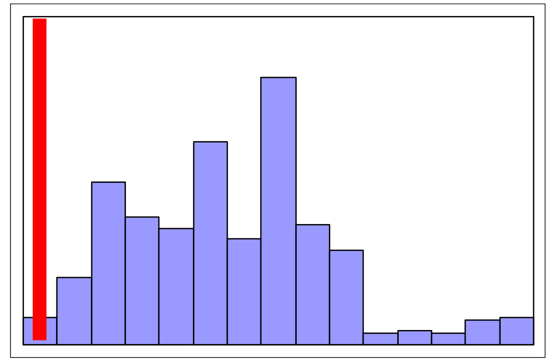

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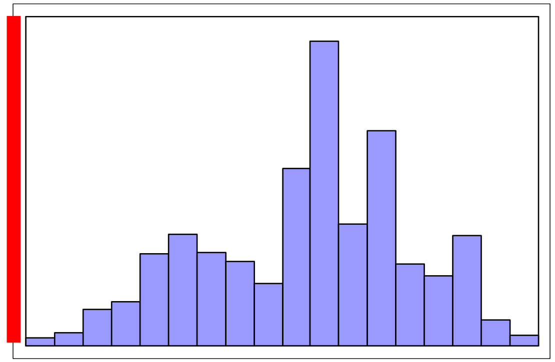

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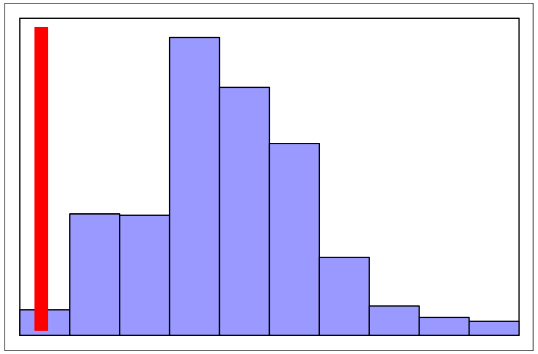
Number of Short-Long Reductions



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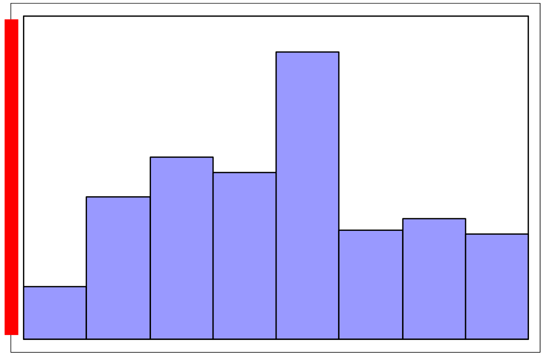

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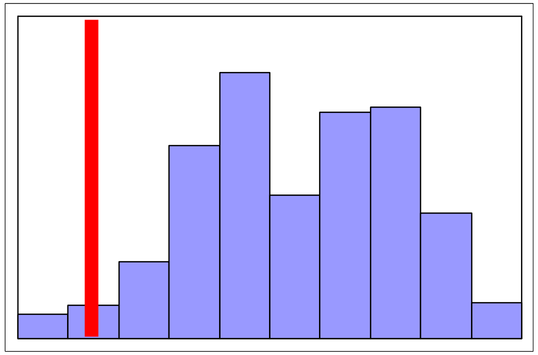

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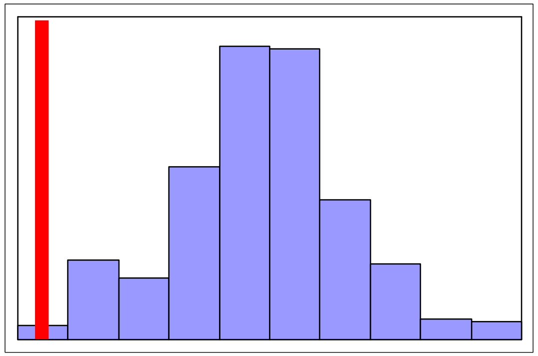

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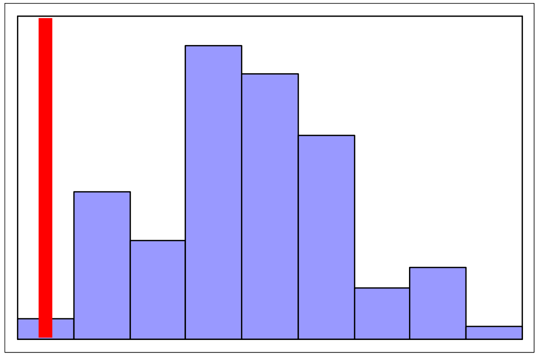

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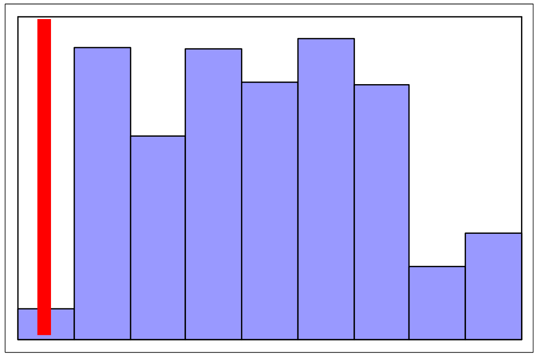

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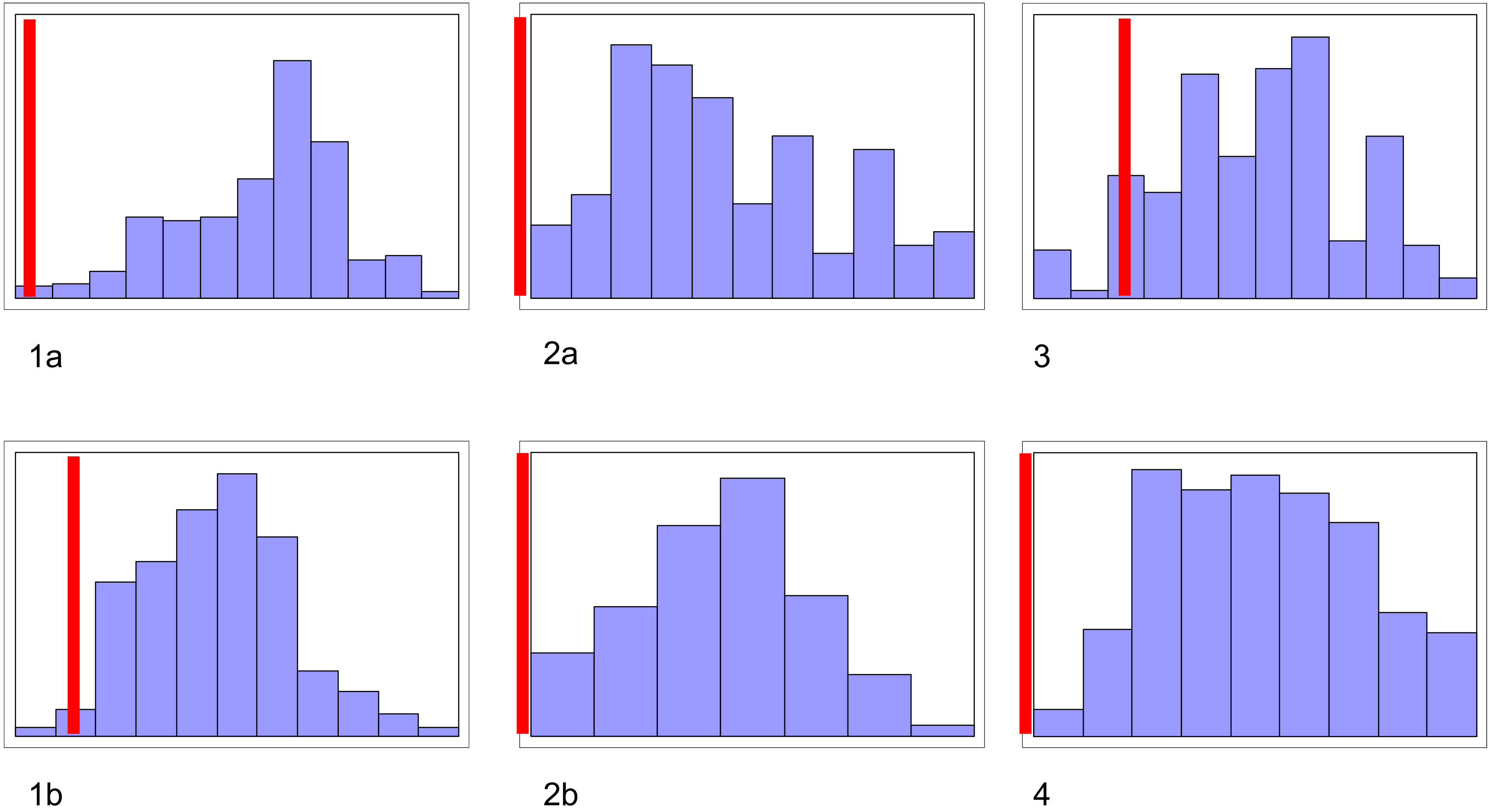
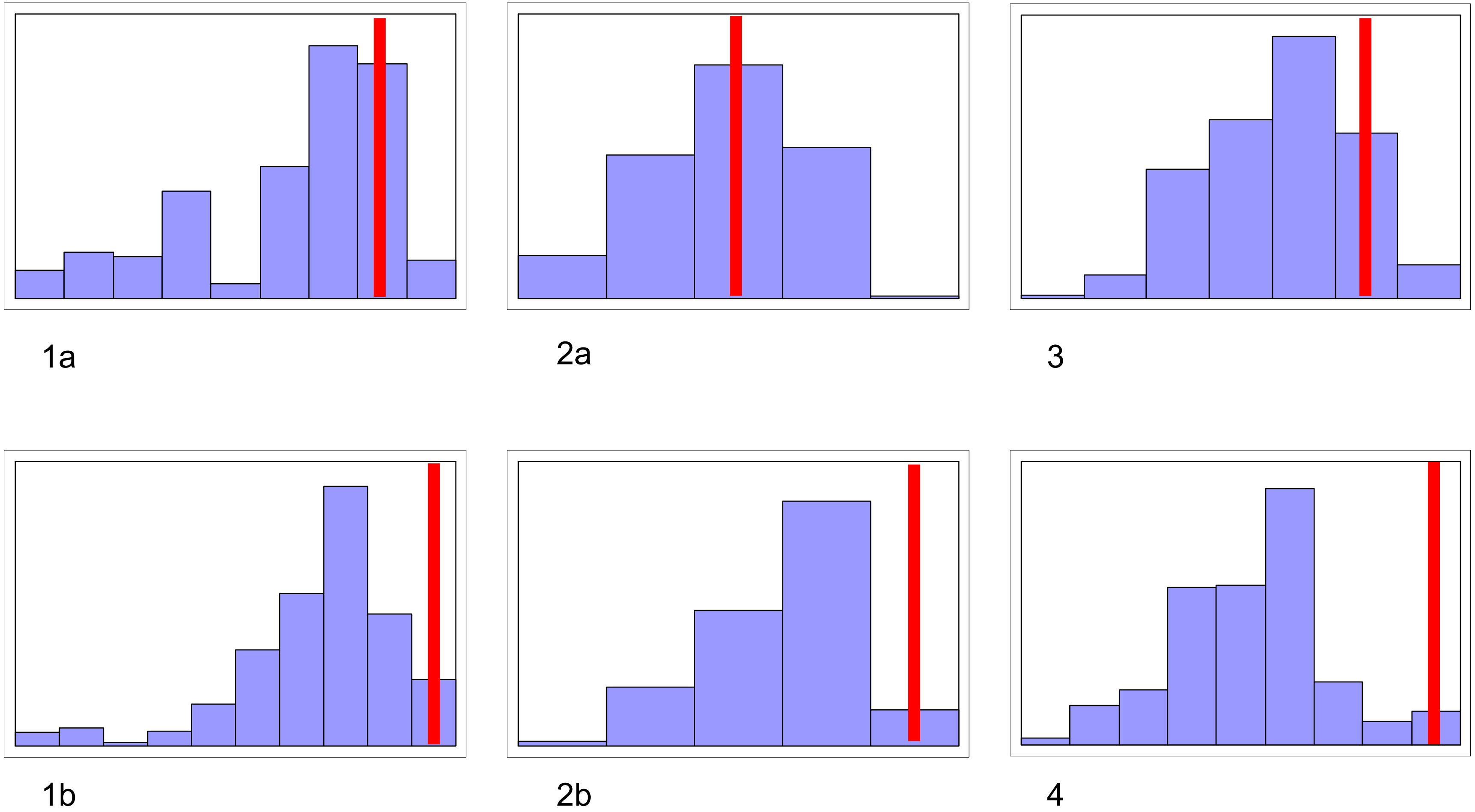
Number of Syncopations



Harmonic Support



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6. Possible Criteria

- Prefer reductions with
  - few syncopations
  - few short-long reductions
  - few reductions in the number of voices
  - low duration ratios
  - high harmonic support

Further Work

- Incorporation of the most obvious selection criteria to prune derivation
- Experimentation on search procedures (with Geraint Wiggins)
- Testing for derivation of published analyses
  - Oster archive (Chopin, Beethoven)
  - *Das Meisterwerk in der Musik*

Further detail at www.lancs.ac.uk/staff/marsdena/research/schenker

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