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Abstract

This paper develops the smallest model of international trade based on differences in factor endowments across countries. We use this model to clarify the result in Helpman and Krugman (1985) that relative country size does not matter for the volume of trade. Relative country size does matter for the volume of trade, holding relative endowments and the size of the world economy constant.

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Keywords: International trade; Heckscher-Ohlin; trade volume.

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1 Introduction

The objective of this paper is to explore the predictions for the volume of trade, of the simplest Heckscher-Ohlin model of international trade based on factor endowment differences across countries. Our key result is that, holding countries' endowment ratios constant relative to one another and the size of the world economy constant, relative country sizes do matter in determining the volume of trade.

This result clarifies the statement in Helpman and Krugman (1985) that "... in some sense relative country size has no effect on the volume of trade" (p. 24). This statement holds along any ray that is parallel to the diagonal of the Dixit-Norman-Helpman-Krugman rectangle[1]. We establish two results. First, a movement along any such ray, does not correspond to a situation where relative factor endowments are constant across countries. Second, we show that if relative endowments are held constant across countries, then the volume of trade increases as countries become more similar in relative size.

The way we proceed is as follows. First, we set up the model. We then demonstrate our two results, before providing some concluding comments.

2 The model

Suppose that there are two countries in the world, Home and Foreign, and two goods, x and y. There are two types of sector-specific labour in the economy, types 1 and 2, where type 1 labour is the sector-specific labour used in producing good x, while type 2 labour is the sector-specific labour used in producing good y. There are identical preferences and technologies across countries, and free trade in goods but not in factors of production. All markets are perfectly competitive. Choose units such that the output of each good is equal to the labour used in producing that good:

$$Q_x = L_1 \qquad Q_y = L_2 \tag{1}$$

[1] First popularised by Dixit and Norman (1980), then used in a variety of contexts by Helpman and Krugman (1985).

The representative consumer's utility function takes the following Cobb-Douglas form:

$$U = \log c_x + \log c_y \tag{2}$$

The utility function implies that the representative consumer will spend equal shares of his income on each type of good. Each country's endowment of the two types of labour is:

$$\left.\begin{array}{l} L_1^H + L_1^F = 2 \\ L_2^H + L_2^F = 2 \end{array}\right\} \qquad \frac{L_1^H}{L_2^H} > \frac{L_1^F}{L_2^F} \tag{3}$$

That is, Home is relatively abundant in type 1 labour, while Foreign is relatively abundant in type 2 labour. World endowment of each type of labour is equal to 2, and therefore so is world output of each type of good. Given identical expenditures on each type of good, prices of both goods are equalised and normalised to 1. This also implies that wages are equal to 1 for both types of labour.[2]

National incomes are equal to $w_1^H L_1^H + w_2^H L_2^H$ for Home, and $w_1^F L_1^F + w_2^F L_2^F$ for Foreign. Finally, the volume of trade can be obtained by the difference between expenditure on each good and value of production of each good. Following Helpman and Krugman (1985), when Home is relatively abundant in type 1 labour, the volume of trade is defined as:

$$VT = p_x \left(Q_x^H - s^H \overline{Q_x} \right) + p_y \left(Q_y^F - s^F \overline{Q_y} \right) \tag{4}$$

where $\overline{Q_x}$ and $\overline{Q_y}$ are the world output of each good, and s^H and s^F are the shares of Home and Foreign in world income. Given our normalisations, the volume of trade reduces to:

$$VT = \frac{2\left(L_1^H L_2^F - L_1^F L_2^H \right)}{L_1^H + L_2^H + L_1^F + L_2^F} \tag{5}$$

This shows the standard result, that the volume of trade decreases the more similar are countries' relative factor endowments; if for example $\frac{L_1^H}{L_2^H} = \frac{L_1^F}{L_2^F}$, then the volume of trade is equal to zero.

[2]This is the complete general equilibrium solution of the model. The Appendix shows the autarkic equilibrium.

3 The volume of trade

Helpman and Krugman (1985) show that the volume of trade is constant along a ray that is parallel to the diagonal linking the origins of the two countries in the Dixit-Norman-Helpman-Krugman (DNHK) rectangle. A natural question to ask is, what is the implication of this constant-trade-volume ray on relative endowments?

To answer this question, consider the DNHK rectangle Figure 1, where O^H is Home's origin, O^F is Foreign's origin, $O^H L_1$ is the world endowment of type 1 labour, and $O^H L_2$ the world endowment of type 2 labour. Suppose that the distribution of endowments between the two countries is at point E, so that the line CED is the constant-trade-volume line which passes through the endowment point, and EF is the volume of trade.

Given the parameters of the model, the equation of the constant-trade-volume line CED is $VT = L_1^H - L_2^H$. Therefore, to investigate what happens to the endowment ratio of Home along this ray, we first totally differentiate this expression, holding the volume of trade constant:

$$dL_1^H - dL_2^H = 0 \tag{6}$$

A movement along this ray changes relative endowments according to the following proportions:

$$\begin{aligned} \left.\frac{d\left(\frac{L_1^H}{L_2^H}\right)}{\left(\frac{L_1^H}{L_2^H}\right)}\right|_{dL_1^H = dL_2^H} &= d\log L_1^H - d\log L_2^H \\ &= \frac{dL_1^H}{L_1^H} - \frac{dL_2^H}{L_2^H} \\ &= \frac{\left(L_2^H - L_1^H\right) dL_1^H}{L_1^H L_2^H} \end{aligned} \tag{7}$$

since $dL_1^H - dL_2^H = 0$. The analogous expression for Foreign is:

$$\left.\frac{d\left(\frac{L_1^F}{L_2^F}\right)}{\left(\frac{L_1^F}{L_2^F}\right)}\right|_{dL_1^F = dL_2^F} = \frac{\left(L_2^F - L_1^F\right) dL_1^F}{L_1^F L_2^F} \tag{8}$$

Since (7) is not in general equal to (8), we can conclude that a movement along the constant-trade-volume line CED does not represent a proportional change in both countries' relative endowment ratio.

It remains to derive the curve that represents a constant relative endowment ratio between the two countries. The equation of this curve satisfies the relation:

$$\frac{\left(\frac{L_1^H}{L_2^H}\right)}{\left(\frac{L_1^F}{L_2^F}\right)} = \beta$$

where β is a constant. Rewriting this gives the equation of this curve:

$$L_1^H L_2^F = \beta L_1^F L_2^H \tag{9}$$

Then the volume of trade along this curve is

$$VT = L_1^H - L_2^H = \frac{\beta L_1^F L_2^H}{L_2^F} - L_2^H = \left[\left(\frac{\beta L_1^F}{L_2^F}\right) - 1\right] L_2^H \tag{10}$$

Figures 2 and 3 show the properties of such a constant-endowment-ratio curve, for a value of $\beta = 10$ (Home's relative endowment of type 1 to type 2 labour is ten times that of Foreign). Figure 2 shows how Home's endowment of type 2 labour varies with its endowment of type 1 labour in order to preserve the relative endowment ratio. Figure 3 shows the volume of trade and the difference in national incomes as we move along the constant-endowment-ratio curve. The volume of trade is maximised when the two countries' incomes are most similar to one another.

4 Conclusions

This paper develops the smallest trade model based on factor endowments, which is then used to clarify the cases when relative country size has no impact on the volume of trade. Relative country size does not matter for the volume of trade when we move along any constant-trade-volume line, which is parallel to the diagonal of the DNHK rectangle. However, movement along this line does not preserve constant endowment ratios across countries. We derive the expression for the curve representing constant endowment ratios, and show that the volume of trade along this curve does depend on relative country size.

The strong assumptions we make especially on the technology side allow us to solve the model very simply. We believe that relaxing this assumption to allow for the use of both factors of production in both industries, should not alter the basic conclusion, which is that more similar country sizes lead to larger trade volumes, when relative factor endowments are controlled for.

Simple though it may be, this result has direct practical implications. A recent paper by Debaere (2005) seeks to empirically test the model of international trade based on monopolistic competition by relating trade shares of GDP to similarity of country GDPs. The volume of trade equation (10) is also an expression for the trade share, since it was derived holding world GDP constant. What it shows, however, is that, after controlling for relative factor endowments, increasing trade shares as countries' GDPs become more similar to one another, is also a prediction of the simple model of trade based on factor endowments, hence cannot be used to distinguish between this model and the model of monopolistic competition.

5 Appendix A: Autarkic equilibrium

The solution of the model when goods trade is prohibited is as follows (here, we solve for Home; the solution for Foreign follows the same steps). Since the expenditure on each good is the same, relative prices and hence relative wages are:

$$\frac{p_1}{p_2} = \frac{w_1}{w_2} = \frac{L_2^H}{L_1^H}$$

Given the assumptions on technologies, output and hence consumption of each good is equal to the endowment of the type of labour associated with each good.

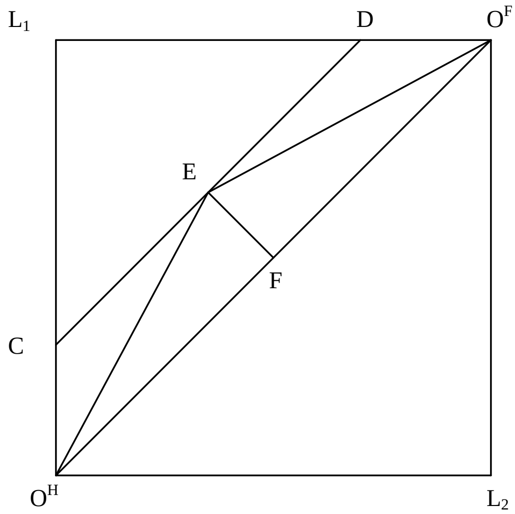
References

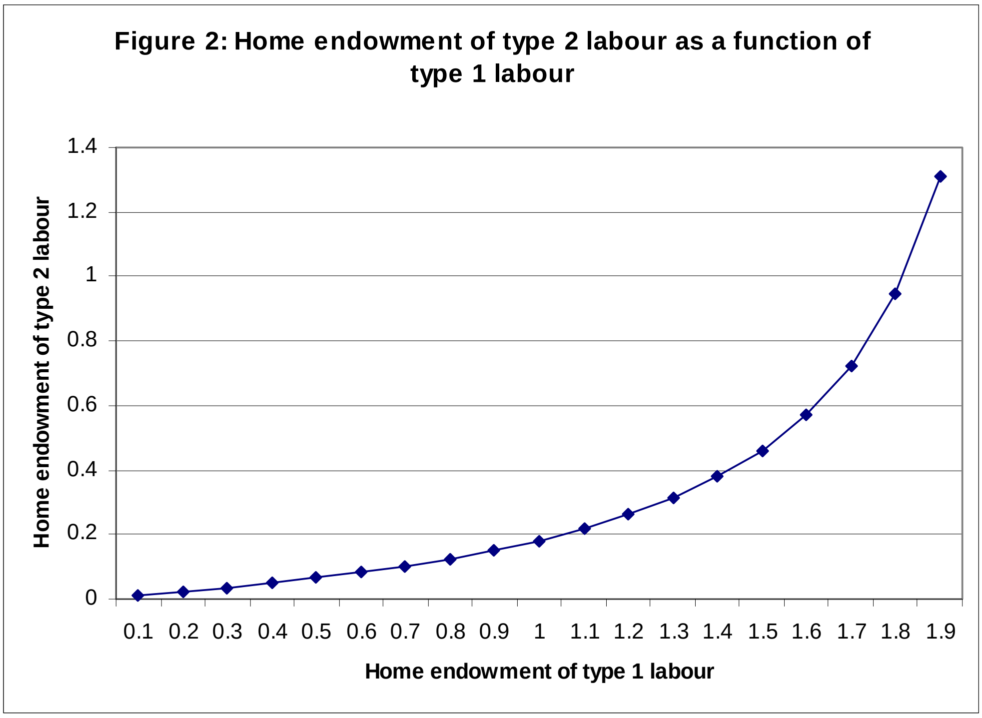
[1] Debaere, P. (2005), Monopolistic Competition and Trade, Revisited: Testing the Model Without Testing for Gravity, Journal of International Economics 66(1), 249-266.

[2] Dixit, A. K. and V. Norman, 1980, Theory of International Trade (Cambridge University Press, Cambridge).

[3] Helpman, E. and P. R. Krugman, 1985, Market Structure and Foreign Trade (MIT Press, Cambridge, MA).

Figure 1: Dixit-Norman-Helpman-Krugman rectangle.





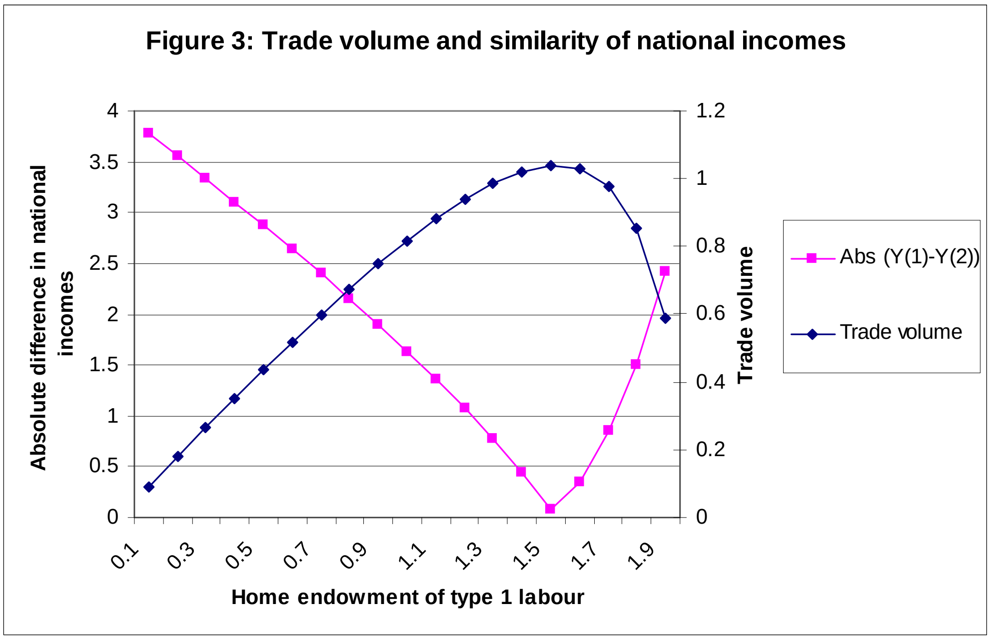


Figure 3: Trade volume and similarity of national incomes
Absolute difference in national incomes
Trade volume
Home endowment of type 1 labour
Abs (Y(1)-Y(2))
Trade volume
0
0.5
1
1.5
2
2.5
3
3.5
4
0
0.2
0.4
0.6
0.8
1
1.2
0.1
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9