Stochastic Epidemic Modelling

STOR-i Summer Research Project

Evie Miller[1] James Neill[2]

[1]Physics Department
Lancaster University

[2]STOR-i Centre for Doctoral Training
Lancaster University

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Introduction

Coronavirus Map. Image Credit: VK Studio/Shutterstock.com

Aims

- Create a simulation of the general stochastic SIR epidemic and its extensions.
- Work on epidemic inference using Bayesian statistics
- Design a model for a ward to include spatial structure

SIR Model

The population is divided into three categories:

- **Susceptible (S)** – Individuals who are not yet infected.
- **Infectious (I)** – Individuals who are currently infected with the pathogen and can transmit it to others.
- **Recovered (R)** – Individuals previously infected with the pathogen, but no longer transmit it, and are now immune.

SIR Model Equations

- **Infection Rate (β)**: Determines how easily the disease spreads.

$$\mathbb{P}(\text{Infection}) = 1 - \exp(-\beta I(t-1))$$

- **Recovery Rate (γ)**: Determines recovery time of infected individuals.

$$\mathbb{P}(\text{Recovery}) = 1 - \exp(-\gamma)$$

- **Basic Reproduction Number (R_0)**

$$R_0 = \frac{\beta}{\gamma}$$

SIR Plot

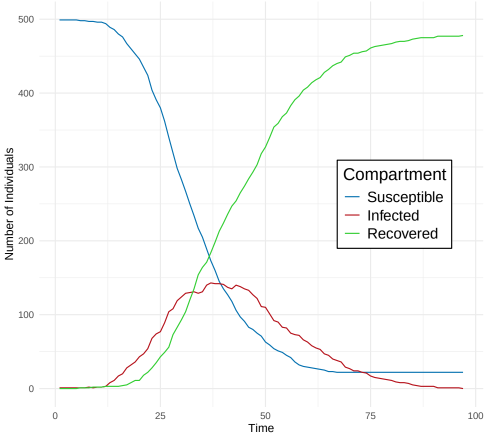
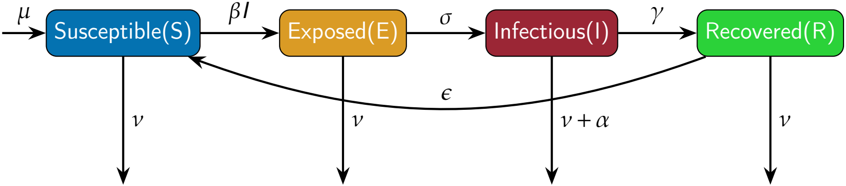


Figure: Stochastic SIR Model with Individual Tracking

Extensions of the SIR Model - SEIRS

The SEIRS model extends the SIR model and exhibits periodicity due to waning immunity over time. It also introduces a latency period where an individual carries the pathogen but is not infectious.



SEIRS Plot with a Varying Population

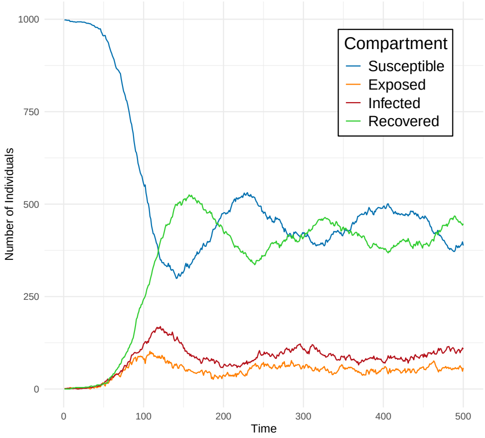


Figure: Stochastic SEIRS Model with Birth and Death rates

Inference with Random-Walk Metropolis MCMC

If given infection time data, Markov chain Monte Carlo (MCMC) methods can be applied to infer the rates of infection and recovery.

Inference with Random-Walk Metropolis MCMC

To estimate β and γ, the Random-Walk Metropolis algorithm is applied:

- **Proposal:** New β and γ values proposed from normal distributions centered at current values, with standard deviations λ_β and λ_γ.
- **Acceptance:** A proposed value is accepted based on:

$$\log(\alpha_\theta) = \log \Pi(\theta_{\text{proposed}}|X) - \log \Pi(\theta_{\text{current}}|X)$$

  where θ is β or γ, and $\Pi(\theta|X)$ is the posterior distribution of θ.
- **Adaptive Tuning:** Every 100 iterations, λ_β and λ_γ are adjusted to maintain a 30% acceptance rate.

Trace Plots and Histograms

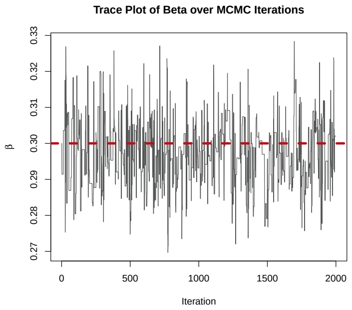


Figure: Beta Trace Plot

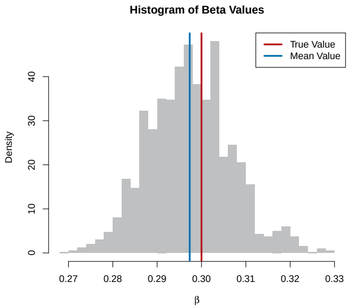


Figure: Beta Histogram

Trace Plots and Histograms

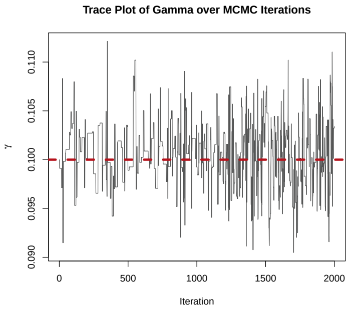


Figure: Gamma Trace Plot

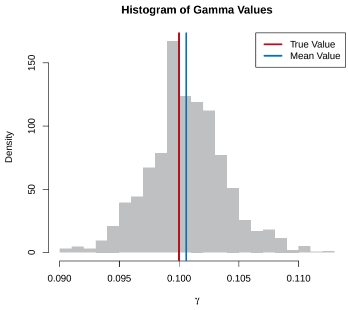


Figure: Gamma Histogram

Ward Layout

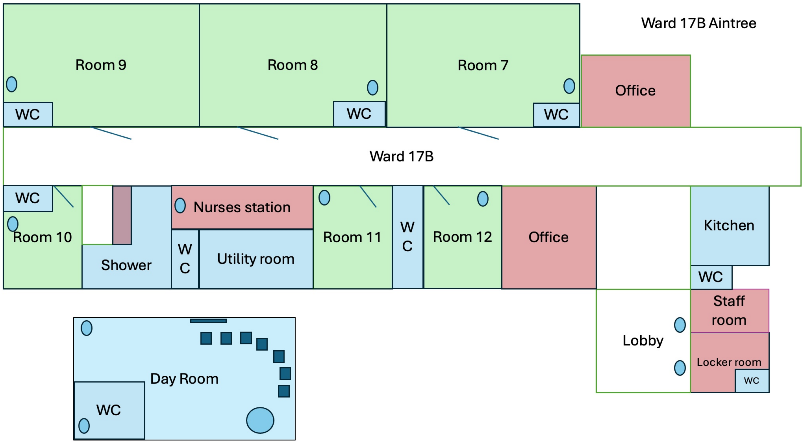


Figure: Layout of Aintree ward 17B

Calculation of Infectious Pressure with Distance

$$\lambda(t)_j = \underbrace{\beta_1 \sum_{i=1}^{N} I(t)_i f(D_{ij})}_{\text{Whole Ward}} + \underbrace{\beta_2 \sum_{i=1}^{N} r_{ij} I_i(t)}_{\text{Individual Room}}$$

- $\lambda(t)$ = infection rate (pressure)
- β = infection parameter
- $I(t)$ = vector of infection status
- $f(D)$ = distance function
- r = indicator if in the same room

Application to Aintree ward 17B

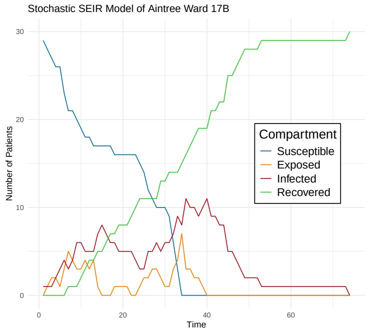


Figure: SEIR Model of Aintree ward 17B with random room allocation, assuming 30 patients.

Figure: Time of infection against Euclidean distance from room of the initially infected individual.

Model Summary

- Discrete Time
- Stochastic
- Markov Chain
- Agent-Based
- Spatial Structure

Conclusions

- **Stochastic epidemic models** provide insight into how infectious diseases spread in various settings.
- The **Random-Walk Metropolis MCMC algorithm** enables estimation of model parameters, such as infection and recovery rates, from observed data.
- The **application to Aintree Ward 17B** demonstrates the utility of these models in hospital settings, aiding in understanding and controlling outbreaks.

References

[1] Held, L., Hens, N., O'Neill, P., Wallinga, J. (2019). *Handbook of Infectious Disease Data Analysis*

[2] Hospital records, Ward 17B at Aintree Hospital.

[3] Gelman, A., Gilks, W. R., and Roberts, G. O. (1997). *Weak convergence and optimal scaling of random walk Metropolis algorithms*

[4] Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., and Teller, E. (1953). *Equation of state calculations by fast computing machines*

[5] Hastings, W. K. (1970). *Monte Carlo sampling methods using Markov chains and their applications*