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Abstract

This paper examines the dynamics of the linkages between Shanghai and Hong Kong stock indices. While the volatility linkage is analysed by a multivariate GARCH framework, the linkage of returns is examined using a copula approach. Eight different copula functions are applied in this study including two time-varying copulas which capture the time varying process of the linkage. The results show significant tail dependence of the returns in the two markets.

JEL classification: G10; G15; F35

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1 Introduction

Linkages between international asset returns are important for fund managers in order to diversify risk (Longin and Solnik, 1995), and also for policymakers to monitor the potential for financial contagion (Bae et al., 2003). There is a widespread agreement that international equity markets are linked to each other.[1] However, the strength, type and regional characteristics of those links are still under scrutiny. Recent evidence of financial contagion suggests that the contagion effect appears more pronounced in developing markets, such as Latin America and Asia (Bae et al., 2003).[2] The staggering increase in market capitalization of the Chinese stock market in the last fifteen years[3] has encouraged the analysis of this market and its links with other developed and developing markets.[4]

Poon and Fung (2000) analyzed spillover effects between Shangai and Hong Kong by fitting an AR(1)-EGARCH(1,1) model with lagged values of the other indices returns and volatilities in the mean and variance equations. Poon found significant volatility spillovers between the two stock markets. Similar result was found by Li (2007) employing a multivariate GARCH (MV-GARCH) model. Li (2007) investigated volatility linkages between China,

[1]For a survey of methods and results on the topic see Heimonen (2002).

[2]The methodology used in the majority of papers in this area is the conditional volatility GARCH framework introduced by Engle and Kroner (1995), Engle (2002) and Tse and Tsui (2002). Worthington and Higgs (2004) reported volatility spillovers from Hong Kong and Singapore to Thailand, and also from Japan to Indonesia, Korea, the Philippines and Thailand. However, it is worth pointing out that lagged domestic volatility had a stronger effect on current domestic volatility than the spillover effect. In a study on central European countries, Kasch-Haroutounian and Price (2001) found spillover effects from the Hungarian equity market to the Polish market, but not the other way round.

[3]The stock market valuation of the Chinese equity market was 260 billion Yuan in 1994 and 27 trillion Yuan in 2008.

[4]Studies on the Chinese market have found evidence of univariate GARCH(1,1) processes (see Yu (1996), and Xu (1999)). However, in the case of a price change limit in the stock market, Friedmann and Sanddorf-Kohle (2002) claimed that the most appropriate model is the MA(1)-GJR GARCH(1,1). Bailey (1994) used single linear equations to investigate linkages between eight individual stocks of the Chinese market. She found little or no correlation with China related shares in Hong Kong and the U.S. stock exchanges. Ma (1996) extended the dataset and analysed the market return within the CAPM framework. Ma found additional evidence of correlation with an international risk factor measured by the international beta from the CAPM model.

Hong Kong and the United States using daily data from January 2001 to August 2005 and found spillover effects from Hong Kong to Shanghai, but not between China and the U.S.

The results of Li (2007) employing the MVGARCH framework suggest significant spillovers between Hong Kong and China. However, a limitation of the MVGARCH family is that it assumes equal weight to small and large returns. This specification will not appropriately capture the differential impact, if it occurs, of abnormal movements due to panic selling which may cause large cross border co-movement (Longin and Solnik, 2001).[5] A method that is able to capture such behaviour is copulas. Copulas have been widely used in many disciplines, such as survival analysis and hydrology (Genest and Favre, 2007). It can also be found in many studies that examine the correlation between variables. Nonetheless, it is probably the use in finance that has accelerated the development of this methodology. Copulas started to be used in risk management, such as credit risk applications and option pricing (Cherubini et al., 2004), and now they are used in studying market co-movement and financial contagions (Rodriguez, 2007) .[6]

In this paper we extend the analysis of the linkages between the Shanghai and the Hong Kong equity markets in two ways. First, we employ a longer data set than previous studies in order to capture the recent falls in the Chinese market. Second, we analyse the dependence between the two markets employing both MVGARCH models and a wide variety of different copulas that allow flexible tail behaviour.[7] Our results suggest the lack of volatility spillover effects using the MVGARCH model. However, the time-varying

[5] In a recent study, Longin and Solnik (2001) applied extreme value theory where a multivariate distribution of stock returns tails was proposed and tested. Using monthly stock indices of the U.S., the U.K., France, Germany and Japan from 1959 to 1996 they found that the correlation of returns increases during bear markets.

[6] Bartram et al. (2007) used copulas to examine the effect of the Euro on the dependence between European stock indices. For a survey of the copula method in finance see Patton (2008).

[7] The only study we are aware of that employs copulas in the analysis of the Chinese stock markets is Ane et al. (2008). Ane et al. (2008) studied the relationship between the Chinese markets of Shanghai and Shenzhen exchanges using the Cook-Johnson (Clayton) copula over the period from January 1996 to December 2003. They found persistent features of dependence between Shanghai and Shenzhen stock returns.

SJC copula provided evidence of tail dependence between the Shanghai and the Hong Kong return series.

The rest of the paper is organised as follows. Section 2 will discuss the MVGARCH model. Section 3 will present the copulas and the methodology used for modeling marginal distributions. Data and its summary statistics will be presented in section 4. Section 5 will discuss the results of the copula and its implications. The last section will provide a brief conclusion.

2 Multivariate GARCH (MVGARCH)

We analyse the returns, R_t, of Shanghai and Hong Kong stock markets which are defined as the first difference of the natural logarithm of each stock index. We initially employ the multivariate GARCH model proposed by Engle and Kroner (1995) where the mean equation is specified as follows,

$$R_t = C + \Theta R_{t-1}^{'} + e_t \quad e_t \sim N(0, H_t) \tag{1}$$

where R_t is 2×1 vector. R_{11} is the Shanghai return series, and R_{21} is the Hong Kong return series. H_t is the variance-covariance matrix which is specified as follows,

$$H_t = \Omega\Omega^{'} + A(e_{t-1}e_{t-1}^{'})A' + BH_{t-1}B' \tag{2}$$

In the setting of the BEKK model, H_t is guaranteed to be positive definite by construction. The conditional variance is not only a function of all lagged conditional variances and squared returns, but also a function of conditional covariances and cross-product returns. The diagonal elements in the parameter matrix B measure the effect of lagged volatility, the off-diagonal elements capture the cross market effects.

3 Copula

3.1 Copula functions

A copula is a function that constructs a joint distribution from n marginal distributions. In other words, the copula contains all the dependence information between marginal distributions. The dependence information is only available in a copula, and not in the marginal distribution. This is proved by Sklar in 1959 (Cherubini et al., 2004) (see Appendix A). Sklar's theorem allows us to take advantage of flexible univariate modeling methods to obtain dependence information in a multivariate distribution.

We consider six bi-variate copula functions that have different dependence features. These fucntions are described in detail in Appendix B. The first two copula functions assemble normal distributions and do not consider the possibility of tail dependence. However, since financial time series usually have fat tails, the last four copula functions under consideration exhibit tail dependence features.[8] Tail dependence measures the joint probability of extreme events. In that case, it is possible to capture, for instance, greater correlation for large price movements than for small price movements. If the return of an investment is assumed to be a white noise process, small movements of returns represent low or no risk to investors. However, when there are large movements in returns, investors will be alerted to the risk. Moreover, some investors may be interested in hedging those risks.

3.2 Copula estimation

All the different types of copula functions considered here can be estimated by the two-stage maximum likelihood method proposed by Patton (2006a). The first stage involves the estimation of the two marginal distributions separately as described in the section below. The conditional marginal distributions are

[8]The dependence is measured by Spearmans's ρ or Kendall's τ statistics. Let us consider $u = F_1(x)$, and $v = F_2(y)$ to be the two marginal distributions. Note that the Gumbel copula exhibits upper tail dependence and zero lower tail dependence, while the Clayton copula has lower tail dependence and zero upper tail dependence. However, either the Student-t and the SJC copulas exhibit both tails dependence.

then used in the second stage of the methodology to estimate the copula function. In this case, the log-likelihood is function is,

$$L(\theta) = \sum_{t=1}^{T} \ln C_t(u, v|w) \tag{3}$$

where u and v are the marginal distributions defined in the Appendix.

3.2.1 Conditional Marginal Models

Conditional volatility GARCH models have been widely used to estimate return series as they can capture stylized facts such as voloatility clustering (Berkowizt and O´ Brien, 2002). The GARCH family of models has been extended to capture very persistent (or integrated) volatility with the IGARCH model. To relax such a restrictive case, equation (4) presents a third type of model that allows for volatility to be fractionally integrated (fractional IGARCH or FIGARCH),

$$h_t = \omega + \phi h_{t-1} + \varphi(1 - L)^d \varepsilon_t^2 \tag{4}$$

where $0 \leq d \leq 1$, where $d = 0$ represents the GARCH model and $d = 1$ the IGARCH. Given that the persistence of volatility is a stylised fact in financial series (Taylor, 1986), and espcially in emerging markets as shown by Ane et al. (2008), we specify an AR(1)-FIGARCH-m as the mean equation

$$r_t = \mu + \alpha r_{t-1} + \beta\sqrt{h_t} + \varepsilon_t \tag{5}$$

We specify Hansen's skewed-t distribution Hansen (1994) for the likelihood estimation, in order to capture skewness and leptokurtosis in the data.[9]

[9] The skewed student's-t distribution is more flexible than student-t distribution. Student-t density is the special case of Skewed Student-t when $\lambda = 0$.

The density function takes the following form,

$$f(z|\eta,\lambda) = \begin{cases} bc\left(1+\frac{1}{\eta-2}\left(\frac{bz+a}{1-\lambda}\right)^2\right)^{-(\eta+1)/2} & z < -a/b, \\ bc\left(1+\frac{1}{\eta-2}\left(\frac{bz+a}{1+\lambda}\right)^2\right)^{-(\eta+1)/2} & z \geq -a/b, \end{cases} \tag{6}$$

where $2 < \eta < \infty$, and $-1 < \lambda < 1$, z is the standardised residual, and the constant a, b, c are given by

$$a = 4\lambda c\left(\frac{\eta-2}{\eta-1}\right),$$

$$b^2 = 1 + 3\lambda^2 - a^2,$$

$$c = \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\pi(\eta-2)}\Gamma(\frac{\eta}{2})}$$

The models are estimated by maximumising the log-likelihood function of the skewed student-t distribution,

$$\begin{aligned} l =& n\log\Gamma(\frac{\eta+1}{2}) - \frac{n}{2}\log(\eta) - n\log(\Gamma(\frac{\eta}{2})) - n\log(\sigma) \\ &- \frac{\eta+1}{2}\sum_{i=1}^{n}\log(1+\frac{\varepsilon_i^2}{\eta\sigma^2}) + \sum_{i=1}^{n}\log(f(z_i|\eta,\lambda) \end{aligned} \tag{7}$$

The accuracy of the result obtained from a likelihood estimation relies heavily on the specification of the density function. In order to test that the density is correctly specified, we employ Kolmogorov-Smirnov (KS) test.[10]

[10]Kolmogorov-Smirnov (KS) test is a non-parametric goodness of fit test that tests if a set of data comes from the hypothesised continuous distribution. Thus, the test has a null hypothesis H_o: the data follows the specified distribution; and H_a: The data does not follow the specified distribution. The test statistic is,

$$D = max_{1\leqslant i\leqslant N}(F(Y_i) - \frac{i-1}{N}, \frac{i}{N} - F(Y_i))$$

4 The Data

Daily data for the Shanghai and Hong Kong composite stock indices are obtained from Thomson One Banker from June 1, 1996 to October 1, 2008. We use both indices denominated in Chinese Yuan. Observations in both series are removed if there is a missing value in one of the series due to holidays, as in Li (2007). As of January 2008, Shanghai stock exchange is the largest exchange in mainland with a market value of 22 trillion Yuan and 5.6 trillion of tradable volume, while Shenzhen exchange has a market value of 5 trillion and 2.5 trillion in tradable volume. There figures are based on A-shares which were initially only available to domestic investors but since the end of 2002 are also opened to foreign investors through the Qualified Foreign Institutional Investor (QFII) scheme. The other class of shares are called B-shares. These shares are also denominated in Yuan, but subscribed to trade in either US dollars or Hong Kong dollars. B-shares were only available to foreign investors before 2001, but since then domestic investors who have US dollar or Hong Kong dollar can trade B-shares as well.

The summary statistics of the returns, defined by changes in the logarithms of these indices times 100, are shown in Table 1. Shanghai series is negatively skewed, while Hong Kong series is positively skewed. Both are leptokurtic and have a very low first–order autocorrelation coefficient. Figures 1 and 2 plot the Shanghai and Hong Kong return series and it becomes apparent that the Shanghai index has more outliers than the Hong Kong counterpart.

5 Estimation results

5.1 MVGARCH model

We specify the mean and the variance equations as in (1) and (2) for the MVAGRCH model. The mean equation includes any possibility of mean

where $F(Y_i)$ is the theoretical cumulative distribution of the specified distribution. The statistics D is compared with tabulated critical values.

spillovers and we use the joint student-t density functions due to heavy tails in the return series.[11] We present the MVAGRCH results in Table 2. The BEKK model provides evidence of return linkages in the mean equation as Θ_{12} is significant. This means that return spillovers from Hong Kong to Shanghai, but not from Shanghai to Hong Kong given that Θ_{21} is insignificant. The positive coefficient implies that the return of Hong Kong is transmitted with the same sign to Shanghai in the next day, but not the other way round. It is worth pointing out that the linkage is weak as the spillover is only 4%. On the other hand, we didn't find any evidence of volatility linkages between the two markets as all the off-diagonal terms in the variance equation are insignificant. These conclusions are different from that of Li (2007) who found no linkage in return series, but found volatility spillover from Hong Kong to Shanghai. The difference might arise from the fact that our study uses a more updated data set that includes the most recent stock market crash.

5.2 Copula models

5.2.1 Result of Conditional Marginal Models

In this section we present the results of two conditional marginal models. First, the FIGARCH-m results are shown in table 3. Both return series display similar patterns. The value of the variance equation parameters are very close. The fractional parameter d is significant in both cases with values 0.59 for Shanghai and 0.57 for Hong Kong. The parameter φ is again similar and indicates a strong effect from past squared conditional returns. λ and η are the degrees of freedom and skewness parameters. The estimated value of these two parameters for Shanghai are very close to those found in Ane et al. (2008). The difference between Shanghai and Hong Kong is that in the latter case returns do not have volatility effect in the mean equation while in Shanghai investors would expect higher returns the higher the volatility.

[11] Initial values of the diagonal parameters are obtained from univariate model estimation and off-diagonal parameter initial values are set to zero.

5.2.2 Copula estimation

The conditional marginal densities estimated above are now used to estimate the copula functions and table presents the log-likelihood estimates are presented in table 4.[12] Considering first the constant parameter copula function we find that the Symmetrised Joe-Clayton (SJC) and the Student-t copulas appears to have the highest log-likelihood figures. Moreover, there seems to be an improvement in the log-likelihood when the parameters in the copula functions are allowed to change over time. In particular, the time-varying SJC copula is sigificanytly[13] improved over the constant SJC copula and implies that the dependence between the two markets changes over time.

To further examine the tail dependence between markets table 5 shows the estimates of the time-varying SJC copula. The coefficients indicate that there is different comovement at both the lower and upper tails. α and β are -7.18(-18.28) and -19.45(-14.27) at the lower(upper) tail and significantly different from zero. The fact that the parameters governing time-varying α and β are greater than the constant parameter ω implies strong time-varying effect. The implied correlation coefficient from this estimates is shown in Figure 3 and changes over time reaching a maximum of 0.5 in some cases. In this sense the copula approach provides additional information to the MVGARCH results by showing the dependence at the tails.

5.3 Robust analysis: The exchange rate effect

The data analysed are both dominated in Chinese currency. However, Hong Kong stock markets are traded in Hong Kong dollars. Thus, there exists a possibility of an exchange rate effect. In order to examine the exchange rate effect, we analyse the Hong Kong series in its native currency. Table 6 compares the summary statistic of Hong Kong index return with different currency denomination and they change. As a further check we estimate the copula functions using the Hong Kong series dominated in HK dollars and

[12]All copula functions are estimated in Matlab. The authors are grateful to Andrew Patton for making the codes publicly available.

[13]Indicated by likelihood ratio tests with 6 d.f. at 5% significance level

we do not find any significant change of results.[14]

6 Conclusion

In this article we have examined the linkages between the Shanghai and Hong Kong stock indices using two different methodologies. On the one hand, the MVGARCH model results suggest that there are spillover effects in the mean of returns but not in the volatility. The conditional marginal models estimated by FIGARCH with skewed student-t density showed that volatilities of the two indices returns are persistent. However, the parametric GARCH methodology does not allow for the dependence in volatility to differ according to the size and sign of changes in returns. In order to capture such effect we use six different copula functions and found evidence of volatility linkages at both tails. The Symmetrised Joe-Clayton Copula (SJC) was found to provide the better fit among all of them and we also estimate it using time-varying parameters. The results again imply significant tail dependence which has varied over time in the last decade.

[14] Results are availabe from the authors upon request.

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Appendix A. Sklar's Theorem

Let $F_1(x)$ be the marginal distribution of x, $F_2(y)$ be the marginal distribution of y, and $H(x,y)$ be the joint distribution of (x,y). Then for every $(x,y) \in \Re * \Re : C(F_1(x), F_2(y))$ is a joint distribution with margins $F_1(x), F_2(y)$

$$C(F_1(x), F_2(y)) = H(x,y) \tag{8}$$

conversely, if $H(x,y)$ is a joint distribution function with margins $F_1(x), F_2(y)$, there exists a copula C, such that

$$H(x,y) = C(F_1(x), F_2(y)) \tag{9}$$

the copula is unique if $F_1(x), F_2(y)$ are continuous, otherwise uniqueness is not guaranteed.

Appendix B. Copula Functions

B.1 Gaussian Copula

The Gaussian copula describes the bi-normal joint distribution which is the most basic distribution function in finance.

$$C_{nc}(u,v|\rho) = \frac{1}{\sqrt{1-\rho^2}} exp\{\frac{-(u^2+v^2-2\rho uv)}{2(1-\rho^2)} + \frac{1}{2}(u^2+v^2)\} \tag{10}$$

where the parameter ρ is the pairwise correlation. The Gaussian copula was used in Bartram et al. (2007) who investigated the dependence in European financial markets. They found a significant change of dependence structure in European equity market after the introduction of Euro.

B.2 Plackett copula

The Gaussian copula restricts ρ to be between 0 and 1. Hence it does not allow negative dependence. The Plackett copula (PC) is designed to relax this restriction.

$$C_{pc}(u,v|\theta) = \{1+(\theta-1)(u+v)-[(1+(\theta-1)(u+v))^2-4uv\theta(\theta-1)]^{1/2}\}\times[2(\theta-1)]^{-1} \tag{11}$$

where $\theta > 1$ implies positive dependence, $\theta < 1$ implies negative dependence and $\theta = 1$ implies independence. The dependence is described by Spearmans's ρ as in (Nelsen, 1998),

$$\rho = \begin{cases} \frac{\theta+1}{\theta-1} - \frac{2\theta}{(\theta-1)^2}\ln(\theta), & \text{if } \theta \neq 1; \\ 0, & \text{if } \theta = 1. \end{cases} \tag{12}$$

B.3 Gumbel Copula

The Gumbel copula exhibits upper tail dependence and zero lower tail dependence.

$$C_{gc}(u, v|\alpha) = \exp\{-[(-\ln u)^\alpha + (-\ln v)^\alpha]^{\frac{1}{\alpha}}\} \tag{13}$$

The upper tail dependence equals $2 - 2^{1/\alpha}$

B.4 Clayton Copula

Introduced by Clayton (1978) the Clayton copula is the opposite of the Gumbel copula. It has lower tail dependence and zero upper tail dependence.

$$\max[(u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha}, 0] \tag{14}$$

$$\tau = \alpha/(\alpha + 2)$$

The lower tail dependence equals $2^{-1/\alpha}$

B.5 Student's-t Copula

The previous two copula functions model one tail behaviour. Student's-t copula exhibits dependence in both tails and the dependence is symmetric.

$$C_{tc}(u, v) = \frac{\Gamma(\frac{\nu+2}{2})}{\Gamma(\frac{\nu}{2})\nu\pi\sqrt{1-\rho^2}} \left(1 + \frac{t^{-1}(u)^2 + t^{-1}(v)^2 - 2\rho t^{-1}(u)t^{-1}(v)}{\nu(1-\rho^2)}\right)^{-\frac{\nu+2}{2}} \tag{15}$$

where ν is the number of degrees of freedom. If ν becomes large Student's t copula becomes Gaussian copula.

B.6 Symmetrised Joe-Clayton Copula

Student's-t copula exhibits symmetric dependence in both tails. However, it might be reasonable to assume that the dependence will be different between bear and bull markets. In other words, the dependence could be asymmetric. Joe (1997) introduced the Joe-Clayton copula to capture such behaviour,

$$C_{JC}(u,v|\tau^U,\tau^L) = 1-(1-\{[1-(1-u)^\kappa]^{-\gamma}+[1-(1-v)^\kappa]^{-\gamma}-1\}^{-1/\gamma})^{1/\kappa} \quad (16)$$

where $\kappa = 1/log_2(2-\tau^U)$, $\gamma = -1/log_2(\tau^L)$ and $\tau^U \in (0,1)$ $\tau^L \in (0,1)$. τ^L and τ^U are the two parameters of this copula that measures lower and upper tail dependence, respectively. One small problem is that when τ^L and τ^U equal, there is still asymmetry. However, Patton (2006b) modified the Joe-Clayton copula and obtained the symmetrised Joe-Clayton copula,

$$C_{SJC}(u,v|\tau^U,\tau^L) = 0.5(C_{JC}(u,v|\tau^U,\tau^L)+C_{JC}(1-u,1-v|\tau^L,\tau^U)+u+v-1) \quad (17)$$

Copula functions are complex and it is not useful to plot a copula function. Instead, we plot the effect of the copula functions, that is, we plot the joint probability distribution against its marginal distributions. We show these plots from Figure 4 to 6. The plot shows different dependence structures implied by different copulas. For instance, the Clayton copula is showing the lower tail dependence while the Gumbel copula is showing upper tail dependence. A negative dependence is simulated by the Plackett copula. Student's t copula displays the symmetric tail dependence, whilethe SJC copula can have different tail dependence.

B.7 Conditional Copula

As previously discussed, the Sklar's theorem separates the joint distribution into n marginal distributions and a copula. This potentially provide the foundation to allow us to build on the success of univariate modeling methods to study multivariate distributions. This involves specifying a model for the marginal distributions and a copula. In economics and finance, many series are conditional on some other variables. An extension to conditional distribution of Sklar's theorem is therefore required and this is provided by Patton (2006b).

Let w be an information set, $F_1(x|w)$ be the conditional marginal distribution of x conditional on w , $F_2(y|w)$ be the conditional marginal distribution of y conditional on w, and $H(x, y|w)$ be the joint conditional distribution of (x, y) conditional on w. Then for every $(x, y) \in \Re * \Re : C(F_1(x), F_2(y))$ is a joint distribution with margins $F_1(x), F_2(y)$

$$C(F_1(x|w), F_2(y|w)|w) = H(x, y|w) \tag{18}$$

conversely, if $H(x, y|w)$ is a joint conditional distribution function with margins $F_1(x|w), F_2(y|w)$, there exists a copula C, such that

$$H(x, y|w) = C(F_1(x|w), F_2(y|w)|w) \tag{19}$$

the copula is unique if $F_1(x|w), F_2(y|w)$ are continuous, otherwise this is not guaranteed. Note that w is the same for both marginal distributions. In other words, the two marginal models need to be conditional on the same information set. Therefore, in empirical estimation, the same conditional variables should be used. However, conditional variables are allowed to be insignificant.

B.8 Time-varying copulas

The dependence structure may change over time due to changes in policy, such as changes in monetary rules (Sims and Zha, 2006) or the introduction of a common currency such as the Euro (Bartram et al., 2007; Patton, 2006b). Therefore, the copula may not be constant (Busetti and Harvey, 2007). There are two ways to handle a changing copula. One is to specify a mixture copula function. The difficulty of this option is that with so many copula functions, it is difficult to specify the mixture. The alternative way is to apply a time-varying copula that is a copula with time-varying parameters as in Patton (2006b). Equation (20)shows the time evolving parameter for the time-varying normal copula. τ_t^U and τ_t^U in equation (21) and (22) are the time-varying parameters for time-varying SJC copula

$$\rho_t = f\left(\omega + \beta\rho_{t-1} + \alpha\frac{1}{n}\sum_{j=1}^{n}\Phi^{-1}(u_{t-j})\Phi^{-1}(v_{t-j})\right) \tag{20}$$

where $f(x) = \tanh(\frac{x}{2}) = (1-e^{-x})(1+e^{-x})^{-1}$, this transformation will guarantee ρ to be in $(-1,1)$.

$$\tau_t^U = f\left(\omega^U + \beta^U\tau_{t-1}^U + \alpha^U\frac{1}{n}\sum_{j=1}^{n}|u_{t-j} - v_{t-j}|\right) \tag{21}$$

$$\tau_t^L = f\left(\omega^L + \beta^L\tau_{t-1}^L + \alpha^L\frac{1}{n}\sum_{j=1}^{n}|u_{t-j} - v_{t-j}|\right) \tag{22}$$

where $f(x) = (1+e^{-x})^{-1}$ is the logistic transformation, which will keep τ^U and τ^L in (0,1) at all times. $|u_{t-j} - v_{t-j}|$ is a forcing variable used by Patton (2006b) as an innovation term. Different forcing variables have been tried by Patton (2006b) and Bartram et al. (2007). and this one is the preferred one.

Table 1: Summary statistics

	Mean	Standard deviation	Skewness	Kurtosis	$AR(1)^a$	$AR(2)^a$
Shanghai	0.04	1.82	-0.21	7.81	0.009	0.005
Hong Kong	0.01	1.78	0.51	16.04	0.008	0.040

The table shows summary statistics of the returns of the Shanghai and Hong Kong stock market indices. Indices are denominated in Chinese Yuan. The sample period covers June 1, 1996 to October 1, 2008 and has 2878 daily observations in each series excluding holidays. [a] AR(i) represents the ith-lag autocorrelation coefficient of returns.

Table 2: BEKK estimation

	Shanghai			Hong Kong	
	Estimate	Standard Error		Estimate	Standard Error
C_{11}	0.039	0.026	C_{21}	0.047*	0.023
Θ_{11}	0.016	0.020	Θ_{21}	-0.007	0.014
Θ_{12}	0.042*	0.015	Θ_{22}	0.005	0.018
Ω_{11}	0.736*	0.040	Ω_{21}	0.503*	0.043
A_{11}	0.301*	0.031	A_{21}	-0.009	0.013
A_{12}	-0.010	0.017	A_{22}	0.192*	0.027
B_{11}	0.933*	0.014	B_{21}	0.007	0.005
B_{12}	0.002	0.004	B_{22}	0.975*	0.006
	Statistic	p-value		Statistic	p-value
Q(12)	48.73	0.00		24.51	0.22
$Q^2(12)$	7.08	0.93		41.10	0.01

* indicates significance at 5% level

Table 3: Result of conditional marginal models

	Shanghai		Hong Kong	
	Estimate	Standard Error	Estimate	Standard Error
μ	-0.079*	0.041	0.055*	0.024
β	0.042*	0.013		
α	0.009	0.018	0.013	0.018
ω	0.439*	0.086	0.210*	0.054
ϕ	0.136*	0.073	0.158*	0.0482
φ	0.600*	0.131	0.695*	0.070
d	0.599*	0.119	0.574*	0.080
λ	4.138*	0.080	5.567*	0.119
η	-0.058*	0.024	-0.006	0.022
	Statistic	p-value	Statistic	p-value
Box-Pierce Q(12)	43.167	0.000	21.935	0.292
Box-Pierce $Q^2(12)$	8.040	0.992	16.564	0.681
KS test	0.016	0.493	0.008	0.025
Log Likelihood	-5291.61		-5102.62	

* indicates significance at 5% level

Table 4: Log-Likelehood of Copula estimation

	Copula	Log likelihood	AIC	BIC
Constant	SJC	47.4648	-94.9282	-94.9241
	Student t	45.8692	-91.7369	-91.7328
	Gumbel	42.0931	-84.1856	-84.1835
	Gaussian	41.5751	-83.1496	-83.1475
	Plackett	35.0474	-70.0942	-70.0921
	Clayton	32.7873	-65.5740	-65.5719
Time-varying	SJC	55.5064	-111.0086	-110.9962
	Gaussian	49.8858	-99.7695	-99.7633

Table 5: Time-varying symmetrised Joe-Clayton copula

Parameter	Coefficient	Standard Error
ω^L	2.590*	1.19*
α^L	-7.18*	3.65*
β^L	-19.44*	5.45*
ω^U	1.51*	0.81*
α^U	-18.28*	7.78*
β^U	-14.27*	3.31*

Standard errors are calculated from Hessian matrix.
* indicates significance at 5% level

Table 6: Hong Kong return series in HK dollar and Chinese Yuan domination

	Mean	Standard deviation	Skewness	Kurtosis	$AR(1)^a$	$AR(2)^a$
HK dollar	0.02	1.78	0.49	16.09	0.01	0.04
Chinese Yuan	0.01	1.78	0.51	16.04	0.01	0.04

[a] AR(i) represents the ith-lag autocorrelation coefficient of returns.

Figure 1: Shanghai index return June 1, 1996 - October 1, 2008

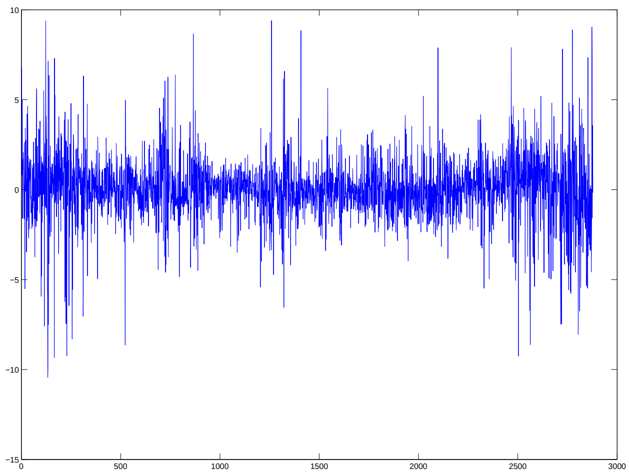

Figure 2: Hong Kong index return June 1, 1996 - October 1, 2008

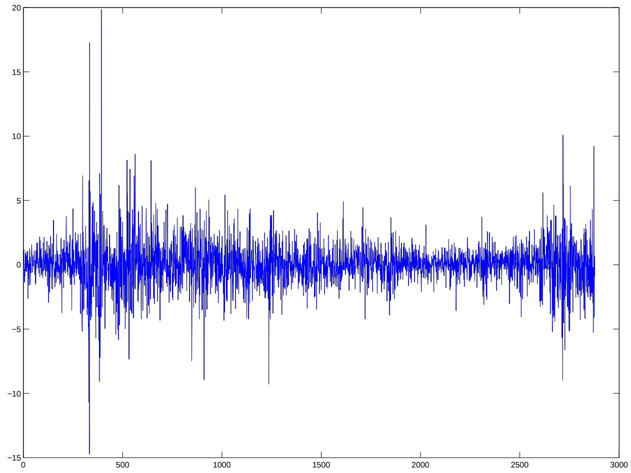

Figure 3: Correlation implied by time-varying SJC

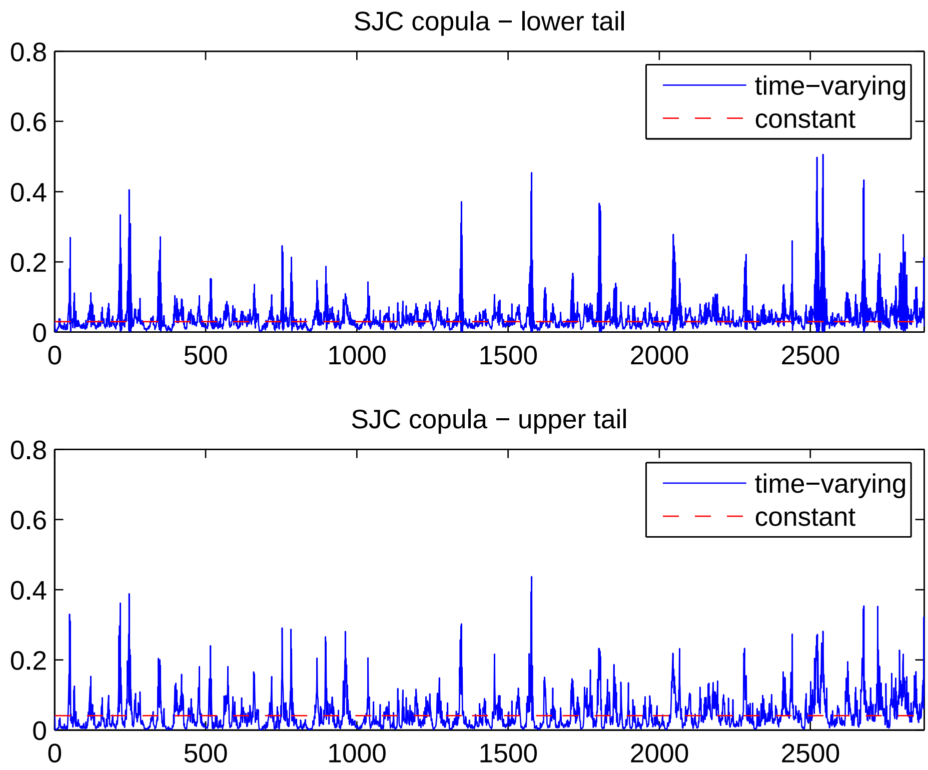

Figure 4: Copula simulation 3 Dimensions view

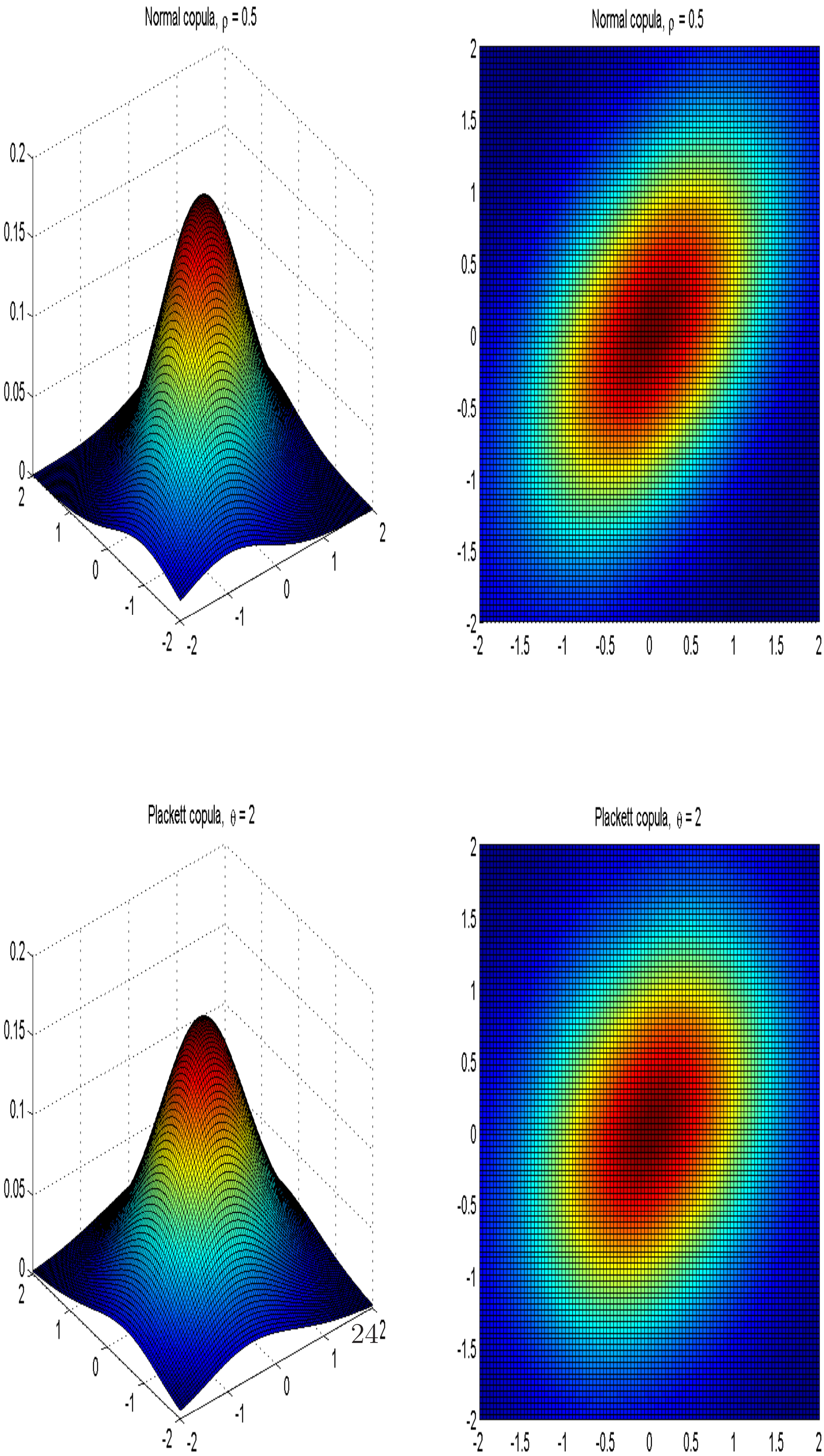

Figure 5: Copula simulation 3 Dimensions view

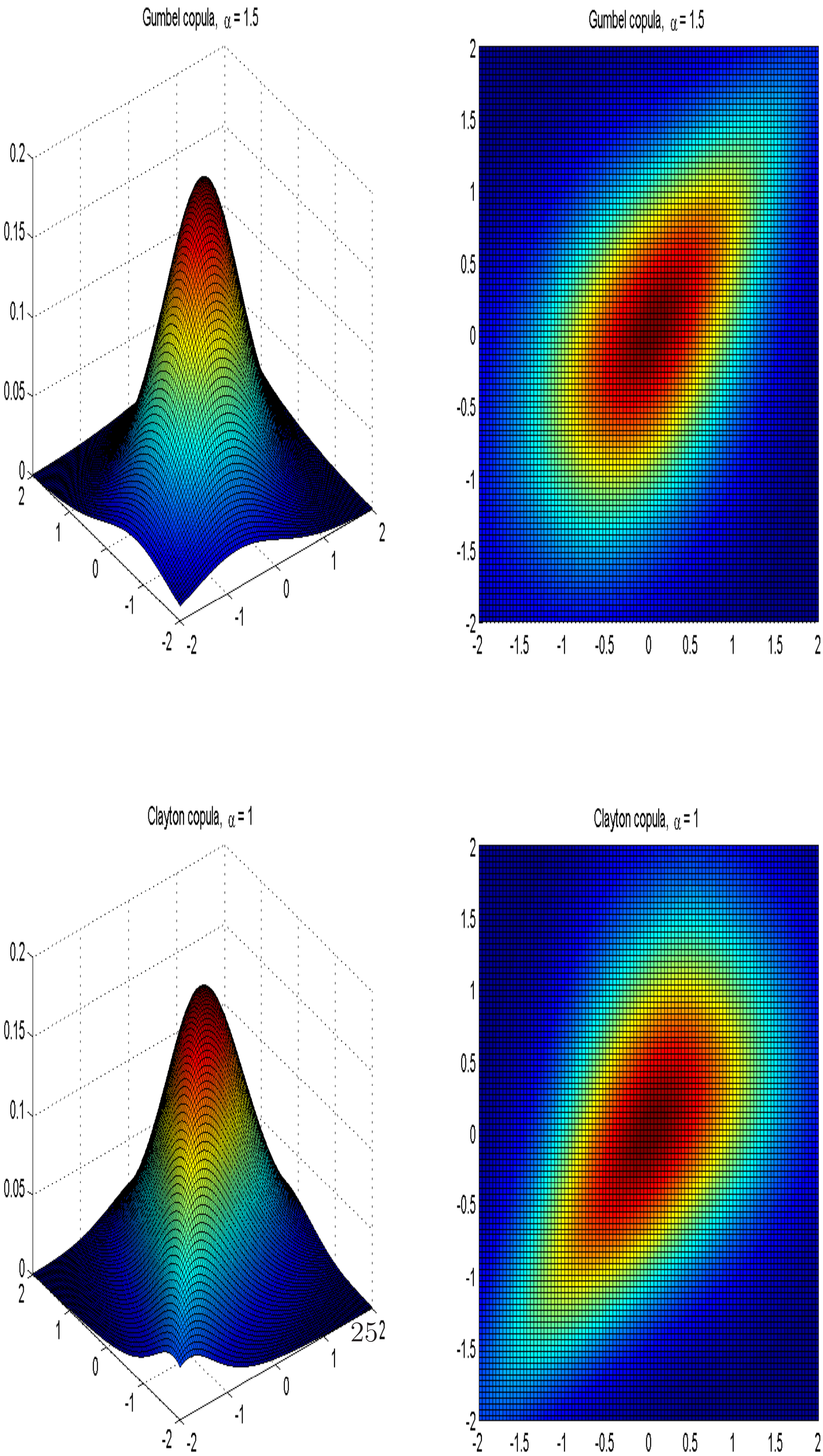

Figure 6: Copula simulation contour plot