Product Grouping Methods for Markdown Pricing

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What is Markdown Pricing?

A markdown is a permanent reduction in price taken when a product in a store reaches the end of its lifetime.

Important Examples:

- Food in a supermarket approaching its expiry date
- Obselete Technology
- Out of fashion or out of season clothing

The Markdown Pricing Problem

Goal: Choose optimal series of price reductions to maximise revenue

For each markdown period:

1. Estimate demand at each possible sale price (price response function)
2. Choose price which would maximise revenue

The Problem with Markdown Data

Problem:

- Sales data only available for a small number of closely-spaced prices
- Difficult to fit a demand model for each product individually

Solution:

- Group different products together based upon price response functions for a non-markdown period
- Combine the markdown data for the different products in each group

Project Goals

Project Goal: develop ways to apply different clustering methods to the product grouping problem

Two approaches:

1. K-Means Clustering
2. Bayesian Hierarchical Clustering

Simulating the Data

Want to generate price and demand data for a number of fictitious products. To do this, use constant-elasticity price response function:

$$d(p) = Ap^e$$

- A = market-size
- e = elasticity

1. Generate values for e and A for distinct product groups
2. Use those to generate values of e and A for each individual product within a group
3. Use $d(p) = Ap^e + \epsilon$ to generate demand for range of prices

What is K-Means Clustering?

K-Means is a clustering method which groups a set of observations into a predefined number of clusters.

The mean of the observations in each group is the **centroid**.

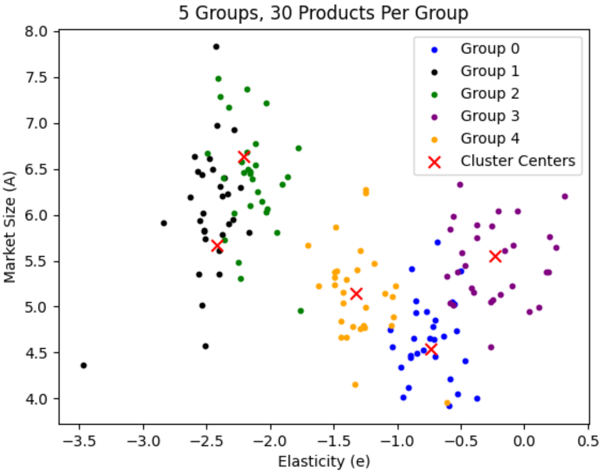
Objective: find centroid positions $\boldsymbol{\mu}_i$ which minimise:

$$\sum_{i=1}^{k} \sum_{\mathbf{x} \in S_i} ||\mathbf{x} - \boldsymbol{\mu}_i||^2$$

Applying K-Means to Product Grouping

1. Suppose $d(p) = Ap^e \implies log(d) = log(A) + elog(p)$
2. Perform linear regression on $log(d)$ and $log(p)$ data to estimate e and A for each product
3. Apply the K-Means Clustering Algorithm to the (e, A) observations

Results on Simulated Data



How can we choose optimal number of clusters?

Elbow Method:

- Plot within cluster sum of squares against number of clusters
- Choose the point at which the rate of decrease changes

Rand Index:

- Metric to compare predicted clustering to known truth:

  $\text{RI} = \frac{\text{TP}+\text{TN}}{\text{TP}+\text{TN}+\text{FP}+\text{FN}}$

- eg: true = (A,B,C), (D,E,F), predicted = (A,B,D), (C,E,F)
- Plot RI against number of clusters and choose maximum point

Comparison of Methods

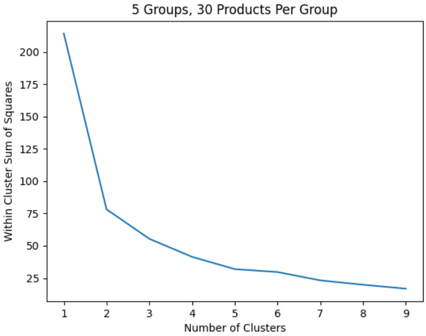


Figure: Elbow Method

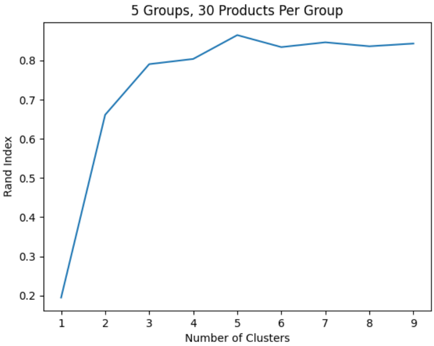


Figure: Rand Index

Limitations of K-Means Clustering

- Analysis requires us to assume a certain parametric form for the price-response function (eg constant-elasticity)
- Choosing the optimal number of clusters is difficult and subjective

Gaussian Process Regression

Gaussian Process: A non-parametric model which can be used in regression to define a probability distribution over functions.

GP Regression:

1. Prior:

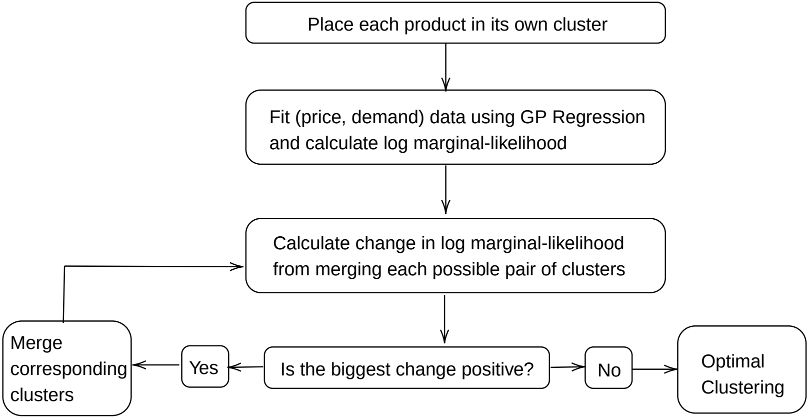
   $$p(f) \sim GP(\mu(x), k(x, x'))$$

2. Posterior:

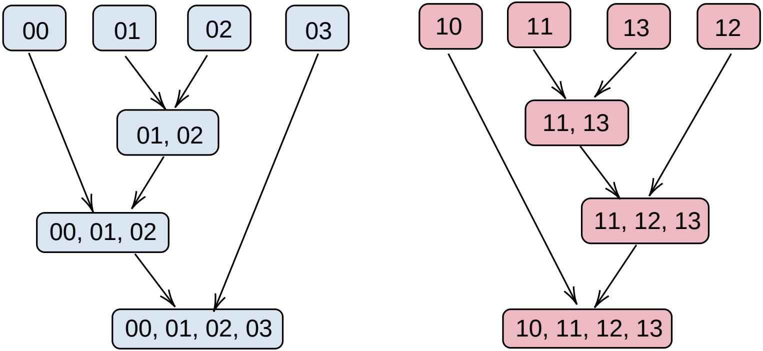
   $$p(f|y) = \frac{p(y|f)p(f)}{p(y)}$$

3. Maximise $\log p(y)$

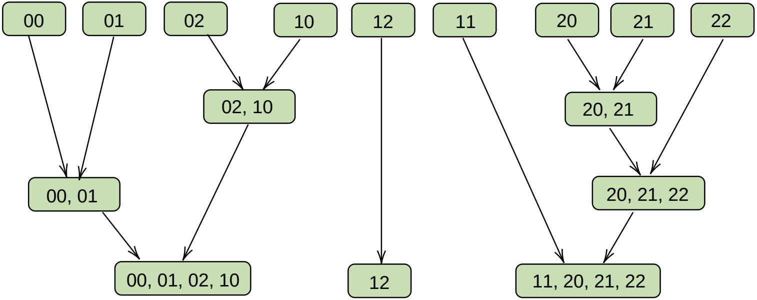
Bayesian Hierarchical Clustering Method



Results on Simulated Data



Results on Simulated Data (Continued)



Time Complexity

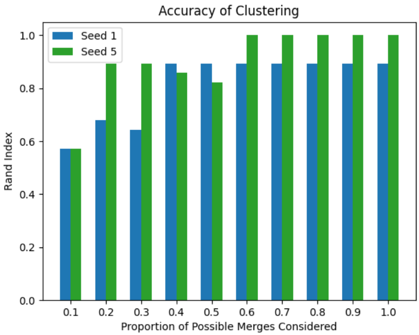
Problem: Time Complexity is $\mathcal{O}(D^3 N^3)$

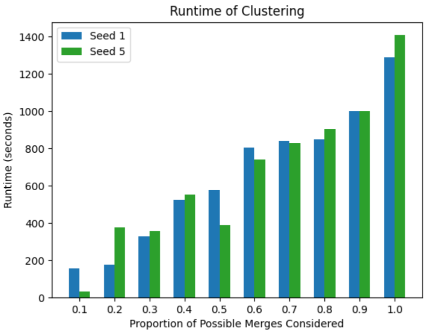
Potential Solution:

Take a random sample of possible merges at each step, eg:

- Suppose 2 Groups, 2 Products Per Group
  Clusters at start: [00], [01], [10], [11]
- Possible Merges at Start:
  (00,01), (00,10), (00,11), (01,10), (01,11), (10,11)
- Choose random sample of 50% of these merges to try:
  eg (00,01), (00,11), (10,11)

Experiment: 2 Groups, 4 Products Per Group





Further Work

- Extend K-Means Clustering method to different parametric models for demand (not constant-elasticity)
- Develop more sophisticated approximate methods to speed up hierarchical clustering
- Test hierarchical clustering on larger number of products
- Integrate product grouping methodology into rest of the markdown pricing process

Thank You for listening!

Any questions?

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