



**Lancaster University**  
MANAGEMENT SCHOOL

**Lancaster University Management School**  
**Working Paper**  
**2009/020**

**Real Exchange Rates and Time-Varying Trade Costs**

E Pavlidis, Ivan Paya and David Peel

The Department of Economics  
Lancaster University Management School  
Lancaster LA1 4YX  
UK

© E Pavlidis, Ivan Paya and David Peel  
All rights reserved. Short sections of text, not to exceed  
two paragraphs, may be quoted without explicit permission,  
provided that full acknowledgement is given.

The LUMS Working Papers series can be accessed at <http://www.lums.lancs.ac.uk/publications/>  
LUMS home page: <http://www.lums.lancs.ac.uk/>

# Real Exchange Rates and Time-Varying Trade Costs

Efthymios G. Pavlidis Ivan Paya\* David A. Peel

Lancaster University Management School, Lancaster, LA1 4YX, UK

## Abstract

Previous empirical work on the Purchasing Power Parity does not explicitly account for time-varying trade costs. Motivated by the recent gravity literature we incorporate a micro-founded measure of trade costs into two nonlinear regression models for the real exchange rate. Using data for the dollar-sterling real exchange rate from 1830 to 2005, we provide significant evidence in favor of a positive relation between the level of trade costs and the degree of persistence of the real exchange rate.

**Keywords:** Real Exchange Rates, Time-Varying Trade Costs, Smooth Transition Nonlinearity, Persistence, Simulation Methods.

**JEL Classification:** F41, C22, C52

---

\*Correspondence to: Ivan Paya, Lancaster University Management School, Lancaster, LA1 4YX, UK. Tel: +44 (0) 1524 593504. Fax: +44 (0) 1524 594244 E-mail: i.paya@lancaster.ac.uk

# 1 Introduction

Trade costs can exhibit significant economic magnitudes and can play an essential role in addressing several major puzzles in international economics (Obstfeld and Rogoff, 2000; Anderson and van Wincoop, 2004). In the Purchasing Power Parity (PPP) framework, equilibrium models of real exchange rate determination demonstrate how trade costs induce nonlinear but mean reverting adjustment toward PPP and, hence, provide a possible explanation for the well-documented persistence in the real exchange rate (Dumas, 1992; O'Connell and Wei, 2002; Taylor and Taylor, 2004). For example, O'Connell and Wei (2002) extend the iceberg model of trade to allow for fixed as well as proportional costs of arbitrage. As a consequence, the tendency of the real exchange rate to return to the equilibrium rate will become apparent only for misalignments which cover the level of transactions costs and imply arbitrage opportunities. Small misalignments, close to equilibrium and within the transactions band, will be left uncorrected so that the real exchange rate will exhibit near unit root behavior.

In a number of empirical contributions trade costs are assumed constant and the implied type of nonlinear behavior of the real exchange rate is modeled by the Exponential Smooth Transition Autoregressive (ESTAR) model (see, e.g., Michael et al., 1997; Kilian and Taylor, 2003; Taylor, Peel and Sarno, 2001). However, it can be argued that this assumption is too restrictive over long time periods.<sup>1</sup> In a recent study, inspired by the gravity literature, Jacks et al. (2008) present an aggregate micro-founded model which allows the construction of long span trade costs series. The authors illustrate that trade costs related to the exchange of goods across countries, far from being constant, have exhibited substantial and nonmonotonic changes from 1870 to 2000.<sup>2</sup> This finding has potentially important implications concerning the behavior of the real exchange rate. Because

---

<sup>1</sup>Clemens and Williamson (2001) and Mohammed and Williamson (2004) among others illustrate that tariffs and global freight rates have fluctuated substantially in the last century. These studies focus on specific impediments of trade costs and, therefore, provide indirect evidence of time-varying trade costs. A survey on recent developments in the measurement of total trade costs and their components is provided by Anderson and van Wincoop (2004).

<sup>2</sup>Consequently, the effect of trade costs cannot be approximated by deterministic trends.

trade costs vary in time so does the speed of mean reversion for a given PPP deviation (see, e.g., Dumas, 1992; Sercu et al., 1995). Intuitively, when trade costs increase (decrease) the trade costs band—in which no trade takes place—widens (narrows) and the real exchange rate process becomes more (less) persistent. Hence, the persistence of the real exchange rate does not only depend on the size of the deviation but also on the level of trade costs at each particular point in time. Neglecting significant changes in trade costs leads to underestimating/overestimating the degree of persistence and the time required for the process to absorb shocks at specific periods.

The contribution of this paper is to report estimates and the properties of two smooth transition regression models of the real exchange rate which incorporate time-varying trade costs. The models are fitted to a long span of data (1830-2005) for the dollar-sterling real exchange rate and the trade costs index for the United Kingdom-United States country pair. Our choice is based on the fact that the relationship between trade frictions and the persistence of the real exchange rate should become apparent over long time periods in which large fluctuations of trade costs occur.

The rest of the paper is structured as follows. In Section 2, we present the trade costs measure of Jacks et al. (2008). Section 3 outlines our nonlinear models of the real exchange rate. Section 4 deals with the description of the data and the empirical results. A summary and concluding comments are offered in the last section.

## **2 Trade Costs**

“Trade costs, broadly defined, include all costs incurred in getting a good to a final user other than the marginal cost of producing the good itself” (Anderson and van Wincoop, 2004, p. 691). Obviously, trade costs break down into a vast number of components such as transportation costs (freight rates and time costs), policy barriers (tariffs and nontariff barriers), informational costs and costs associated with the use of different currencies. The fact that several of these components are unobservable and data limitations pose serious problems in obtaining accurate estimates of

the magnitude of total trade costs by direct atheoretical measures. The gravity literature circumvents this obstacle on the basis of theoretical models which enable measuring the degree of trade restrictiveness by extracting information from trade flows.

In this framework, Jacks et al. (2008) present a micro-founded measure of aggregate bilateral trade costs that captures trade frictions. The key idea in the derivation of their measure is that changes in trade barriers have an effect on both international and intranational trade. By establishing a relationship between countries' average international trade barriers and intranational trade, trade costs can be obtained directly from observable trade data without imposing a particular trade cost function (Novy, 2008).

Consider a world consisting of  $N$  countries and a continuum of differentiated goods. Anderson and van Wincoop (2003) derive the following gravity equation of international trade

$$x_{i,j} = \frac{y_i y_j}{y_w} \left( \frac{t_{i,j}}{\Pi_i P_j} \right)^{1-\sigma}, \quad (1)$$

where  $x_{i,j}$  are nominal exports from country  $i$  to  $j$ . Income levels of country  $i$ , country  $j$  and world income are denoted by  $y_i$ ,  $y_j$  and  $y_w$ , respectively. The elasticity of substitution,  $\sigma$ , is assumed to be constant and greater than unity. The cost of importing a good or, equivalently, the trade cost barrier (one plus the tariff equivalent) is  $t_{i,j} \geq 1$ . Finally, the price indices (or outward and inward multilateral resistance variables)  $\Pi_i$  and  $P_j$  for countries  $i$  and  $j$  represent the average trade restrictiveness of the countries. Novy (2008) uses Equation (1) to obtain a bidirectional gravity equation, which includes inward and outward multilateral resistance variables for both countries,

$$x_{i,j} x_{j,i} = \left( \frac{y_i y_j}{y_w} \right)^2 \left( \frac{t_{i,j} t_{j,i}}{\Pi_i P_j \Pi_j P_i} \right)^{1-\sigma}. \quad (2)$$

In turn, the author makes use of the fact that intranational trade, like international trade, depends on the magnitude of trade barriers,  $x_{i,i} = ((y_i y_i)/y_w)(t_{i,i})/(\Pi_i P_i)^{1-\sigma}$ , so as to control for multilateral

resistance. Substituting into the bidirectional gravity equation yields

$$x_{i,j}x_{j,i} = x_{i,i}x_{j,j} \left( \frac{t_{i,j}t_{j,i}}{t_{i,i}t_{j,j}} \right)^{1-\sigma}. \quad (3)$$

The geometric average of the tariff equivalent can now be obtained by

$$\tau \equiv \left( \frac{t_{i,j}t_{j,i}}{t_{i,i}t_{j,j}} \right)^{\frac{1}{2}} - 1 = \left( \frac{x_{i,i}x_{j,j}}{x_{i,j}x_{j,i}} \right)^{\frac{1}{2(\sigma-1)}} - 1. \quad (4)$$

The above equation states that a drop in trade flows between countries with respect to trade flows within countries is associated with higher trade costs. Note that the micro-founded measure evaluates bilateral trade costs against the domestic trade cost benchmark. Further, it enables the construction of long span trade costs series since its estimation only requires data for bilateral exports and intranational trade. The latter variable can be approximated by subtracting aggregate exports from a country's Gross Domestic Product (GDP) (Jacks et al., 2008).

### 3 Nonlinear Adjustment & Time-Varying Trade Costs

Let us define the log real exchange rate as  $q_t = s_t - p_t + p_t^*$ , where  $s_t$  is the logarithm of the spot exchange rate (the domestic price of foreign currency),  $p_t$  is the logarithm of the domestic price level and  $p_t^*$  the logarithm of the foreign price level.

#### 3.1 The ESTAR Model

A widely employed nonlinear econometric model that can capture the behavior of the real exchange rate in the presence of constant trade costs is the Exponential STAR (ESTAR) model advocated by Teräsvirta (1994). The appealing feature of the ESTAR model is that it allows transitions between a continuum of regimes to occur smoothly and symmetrically. In this setting, the speed of mean reversion is an increasing function of the size of the absolute deviation from equilibrium. This

property is suggested by the analysis of Dumas (1992) and demonstrated by Berka (2005). In addition, Teräsvirta (1994) argues that if an aggregated process is observed, regime changes may be smooth rather than discrete as long as heterogeneous agents do not act simultaneously even if they individually make dichotomous decisions. All the above favor the use of ESTAR models over Threshold Autoregressive (TAR) models, in which changes of persistence occur abruptly.<sup>3</sup>

A STAR model for the process  $\{q_t\}$  may be written as

$$q_t - \mu = \sum_{p=1}^{\bar{p}} \phi_p (q_{t-p} - \mu) G_j(\cdot) + \epsilon_t, \quad (5)$$

where  $\mu$  is a constant representing the long run equilibrium,  $\epsilon_t$  is a white noise process with mean 0 and variance  $\sigma_\epsilon$ , and  $G_j(\cdot)$  is the transition function. For a given AR structure,  $\sum_{p=1}^{\bar{p}} \phi_p$ , the transition function,  $G_j(\cdot)$ , specifies the degree of persistence of the real exchange rate at each point in time. In the presence of constant trade costs, the transition function for the ESTAR model is given by

$$G_1(q_{t-d}) = \exp(-\gamma^2 (q_{t-d} - \mu)^2). \quad (6)$$

where  $q_{t-d}$  is the transition variable and  $\gamma > 0$  is the smoothness (or transition) parameter. The exponential transition function  $G_1$  is particularly applicable because it implies symmetric adjustment for positive and negative deviations from the equilibrium. Furthermore, the speed of adjustment is increasing with the smoothness parameter  $\gamma$  and the absolute value of the past deviation from the equilibrium. For expositional reasons, we assume that  $\sum_{p=1}^{\bar{p}} \phi_p = 1$  throughout this section. In this case, at the equilibrium  $G_1(\cdot) = 1$  and the real exchange rate behaves as a unit root process,  $q_t = \sum_{p=1}^{\bar{p}} \phi_p (q_{t-p} - \mu) + \epsilon_t$ . Whilst, for nonzero deviations  $G_1(\cdot) \in [0, 1)$  and the process becomes mean reverting. Finally, if  $|q_{t-d} - \mu| \rightarrow \infty$  the function value approaches zero and the process is white noise,  $q_t = \epsilon_t$ . The speed of transition between regimes is specified by the smoothness parameter  $\gamma$ . If  $\gamma$  is equal to zero the real exchange rate behaves as a linear unit root process irre-

---

<sup>3</sup>Note also that the incorporation of trade costs in TAR models is not straightforward.

spectively of the regime. Whilst, if  $\gamma \rightarrow \infty$  the process becomes white noise. Intermediate values of  $\gamma$  imply smooth adjustment of the real exchange rate.

Let us consider two deviations from PPP which have the same size but occur at different time periods,  $|q_{t_1-d} - \mu| = |q_{t_2-d} - \mu| \neq 0$  with  $t_1 < t_2$ . The fact that  $\gamma$  is constant in the typical ESTAR model implies that the real exchange rate will exhibit the same degree of persistence at time  $t_1$  and  $t_2$ . Conditional on the assumption of constant trade costs this is an attractive property. However, if trade costs vary in time so will the speed of adjustment. An increase (decline) in trade costs,  $\tau$ , during the two time periods,  $\tau_{t_1-d} \neq \tau_{t_2-d}$ , will induce higher (lower) persistence of the real exchange rate and, therefore, a decrease (increase) of the  $\gamma$  parameter. Hence, time varying trade costs can be incorporated into Equation (6) by allowing  $\gamma$  to change over time depending on  $\tau_{t-d}$ . By assuming a linear relationship between the value of the smoothness parameter and trade costs, the transition function for the Time Varying Trade Costs ESTAR (TVTC-ESTAR) is given by

$$G_2(q_{t-d}, \tau_{t-d}) = \exp\left(-(\gamma - \gamma_\tau \tau_{t-d})^2 (q_{t-d} - \mu)^2\right), \quad (7)$$

where the coefficient,  $\gamma_\tau$ , on trade costs is greater than zero and  $\gamma \geq \gamma_\tau \tau_{t-d} \forall t$ . The above equation allows both the degree of trade restrictiveness and the size of the deviation from the equilibrium to determine the speed of adjustment of the real exchange rate at a particular point in time (see Figure 1).

### 3.2 The QLSTAR Model

An alternative model to the ESTAR that captures the theoretical insights of the authors above and allows us to parsimoniously encompass the influence of fixed and proportional time-varying trade costs is the Quadratic Logistic Smooth Transition Autoregressive (QLSTAR) model of Jansen and



Teräsvirta (1996). The transition function of the QLSTAR model is given by

$$G_3^*(q_{t-d}) = 1 - \left(1 + \exp\left(-\gamma^2(q_{t-d} + c_1)(q_{t-d} + c_2)\right)\right)^{-1}, \quad (8)$$

where  $c_1 = -\mu - c$  and  $c_2 = -\mu + c$  with  $c > 0$  are the band coefficients. The quadratic logistic transition function  $G_3^*(\cdot)$  is particularly applicable because it, as the exponential function, implies symmetric adjustment for positive and negative deviations from the equilibrium. Further, the QLSTAR model specified by Equation (8) can approximate ESTAR models but also nests three regime Threshold Autoregressive (TAR) models and linear AR models. In contrast to TAR and ESTAR models, the QLSTAR allows the type of adjustment (smooth or discrete) between regimes to be specified by the data and, at the same time, can approximate narrow and wide “bands of inaction”. Hence, the model allows for both fixed and proportional costs. Overall, the model is particularly applicable when one is agnostic about the range of the “band of inaction” and the type of transition.

Suppose that regime changes occur abruptly rather than gradually (see Sercu et al., 1995), which favors the use of TAR over ESTAR models. If  $\gamma \rightarrow \infty$  and  $q_{t-d} < c_1$  or  $q_{t-d} > c_2$  the transition function value equals zero and  $q_t$  becomes white noise. Whilst, inside the “band of inaction”,  $c_1 < q_{t-d} < c_2$ ,  $G_3^*(\cdot)$  equals one and  $q_t$  behaves as a unit root process. Note that an increase in trade costs will widen the “band of inaction” and, therefore, result in higher absolute values of the band coefficients,  $c_1$  and  $c_2$ . At the other extreme, when  $\gamma = 0$  the model becomes linear. For moderate values of  $\gamma$ , the QLSTAR model can approximate both ESTAR and TAR models. The speed of mean reversion increases with the absolute deviation from the equilibrium. If  $|q_{t-d} - \mu| \rightarrow \infty$  the process approaches the white noise regime (outer regime). Whilst, in the inner regime,  $q_{t-d} - \mu = 0$ , the degree of persistence is given by the maximum value of the transition function  $G_3^*$

$$G_3^*(\mu) = 1 - \left(1 + \exp\left(\gamma^2 c^2\right)\right)^{-1}, \quad (9)$$

which is determined by the transition parameter  $\gamma$  and the coefficient  $c$ . Consequently, changes in  $\gamma$  or  $c$  lead to different degrees of persistence at the equilibrium. Due to the fact that there is no *a priori* reason why changes in trade costs should alter the degree of persistence in the inner regime, we modify Equation (8) as follows

$$G_3(q_{t-d}) = 1 - \left( 1 + \exp \left( -\frac{\gamma^2}{c^2} (q_{t-d} + c_1)(q_{t-d} + c_2) \right) \right)^{-1}. \quad (10)$$

The maximum value of  $G_3(\cdot)$ , which again occurs at the equilibrium rate, is

$$G_3(\mu) = 1 - (1 + \exp(\gamma^2))^{-1}, \quad (11)$$

and is independent of the value of the band coefficient. The above modification enables the incorporation of time-varying trade costs in the QLSTAR model in a straightforward manner. The transition function for the Time-Varying Trade Costs QLSTAR (TVTC-QLSTAR) is given by

$$G_4(q_{t-d}, \tau_{t-d}) = \left[ 1 - \left( 1 + \exp \left( -\frac{\gamma^2}{(c + c_\tau \tau_{t-d})^2} (q_{t-d} + c_3)(q_{t-d} + c_4) \right) \right)^{-1} \right] + \epsilon_t, \quad (12)$$

where  $c_3 = -\mu - c - c_\tau \tau_{t-d}$  and  $c_4 = -\mu + c + c_\tau \tau_{t-d}$  with  $c_3 < c_4$  are the time-varying band coefficients,  $c$  is a positive constant,  $c_\tau \geq 0$  is the coefficient on trade costs  $\tau$ .<sup>4</sup> Controlling for  $\gamma$ , the speed of mean reversion decreases with the absolute value of the band coefficients  $c_1$  and  $c_2$ , and increases with the past deviation from the equilibrium rate (see Figure 1).<sup>5</sup> We examine the impact of trade costs on the speed of mean reversion of the real exchange rate in the next section.

---

<sup>4</sup>We have scaled the trade costs index so as to have a minimum value of zero. Consequently,  $c$  reflects the lowest level of trade costs in time.

<sup>5</sup>Note that dividing the smoothness parameter  $\gamma^2$  by  $(c + c_\tau \tau_{t-d})^2$  also implies that changes in the persistence of the process become more abrupt as  $\tau_{t-d}$  decreases. This behavior is in line with the presence of both fixed and proportional costs which move together in time (O'Connell and Wei, 2002).

## 4 Empirical Results

Our data set consists of annual observations for the dollar-sterling real exchange rate and the corresponding trade costs index from 1830 to 2005. For the construction of the real exchange rate we use the International Financial Statistics database to update the nominal exchange rate and the price indexes analyzed in Lothian and Taylor (1996). International trade data are obtained by Mitchell (2008b,a) and GDP series for the United States and the United Kingdom are taken from Officer (2008) and Johnston and Williamson (2008), respectively. Figure 2 shows the demeaned real exchange rate and the trade costs series. In line with Jacks et al. (2008), the latter exhibits significant fluctuations throughout the period. Specifically, until the beginning of the 20th century trade costs were relatively low. Subsequently, the war and interwar periods were associated with a remarkable increase of bilateral trade costs with respect to intranational domestic costs. During this time interval the series displays two peaks, the first in 1935 following the Great Depression, and the second in 1946 at the end of the second World War and the establishment of the Bretton Woods system. A gradual decline has occurred since then.

### FIGURE 2

After running a battery of linearity tests on the real exchange rate series, which indicate the presence of smooth transition nonlinearity, we examine whether trade costs are an important constituent of the nonlinear adjustment mechanism of the real exchange rate.<sup>6</sup> The results for the nonlinear models with constant and time-varying trade costs are reported in Table 1.<sup>7</sup> Overall, all

---

<sup>6</sup>Specifically, we employ the testing procedures proposed by Teräsvirta (1994), Harvey and Leybourne (2007), and Kapetanios et al. (2003). The first two are general procedures for testing linearity against smooth transition nonlinearity. The main difference between them lies in the fact that the null critical values for the test of Teräsvirta (1994) are based on the assumption of an  $I(0)$  process, whilst, the test of Harvey and Leybourne (2007) allows for both  $I(0)$  and  $I(1)$  processes. We find that the hypothesis of linearity can be rejected at the 5 and 10 percent significance levels, respectively. Finally, the test of Kapetanios et al. (2003) shows that the null hypothesis of a unit root in the real exchange rate against the alternative hypothesis of a globally stationary exponential smooth transition autoregressive process can be rejected at all conventional levels of significance. The results are available upon request.

<sup>7</sup>The models are fitted to the demeaned real exchange rate. The lag length of the autoregressive part and the variables which enter the transition function are specified on the basis of residual diagnostics and, subsequently, the statistical significance of the coefficients of the models. In the estimation procedure we impose the restriction  $\phi_1 = 1$ .

models provide a parsimonious fit to the real exchange rate. However, the incorporation of time-varying trade costs leads to a radically different adjustment process. The statistical significance of the coefficient  $\gamma_\tau$  and the band coefficient  $c_\tau$  of the TVTC-ESTAR and TVTC-QLSTAR models, respectively, indicates that movements in trade costs can help explain changes in the level of persistence of the real exchange rate.<sup>8</sup> An increase in trade costs widens the “band of inaction” and reduces the speed of mean reversion for a given PPP deviation.

### TABLE 1

Figure 3 displays the transition functions of the time-varying trade costs models for three representative time periods, namely 1900, 1950 and 2000, which correspond to relatively low, large and moderate levels of trade costs, respectively. At those time periods, for the TVTC-ESTAR model, a PPP deviation of 0.4, which is roughly the maximum realized deviation, would suggest that the real exchange rate behaves similar to an AR process with coefficient around 0.3, a near unit root and an AR process with coefficient around 0.5. For the TVTC-QLSTAR model, the same PPP deviation would suggest that the real exchange rate behaves similar to a white noise, a near unit root and an AR process with coefficient around 0.2. On the other hand, according to the estimated ESTAR and QLSTAR models with constant trade costs the real exchange rate would behave as an AR process with coefficient of about 0.7 and 0.5, respectively, at all points in time. It appears that the inability of ESTAR models to approximate a wide “band of inaction” results in finding substantially higher persistence for large deviations than that implied by the QLSTAR.

### FIGURE 3

---

This choice is based on the fact that the AR coefficient is not statistically different from unity in the estimated ESTAR models with constant and time-varying trade costs and in the TVTC-QLSTAR model. Further, the results for the unrestricted models are qualitatively the same. For the standard QLSTAR model imposing the restriction  $\phi_1 = 1$  allows convergence of the nonlinear least squares algorithm. Note that this restriction does not necessarily imply a unit root behavior of  $\{q_t\}$  in the inner regime when QLSTAR models are applied since the maximum value of the transition function may differ from unity.

<sup>8</sup>Paya and Peel (2006) emphasize that the high degree of persistence of both the dependent and explanatory variables (such as the trade costs series) that enter the transition function may give rise to a spurious regression problem. To this end, we report the bootstrap  $p$ -values for the coefficients on trade costs. The null Data Generating Process (DGP) in the simulation experiment is given by the fitted ESTAR and QLSTAR models.

Clearly, the assumption of constant trade costs can result in severe overestimation / underestimation of persistence. The difference between the degrees of persistence (as measured by the value of the transition function of the corresponding model) estimated by the time-varying and constant trade costs models are illustrated in Figure 4. Starting with the ESTAR model, overestimation due to the the exclusion of time-varying trade costs occurs with almost the same likelihood as underestimation (55 percent versus 45 percent of the times). On the contrary, the QLSTAR model with constant trade costs appears to underestimate the degree of persistence with respect to the TVTC-QLSTAR in most periods (85 percent of the cases). Overestimation occurs on rare occasions (15 percent of the time) which are usually associated with substantial differences in the speed of mean reversion.<sup>9</sup>

#### FIGURE 4

A natural question that arises in the nonlinear framework is how fast does the process adjust to deviations from the equilibrium under different trade costs levels. In order to examine the time profile of the impact of a shock on the future behavior of the series we adopt the Generalized Impulse Response Functions (GIRF) proposed by Koop et al. (1996). The GIRF is defined as the average difference between two realizations of the stochastic process,  $q_{t+h}$ , which start with identical histories up to time  $t - 1$ , but only the first realization is hit by a shock of magnitude  $\delta_t$  at period  $t$ .

$$\text{GIRF}(h, \delta_t, \omega_{t-1}) = E [q_{t+h} | \epsilon_t = \delta_t, \omega_{t-1}] - E [q_{t+h} | \omega_{t-1}], \quad (13)$$

where  $h = 1, 2, \dots$  denotes horizon,  $\epsilon_t = \delta_t$  is an arbitrary shock occurring at time  $t$ , and  $\omega_{t-1}$  is the history set of  $q_t$ . Given that the  $\text{GIRF}(h, \delta, \omega_{t-1})$  is a function of  $\delta_t$  and  $\omega_{t-1}$ , which are realizations of random variables, the  $\text{GIRF}(h, \delta, \omega_{t-1})$  itself is a realization of a random variable. It follows that

---

<sup>9</sup>We note that the mean underestimation—the mean of the positive differences between the values of the transition function of the TVTC-ESTAR and the ESTAR—is 0.04 and the maximum value 0.24. While the mean overestimation—the mean of the negative differences between the values of the transition function of the TVTC-ESTAR and the ESTAR—is -0.07 and the minimum value is equal to -0.35. For the QLSTAR models, the mean underestimation is 0.04 and the maximum value 0.28. While the mean overestimation is -0.1 and the minimum value -0.48.

various conditional versions of the GIRF can be defined. In this work we set  $\omega_{t-1} = \mu$ , so that the process is initially at its equilibrium value, and we consider shocks of magnitude  $\delta$  equal to the maximum absolute PPP deviation and half the maximum PPP deviation. Due to the fact that analytic expressions for the conditional expectations involved in (13) are usually not available for  $h > 1$ , we use bootstrap integration methods (see Koop et al., 1996, for a detailed description) to overcome the issue of future shocks intrinsically incorporated in the model. In particular, 1000 repetitions are implemented to average out future shocks, where future shocks are drawn with replacement from the models residuals, and then the results are averaged.

#### FIGURE 5

Figure 5 illustrates the GIRFs for all levels of trade costs and for a maximum impulse response horizon of 20 years. Overall, low levels of trade costs are associated with fast shock absorption for all cases. The absorption time increases with the level of trade costs. For large shocks (maximum PPP deviation), the increase for the TVTC-ESTAR is substantially greater than for the TVTC-QLSTAR model and becomes apparent at a much lower level of trade costs. On the other hand, for moderate shocks (half the maximum PPP deviation), the absorption time for the TVTC-QLSTAR model initially grows faster as the degree of trade restrictiveness increases. However, this situation is reversed for high levels of trade costs. Generally, when the level of trade costs is high shocks fade out extremely slowly for the TVTC-ESTAR model. Put it differently, the transition parameter in the TVTC-ESTAR model approaches zero (infinite band width) falsely suggesting that the real exchange rate series is a unit root process.

To further illustrate this point as well as to make comparisons with the standard STAR models, we compute the half-lives corresponding to the maximum PPP deviation.<sup>10</sup> The results are presented in Table 2. Starting with the standard ESTAR and QLSTAR models, the real exchange rate process would absorb half of the shock in four years. Turning to the time-varying trade costs

---

<sup>10</sup>The half-life is defined as to the minimum horizon beyond which the difference between the impulse responses at all longer horizons and the ultimate response is less than or equal to half of the difference between the initial impact and the ultimate response (van Dijk et al., 2007).

models, we consider three scenarios. Again, we set trade costs equal to their 1900, 1950 and 2000 levels. In the former and latter cases, both the TVTC-ESTAR and TVTC-QLSTAR models suggest that the time required for the process to absorb half of the maximum PPP deviation is only two years, which is half of that corresponding to constant trade costs. Obviously, large deviations of the real exchange rate appear to mean revert much faster (than that implied by the ESTAR and QLSTAR models) during the beginning of the 20th century and the recent floating period. On the contrary, the high level of trade costs around the middle of the 20th century leads to an increase in the half-life of the shock with respect to the constant trade costs benchmark. In particular, the TVTC-QLSTAR and TVTC-ESTAR models imply a half-life of 5 and 20 years, respectively. As above, the large discrepancy between the results of the two models can be attributed to the inability of the ESTAR model to capture the effect of wide “bands of inaction”.<sup>11</sup>

## TABLE 2

In order to examine which model is superior in terms of capturing the effect of time-varying trade costs, we conduct two bootstrap experiments. For each experiment, we employ either the estimated TVTC-QLSTAR or the TVTC-ESTAR model (reported in Table 1), the original trade costs series and the corresponding estimated residuals so as to generate  $B$  artificial samples of size 176.<sup>12</sup> In turn, we fit the alternative model to each artificial sample and compute the  $t$ -statistic for the coefficient on trade costs,  $\tilde{t}_b$ . This provides the empirical distributions for the  $t$ -statistics for  $\hat{\gamma}_\tau$  and  $\hat{c}_\tau$  under the null that the true DGP is given by the alternative model. The probability of obtaining a  $t$ -statistic as extreme as the original is

$$p_b = \frac{1}{B} \sum_{b=1}^B I(\tilde{t} \leq \tilde{t}_b),$$

---

<sup>11</sup>We note that when trade costs reach a maximum, which occurs in 1946, the corresponding half-lives are 12 and 57 years for the TVTC-QLSTAR and TVTC-ESTAR models, respectively.

<sup>12</sup>We set the number of generated samples  $B$  equal to 1000 and initialize the bootstrap DGP by using the first observations of the original real exchange rate series.

where  $I(A)$  is the indicator function, which takes the value of 1 if event  $A$  occurs and 0 otherwise, and  $\tilde{t}$  is the original  $t$ -statistic. When the DGP is the TVTC-ESTAR model, the probability of the  $t$ -statistic for  $\hat{c}_\tau$  exceeding 4.488 is only 13.8 percent. Whilst, when the DGP is given by the fitted TVTC-QLSTAR, there is a 52.1 percent probability that the value of the  $t$ -statistic for  $\hat{\gamma}_\tau$  is greater than 3.145. Hence, it is very likely to obtain a  $t$ -statistic for the coefficient on trade costs in the TVTC-ESTAR model as extreme as the original when the DGP is given by the estimated TVTC-QLSTAR model. However, the opposite is not true.

## 5 Conclusion

In empirical work on the dynamic behavior of the real exchange rates trade costs have typically been assumed constant. Essentially, arbitrage will commence, *ceteris paribus*, when it is profitable and PPP deviations are outside the transactions band. Motivated by the recent gravity literature we construct a long span trade costs index. Further, we develop and estimate two nonlinear models for the real exchange rate which incorporate time-varying trade costs. Our empirical approach is supported by a battery of statistical tests and simulation methods. Our results provide strong evidence in favor of a time-varying “band of inaction”, which widens with the level of trade costs. The persistence of the real exchange rate is found to depend on both the magnitude of trade frictions and the size of the deviation from PPP. For instance, a given shock to the real exchange rate would be absorbed at significantly different speeds in 1950 and 2000 due to the existence of different trade costs levels. Although trade costs appear to have declined substantially since the second World War, their magnitude is still significant. Consequently, our empirical results are also consistent with the documented high persistence of real exchange rates in the post-Bretton Woods era.



## References

- Anderson, James E. and Eric van Wincoop**, “Gravity with Gravititas: A Solution to the Border Puzzle,” *American Economic Review*, 2003, 93 (1), 170–192.
- **and** —, “Trade Costs,” *Journal of Economic Literature*, 2004, 42 (3), 691–751.
- Berka, Martin**, “General Equilibrium Model of Arbitrage Trade and Real Exchange Rate Persistence,” MPRA Paper 234, University Library of Munich, Germany 2005.
- Clemens, Michael A. and Jeffrey G. Williamson**, “A Tariff-Growth Paradox? Protection’s Impact the World Around 1875-1997,” Working Paper 8459, National Bureau of Economic Research 2001.
- Dumas, Bernard**, “Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World,” *Review of Financial Studies*, 1992, 5 (2), 153–80.
- Harvey, David I. and Stephen J. Leybourne**, “Testing for Time Series Linearity,” *Econometrics Journal*, 2007, 10 (1), 149–165.
- Jacks, David S., Christopher M. Meissner, and Dennis Novy**, “Trade Costs, 1870-2000,” *American Economic Review*, 2008, 98 (2), 529–34.
- Jansen, Eilev S. and Timo Teräsvirta**, “Testing Parameter Constancy and Super Exogeneity in Econometric Equations,” *Oxford Bulletin of Economics and Statistics*, 1996, 58 (4), 735–63.
- Johnston, Louis D. and Samuel H. Williamson**, “What Was the U.S. GDP Then?,” Measuring-Worth 2008.
- Kapetanios, George, Yongcheol Shin, and Andy Snell**, “Testing for a Unit Root in the Nonlinear STAR Framework,” *Journal of Econometrics*, 2003, 112 (2), 359–379.

- Kilian, Lutz and Mark P. Taylor**, “Why Is It So Difficult to Beat the Random Walk Forecast of Exchange Rates?,” *Journal of International Economics*, 2003, 60 (1), 85–107.
- Koop, Gary, Hashem M. Pesaran, and Simon M. Potter**, “Impulse Response Analysis in Non-linear Multivariate Models,” *Journal of Econometrics*, 1996, 74 (1), 119–147.
- Lothian, James R. and Mark P. Taylor**, “Real Exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries,” *Journal of Political Economy*, 1996, 104 (3), 488–509.
- Michael, Panos, Robert A. Nobay, and David A. Peel**, “Transactions Costs and Nonlinear Adjustment in Real Exchange Rates: An Empirical Investigation,” *Journal of Political Economy*, 1997, 105 (4), 862–79.
- Mitchell, Brian R.**, *International Historical Statistics: Americas 1750-2005*, New York: Palgrave Macmillan, 2008.
- , *International Historical Statistics: Europe 1750-2005*, New York: Palgrave Macmillan, 2008.
- Mohammed, Saif I. Shah and Jeffrey G. Williamson**, “Freight rates and productivity gains in British tramp shipping 1869-1950,” *Explorations in Economic History*, 2004, 41 (2), 172–203.
- Novy, Dennis**, “Gravity Redux : Measuring International Trade Costs with Panel Data,” The Warwick Economics Research Paper Series (TWERPS) 861, University of Warwick, Department of Economics 2008.
- Obstfeld, Maurice and Kenneth S. Rogoff**, “The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?,” Working Paper 7777, National Bureau of Economic Research 2000.
- O’Connell, Paul G. J. and Shang-Jin Wei**, “The Bigger They Are, the Harder They Fall: Retail Price Differences Across U.S. Cities,” *Journal of International Economics*, 2002, 56 (1), 21–53.

**Officer, Lawrence H.**, “What Was the U.K. GDP Then?,” *MeasuringWorth* 2008.

**Paya, Ivan and David A. Peel**, “A New Analysis of the Determinants of the Real Dollar-Sterling Exchange Rate: 1871-1994,” *Journal of Money, Credit & Banking*, 2006, 38 (8), 1971–1990.

**Sercu, Piet, Raman Uppal, and Cynthia Van Hulle**, “The Exchange Rate in the Presence of Transaction Costs: Implications for Tests of Purchasing Power Parity,” *Journal of Finance*, 1995, 50 (4), 1309–1319.

**Taylor, Alan M. and Mark P. Taylor**, “The Purchasing Power Parity Debate,” *Journal of Economic Perspectives*, 2004, 18 (4), 135–158.

**Taylor, Mark P., David A. Peel, and Lucio Sarno**, “Nonlinear Mean-Reversion in Real Exchange Rates: Toward a Solution to the Purchasing Power Parity Puzzles,” *International Economic Review*, 2001, 42 (4), 1015–1042.

**Teräsvirta, Timo**, “Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models,” *Journal of the American Statistical Association*, 1994, 89 (425), 208–218.

**van Dijk, Dick, Philip H. Franses, and Peter H. Boswijk**, “Absorption of Shocks in Nonlinear Autoregressive Models,” *Computational Statistics and Data Analysis*, 2007, 51 (9), 4206–4226.

Table 1: Estimated Nonlinear Real Exchange Rate Models

---

Panel A, ESTAR

---

$$\hat{q}_t + \frac{0.016}{(0.690)} = \left( q_{t-1} + \frac{0.016}{(0.690)} \right) \exp\left( -\frac{1.505^2}{(7.102)} \left( q_{t-1} + \frac{0.016}{(0.690)} \right)^2 \right).$$

$s = 0.064$ ;  $Q_1 = 0.140$  [0.062];  $Q_5 = -0.127$  [0.227];  $\text{ARCH}_1 = 0.557$  [0.456];  
 $\text{ARCH}_5 = 0.802$  [0.550].

---

Panel B, TVTC-ESTAR

---

$$\hat{q}_t - \frac{0.066}{(3.262)} = \left( q_{t-1} - \frac{0.066}{(3.262)} \right) \exp\left( -\left( \frac{3.552}{(5.130)} - \frac{5.324}{(3.145)} \tau_{t-2} \right)^2 \left( q_{t-2} - \frac{0.066}{(3.262)} \right)^2 \right).$$

$s = 0.063$ ;  $Q_1 = 0.035$  [0.642];  $Q_5 = -0.161$  [0.374];  $\text{ARCH}_1 = 1.538$  [0.217];  
 $\text{ARCH}_5 = 0.538$  [0.747].

---

Panel C, QLSTAR

---

$$\hat{q}_t + \frac{0.014}{(0.656)} = \left( q_{t-1} + \frac{0.014}{(0.656)} \right) \left[ 1 - \left( 1 + \exp\left( -\frac{1.829^2}{(6.700)} / \frac{0.402^2}{(5.853)} \left( q_{t-1} - 0.387 \right) \left( q_{t-1} + 0.416 \right) \right) \right)^{-1} \right].$$

$s = 0.064$ ;  $Q_1 = 0.141$  [0.061];  $Q_5 = -0.126$  [0.219];  $\text{ARCH}_1 = 0.535$  [0.465];  
 $\text{ARCH}_5 = 0.786$  [0.561].

---

Panel D, TVTC-QLSTAR

---

$$\hat{q}_t - \frac{0.059}{(4.064)} = \left( q_{t-1} - \frac{0.059}{(4.064)} \right) \left[ 1 - \left( 1 + \exp\left( -\frac{2.146^2}{(7.811)} / \left( \frac{0.172}{(6.929)} + \frac{0.587}{(4.488)} \tau_{t-2} \right)^2 \right) \right)^{-1} \right].$$

$$\times (q_{t-2} - 0.231 - \frac{0.587}{(4.488)} \tau_{t-2})(q_{t-2} + 0.1128 + \frac{0.587}{(4.488)} \tau_{t-2}))^{-1}].$$

[0.008]
[0.008]

$s = 0.063$ ;  $Q_1 = 0.020$  [0.787];  $Q_5 = -0.154$  [0.426];  $\text{ARCH}_1 = 0.667$  [0.411];  
 $\text{ARCH}_5 = 0.344$  [0.886].

---

Notes: Figures in parentheses and square brackets denote absolute  $t$ -statistics and  $p$ -values, respectively. The  $p$ -values for the coefficients on trade costs  $\hat{\gamma}_\tau$  and  $\hat{c}_\tau$  are obtained through a simulation exercise, where the bootstrap DGPs are the fitted ESTAR and QLSTAR models, respectively. For illustration purposes, we report the summation of the long run equilibrium estimate and the constant part of the band coefficients  $\hat{\mu} \pm \hat{c}$ .  $s$  is the standard error of the regression.  $Q_1$  and  $Q_5$  denote the Ljung-Box  $Q$ -statistic for serial correlation up to order 1 and 5, respectively.  $\text{ARCH}_1$  and  $\text{ARCH}_5$  denote the LM test statistic for conditional heteroskedasticity up to order 1 and 5, respectively.

Table 2: Half-Lives of the Nonlinear Real Exchange Rate Models

Trade Costs Level	ESTAR	QLSTAR	TVTC-ESTAR	TVTC-QLSTAR
1900	4	4	2	2
1950	4	4	12	5
2000	4	4	2	2

Notes: The size of the shock is set equal to the maximum PPP deviation. Half-lives are measured in years.

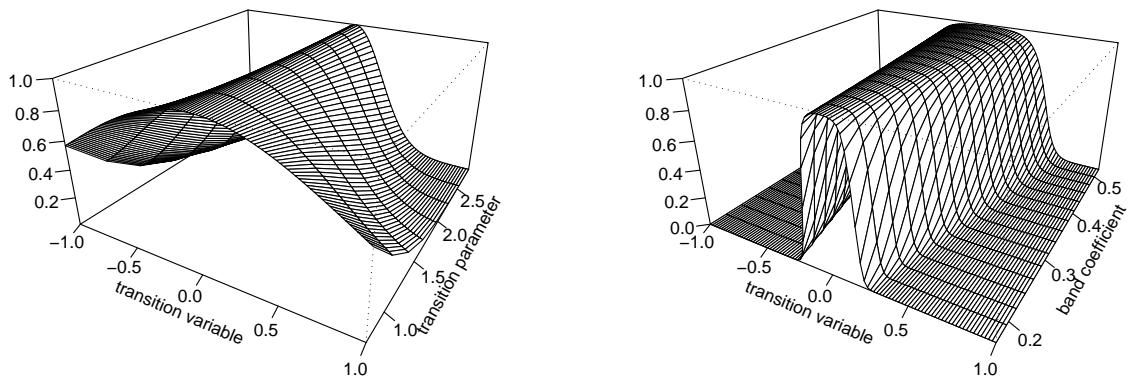


Figure 1: The exponential transition function (left) for  $0.75 \leq \gamma - \gamma_\tau \tau_{t-d} \leq 3$ ,  $q_{t-d} \in \{-1, \dots, 1\}$ , and  $\mu = 0$ . The quadratic logistic transition function (right) for  $\gamma = 2.146$ ,  $q_{t-d} \in \{-1, \dots, 1\}$ ,  $0.17 \leq c + c_\tau \tau_{t-d} \leq 0.52$ , and  $\mu = 0$ .

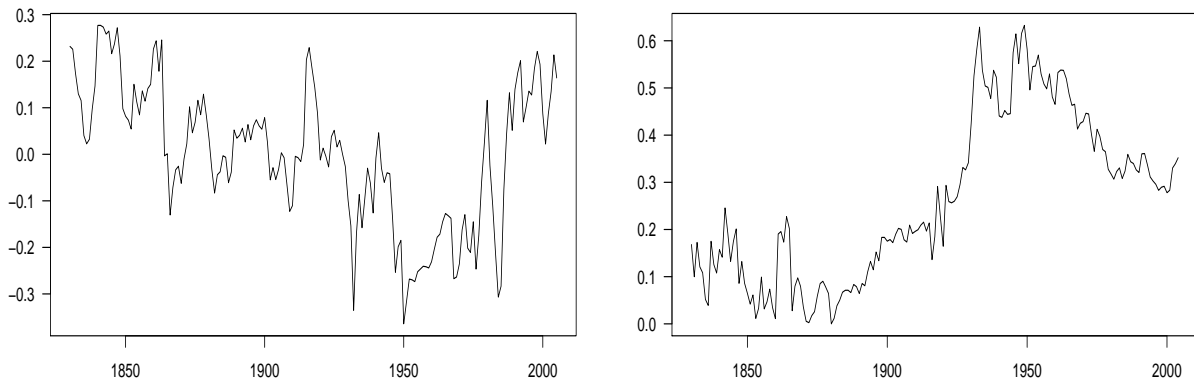


Figure 2: Time series plots of the demeaned dollar-sterling real exchange rate (left) and the United States-United Kingdom trade costs index (right).

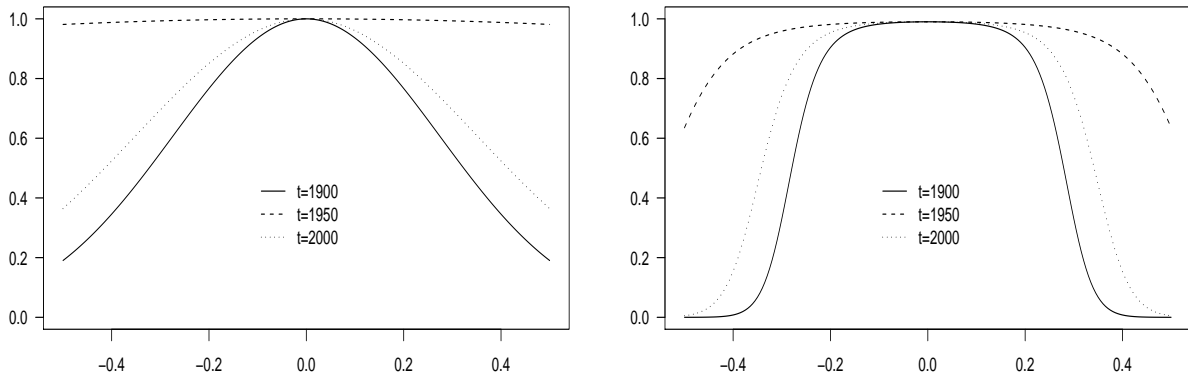


Figure 3: The exponential (left) and quadratic logistic (right) functions corresponding to 1900, 1950 and 2000 trade costs levels.

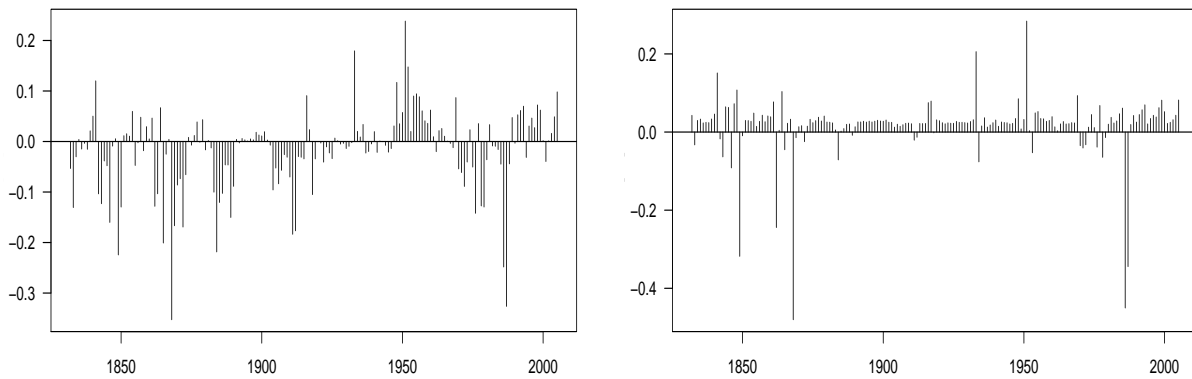


Figure 4: Differences in the degree of persistence between the TVTC-ESTAR and ESTAR models (left) and the degree of persistence between the TVTC-QLSTAR and QLSTAR models (right).

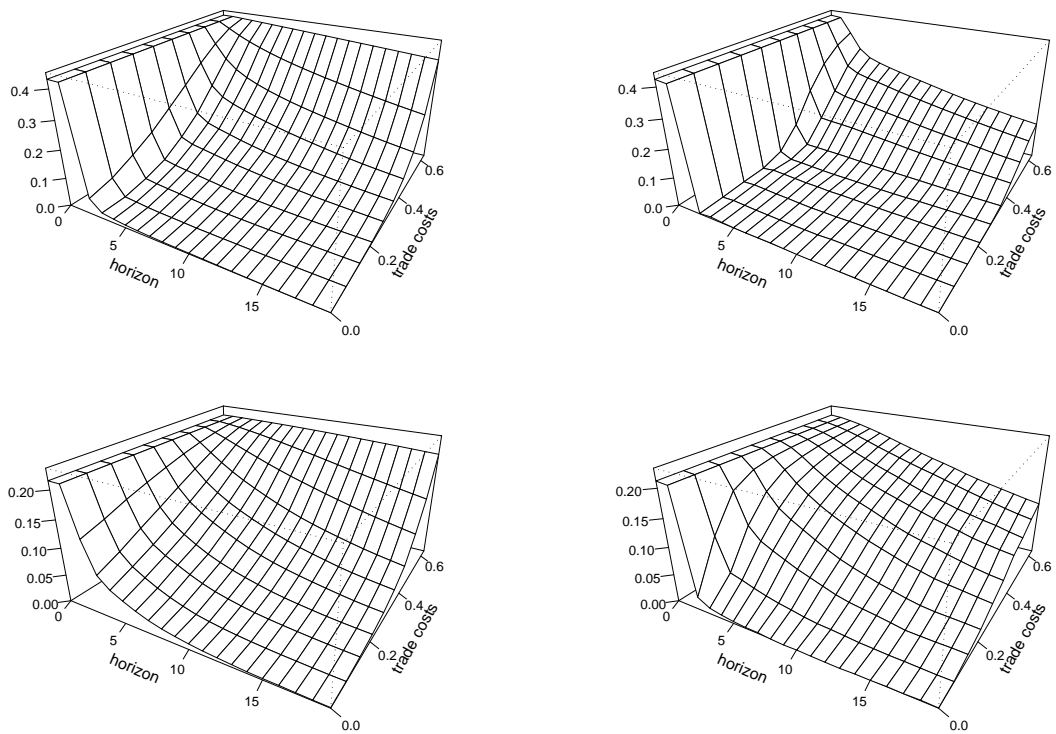


Figure 5: GIRFs for the TVTC-ESTAR (left) and TVTC-QLSTAR (right) models. Top (bottom) graphs correspond to shocks equal to the maximum absolute PPP deviation (half the maximum absolute PPP deviation).