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Funding Higher Education and Wage Uncertainty: Income Contingent Loan versus Mortgage Loan

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Abstract

Individual risk aversion and riskiness of investment in higher education are combined with two alternative loan-based financing systems, income contingent loans (ICL) and mortgage loans (ML), to investigate the effects on graduate lifetime expected utilities. We deal explicitly with the presence of hidden subsidies due to discounting, which is one of the main drawbacks of an ICL. The theoretical model has been calibrated using real data on graduate earnings and their volatility, together with the features of the English HE financing system, which has recently switched from a ML to an ICL system. Higher uncertainty in earnings in general makes an ICL the preferred system for risk averse individuals, while risk neutral individuals prefer mortgage loans.

JEL Classification: D81, I22, H80.

Keywords: Education Choice; Risk Aversion; Uncertainty.

1 Introduction

The combined effect on individual welfare of higher education investment risk, risk aversion and higher education funding systems is of great interest to economists and policy makers. However it remains challenging to model. We know that an individual making schooling decisions is likely to be only imperfectly aware of her abilities, her probability of success, and the earnings that may be obtained after completing her education. There is a large literature, both theoretical and empirical, that addresses these issues. Weiss (1979) finds that risk adjusted average rate of return to schooling sharply decreases as risk aversion increases, Olson et al. (1979) allow for some form of borrowing to finance education and estimate small but positive risk premium for attending college. More recently, Padulla and Pistaferri (2001) extend Olson et al. including both employment risk and wage uncertainty and find that without accounting for risk the returns to education are downward biased. Hartog and Vijverberg (2007) measure the uncertainty associated to post-schooling earnings and find evidence that workers are not only risk averse but also exhibit skewness affection. Belzil and Leonardi (2007) study how risk aversion can explain differences in schooling attainments. The common thread of these studies

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is individual risk aversion and uncertainty of educational outcomes. However fully integrating risk into the analysis of human capital investment where borrowing is possible is still technically problematic. Indeed there is a large and separate literature on funding higher education (e.g. see Chapman 2007 for a comprehensive summary) and in particular based on student loans, such as mortgage loan (ML) and income contingent loan (ICL). The concept of ICLs as a means to fund human capital investments started with Friedman (1975) and since then several works such as Nerlove (1975), Barr (1993, 2005) and Chapman (1997) analyzed the usefulness of ICLs as a source of funding HE. Our contribution is an attempt to integrate the literature on education and risk with the literature on funding higher education. We propose a theoretical model which investigates the effects of two alternative loan schemes, ML and ICL, on the returns to schooling of risk averse individuals receiving stochastic earnings after graduation. Our work is close in spirit to Hartog and Serrano (2007) and Cigno and Luporini (2009). The first develop a human capital model to analyze the effects of stochastic post schooling earnings on the optimal schooling length and they typically find a statistically significant negative effect of risk on investment in HE education. In our analysis we represent uncertainty by the variance of the post schooling earnings but we then compare the individuals outcomes under a ML and an ICL funding schemes. Cigno and Luporini (2009) solve a multi-agency problem to find the conditions under which an ICL can achieve the social optimum, compared to other HE policies such as scholarships and graduate tax. In our work we are mainly concerned with individual welfare and we add theoretical evidence on the effects of ICLs, taking into consideration one of the main drawbacks of an ICL, that is the presence of hidden subsidies. Our interest in the combination of earnings uncertainty and student loan design is motivated by two reasons: the observed uncertainty in the real earnings of graduates; and the reform of the higher education financing system in England[1], with a mortgage loan (ML) and its subsequent replacement by an income contingent loan scheme (ICL). We therefore provide some empirical evidence and discuss some policy implications by calibrating our model using real data on graduate earnings and their volatility, combined with the features of the English financing system.

We present a model in which we assume that students receive a loan from the government to finance the cost of HE, and they repay their debt after graduation according to one of two loan schemes: ICL and ML. Graduates receive uncertain future incomes (affected by a single lifetime shock), and we measure the level of uncertainty considering the variance of the incomes. Our intuition is that if graduates expect high variance in their earnings, an income contingent loan that allows them to repay the debt only when they have the financial resources to do it provides higher welfare, because it provides insurance against uncertainty.

To verify our theoretical intuition we simulate different scenarios where we compare the two loan schemes, observing the changes in individual welfare. We use the British Cohort Study 1970 which provides useful information on family background, individual characteristics and job environment.

There are two important differences between an ICL and a ML: the first is that a ML does not provide insurance against uncertainty and the second is the presence, under an ICL, of a potential hidden subsidy, which arises because the average repayment period could be different to a ML. The first difference does not matter if the individuals are risk neutral, while the second only matters because of discounting. We isolate the second effect by first assuming risk neutral individuals. We notice that the hidden subsidy typically makes an ICL more attractive. If we rule out this possibility, the expected costs under an ICL are always higher than the costs

[1]Higher Education reform (approved in 2004 and effective from 2006/2007).

under a ML. The effect of the English reform is to increase this expected differential in cost. In terms of utility, a ML provides higher expected utility for risk neutral graduates, and in particular for those on low earnings, e.g. females. In presence of risk aversion the model offers interesting policy implications. We consider the case whether there is no hidden subsidy, by fixing the expected repayment period to be the same under an ICL as it would be under a ML, and we notice that the effect of high uncertainty combined with risk aversion makes an ICL the preferred system.

In our simulations we typically find that graduates from low educated parental background, males over females, graduates working in the private sector receive higher utility from an ICL, because of higher variance in their earnings. A ML is the preferred system when earnings exhibit low variance, and when earnings are not too high: something which is typical of the public sector careers.

In the second part of our work, this model is extended to incorporate stochastic changes of earnings over time. In particular, we assume that the growth rate of the earnings follows a Brownian motion, as in Hogan and Walker (2007). However, they solve the problem of education choice with uncertain returns using an options approach. Instead we compute the expected utilities under the two loan schemes using a numerical method. The new framework allows us to generate earnings paths for the entire individual working life, where uncertainty affects the earnings each year. We again find, as in the previous part, that higher uncertainty increases the utility of an ICL. However, the results in this case show the importance of the initial earnings: in fact an ICL is highly preferred by individuals with low initial earnings.

Our model is of wide interest because it indicates policy implications to many countries that have already implemented an ICL funding system or more generally where there exists a debate on the appropriate HE funding system. Given the impact of the UK reform in raising the cost of HE, an ICL becomes the preferred option. Participation in HE is greater under an ICL than under a ML, for low initial income earners and in the case of high earnings uncertainty.

In the next Section we describe the theoretical model, in Section 3 we analyze the case of risk neutrality, in Section 4 we obtain the algebraic form of the expected utilities under risk aversion and show the results of the simulations. In Section 5 we set up the model with the new assumptions on the earnings and show the results of the simulations. Section 6 concludes.

2 Theoretical Model

This section presents the main assumptions of the theoretical model that hold both under a mortgage loan and under an income contingent loan. In general, we do not consider any external effects of education on society as a whole. We analyze which loan scheme yields greatest welfare in terms of individual lifetime expected utility, for risk neutral and risk averse graduates. We also assume non graduates earnings are certain and there is no unemployment.

Individuals go to university for s years full time and education has the same cost for everybody without distinction between courses and subjects. Earnings during the schooling are assumed to be zero. Following Olson, White and Shefrin (1979) consumption is always equal to earnings. Therefore, during university, consumption is also set to zero. There is no private market for loans, and no informal market for loans e.g from parents. There is no insurance market because it is not profitable for the private market to insure the investment in HE, since moral hazard and adverse selection cannot be avoided due to the lack of any collateral.

The only source of financing allowed is a public loan, again equal for all the students, of fixed size and that covers all the costs of attending university. The real interest rate on the loan is zero[2]. For simplicity, consumption at school is zero and equals earnings. The loan finances fees - although we could allow it to finance consumption during schooling. Thus the loan size is the same irrespective of scheme and equal to fees. The government finances a constant cost of higher education, through issuing the same amount of debt regardless of repayment scheme (therefore the subsidy is the same for all the students). The debt is repaid only by the graduates' repayments, there is no opting out choice: when the students enter a scheme he or she cannot leave it before the full repayment of the loan. This implies that there is no default[3], and in the long run all the cost of education is recovered equally by both schemes. The government is risk neutral and does not have any preference over the funding systems. Since the government could bear different costs of providing the loan according to the scheme, we assume a zero real interest rate on the borrowing[4]. Under this assumption the costs for the government are the same under an ICL and a ML, and social welfare depends only on students' utility.

Moreover, when the expected repayment periods under the two systems differ, hidden subsidies arise. The longer is the repayment period, the lower is the present value of the repayments, given a positive subjective discount rate. Therefore the system with the longer repayment period provides a larger hidden subsidy to graduates and it is more attractive than the scheme with shorter repayments. We mainly perform the analysis under the condition of no hidden subsidy, but in one case we also allow for hidden subsidies to establish what difference this makes.

Upon graduation, the individuals start working immediately, and, as in Hartog and Serrano (2007), they obtain an uncertain wage because it is subject to a random shock. For simplicity, the shock has a single lifetime realization, after which earnings remain constant. We can imagine an initial random draw that fixes earnings at a certain level which remains unchanged for all the working life. Let $\tilde{y} > 0$ be the shock with $\mathrm{E}(\tilde{y}) = 1$ and $Var(\tilde{y}) = \sigma^2$.

Individuals cannot insure the wage risk and seek to maximize the expected lifetime utilities. Consumption is equal to earnings and is strictly positive; utility is defined over the individuals' earnings stream.

In this model we focus only on the post graduation period, but it is important in all the following analysis to distinguish between the repayment period and the post repayment period. Graduates must start repaying their educational loan straight after graduation and for T years, after that they receive their entire earnings for the rest of their life, assumed infinite for simplicity. Considering a general repayment scheme, we define R as the general per-period payment. The expected utility is:

$$V = \mathrm{E}\left\{\int_{s}^{T+s} e^{-\rho t} u\left(\tilde{y} - R\right) dt + \int_{T+s}^{\infty} e^{-\rho t} u\left(\tilde{y}\right) dt\right\} \tag{1}$$

where $R < \tilde{y}$, and ρ is the subjective discount rate that measures how much the present is taken in consideration with respect to the future. A loan scheme is described fully by (T, R).

[2]This is not just a simplifying assumption, but a real feature of the income contingent loans as implemented by the national governments in England and Australia. There is only an adjustment to the inflation. In our model, for fair comparison, we assume zero real interest also for a mortgage loan.

[3]The case of the students' default is analyzed in a further work.

[4]The case with positive real interest on the borrowing is also analyzed in this work, but only for risk neutral individuals.

2.1 Mortgage Loan and Income Contingent Loan

We stress that the cost of the loan is equal to the total cost of education C, and it is the same under both systems. The way it is repaid produces different individual utilities because of the random earnings. If we assume no uncertainty and identical repayment rates the two systems are equal in terms of utility.

Under a mortgage loan funding system, the individuals take out a loan equal to C and repays through T equal, fixed and periodical installments φ, at a zero real interest rate. The repayment period is just

$$T = C/\varphi. \tag{2}$$

Under an income contingent loan system, the individuals borrow an amount equal to C, and start to pay back their loan after graduation according to the level of their earnings. Under this scheme, if earnings are below a minimum threshold no payment is due. If earnings increase, a greater portion of the debt is repaid and all the loan is paid off in less time. Therefore, compared to a mortgage loan the ICL repayment period, $\tilde{T}$, is *random*. In our model, for simplicity, we assume no initial threshold and the total cost of schooling is given by a fixed percentage (γ) of the random graduate earnings. Note, γ is chosen ex ante by government, the repayment period $\tilde{T}$ is determined when the income shock is realized.

$$C = \gamma \int_{s}^{\tilde{T}+s} \tilde{y}\, dt \tag{3}$$

therefore the repayment period is:

$$\tilde{T} = \frac{C}{\gamma \tilde{y}} \quad . \tag{4}$$

We can now define the following assumption concerning the expected repayments under the two schemes.

Assumption 1. *With a ML,*

$$R < \tilde{y} \quad \Longleftrightarrow \quad \varphi < \tilde{y}$$

and the expected repayment is

$$\mathrm{E}[P_{ML}] = T \times \varphi = C$$

Assumption 2. *Under an ICL the annual repayment is:*
$\gamma \times \tilde{y}$ *until the loan is repaid, and the expected repayment is*

$$\mathrm{E}[P_{ICL}] = T\gamma\tilde{y} = C$$

Proposition 1. *Expected repayment periods under the two systems*

$$\begin{array}{lll} \text{(a)} & if\ \ \gamma = \varphi & \mathrm{E}[T_{ICL}] > T_{ML} \\ \text{(b)} & if\ \ \gamma = \varphi \times \mathrm{E}[\frac{1}{\tilde{y}}] & \mathrm{E}[T_{ICL}] = T_{ML} \end{array}$$

where $\mathrm{E}[\frac{1}{\tilde{y}}] > 1$

Proof. *See Appendix A.1*

The first proposition highlights one of the main differences between the two schemes: a feature of an ICL is to spread the same cost over a longer repayment period compared to a ML, assuming the same repayment rates. However, this implies the existence of a hidden subsidy because the longer repayment period under ICL compared to ML because of positive discounting and the real interest is zero[5]. Therefore, in point (b) of Proposition 1, we find the condition that rules out the hidden subsidy and allows a comparison of the two schemes on the same basis. In particular, we notice that to have same repayment periods we require a higher repayment rate under an ICL.

3 Risk Neutrality and Expected Costs

When the individuals are risk neutral $u(\tilde{y}) = \tilde{y}$, and we need only consider the costs to compare the two repayment schemes. We work out the present value of the costs, substituting for each scheme the respective repayment period, T and $\tilde{T}$, and discount to $t = 0$.

Proposition 2. *Assuming risk neutral individuals,*

$$V_{ICL} > V_{ML}$$

when $\gamma = \varphi$, *instead*

$$V_{ICL} < V_{ML}$$

when $\varphi = \frac{\gamma}{\mathrm{E}[1/\tilde{y}]}$

Proof. See Appendix A.2

In the Appendix A.2, we prove analytically Proposition 2 when $\gamma = \varphi$. Then, we argue that for the case $\varphi = \frac{\gamma}{\mathrm{E}[1/\tilde{y}]}$ we require a numerical solution[6] which we provide using real data from BCS70 dataset.
The result highlights, in terms of expected utilities, the differences between the two systems raised in Proposition 1. The presence of hidden subsidies completely changes the preferences and makes an ICL more preferred. If we assume the same repayment periods, instead, we would need to increase the ICL repayment rate and then a ML gives higher welfare.
To give a broad intuition of the result in Proposition 2, we assume a general repayment method R, the present value of the education cost is:

$$PVC = \int_0^T R\, e^{-\rho t} dt = \frac{R}{\rho}\,[1 - e^{-\rho T}]$$

Taking the derivatives of PVC with respect to T, we can easily observe that this function is concave[7]. Consider now a loan with a certain repayment period of $T_1 = 10$ years, and an alternative loan with two equal probability repayment periods of 5 and 15 years, T_2; therefore we have $T_1 = ET_2$, which rules out the hidden subsidy, at least in the risk neutral case. The

[5]The subsidy still exists providing a real interest rate lower than ρ.

[6]This requirement is clear looking in the Appendix A.2 at equations 20 and 21 that need to be compared in order to evaluate which system produces higher expected utility. Since we are considering risk neutral individuals we compare only the expected value of the costs.

[7] $\frac{\vartheta^2 PVC}{\vartheta T^2} = -R\rho\, e^{-\rho T} < 0$ for all T.

concavity property implies that the expected present value of the cost of a certain repayment period is lower than the present value of the cost of an uncertain repayment period of the same expected length (even though we are risk neutral):

$$\mathrm{E}[PVC(T_1)] < PVC[\mathrm{E}(T_2)] \quad \Longrightarrow \mathrm{E}[PVC(10)] < PVC(10).$$

We now analyze the second result of Proposition 2, through a numerical calibration. We first estimate $\mathrm{E}[1/\tilde{y}]$ by its sample analogue, then fix γ and get φ accordingly. The method is explained in detail in the Appendix A.2, and we report the results of the simulations in the next paragraph. The details about the data set used and the parameters setting are reported in the Appendix B.

3.1 Equal repayment periods

Looking at the top of Figure 1, we report on the vertical axes the difference between the expected costs $EC_{ICL} - EC_{ML}$, therefore positive values should be interpreted as individual preference for an ML system. Setting $\gamma = 9\%$, we observe that the expected costs under an ICL are always higher than the expected costs under a ML, and the effect of the reform is to increase this gap. This means that when there are no hidden subsidies the benefits of an ICL for risk neutral graduates decrease strongly. Moreover, although the repayment periods are equal between the two systems, they differ across graduates. For all graduates, the repayment period increases from 5.6 to 14.7 years, for male from 5.1 to 13.3 years and for females from 5.7 to 14.9 years. Males are those with the highest earnings and therefore the shortest repayment period. These differences across sex are reflected in the expected costs, and we observe that for males the gap between ICL and ML is lower than for females, although they both prefer a ML.

At the bottom of Figure 1, we keep the cost fixed at £9000, with γ as above, and therefore the repayment periods are unchanged. However we let the subjective discount rate increase and we notice an increase of the gap in the expected costs when the individuals discount their future more. A ML is still preferred.

3.2 Cost for the government

We assumed so far that the government imposes a zero real interest on the education loan. This implies that the total repayments to the government by the students are equal under both schemes. Defining P the total repayment and r its interest rate, C is the cost of the loan to the students and ρ their subjective discount rate, we obtain the net benefit as $NB = \mathrm{E}P - \mathrm{E}C$.[8] Assuming $r = 0$, $EP_{ML} = EP_{ICL}$ and then we can just compare the expected costs to the students to see which system is producing higher social net benefit:

$$NS = NB_{ICL} - NB_{ML} = \mathrm{E}C_{ML} - \mathrm{E}C_{ICL}.$$

We now allow a positive interest rate for the government, we maintain the hypotheses of risk neutrality, for simplicity, but we rule out the hidden subsidies. This implies equal expected repayments periods under the two loan schemes and requires to set $\varphi = \frac{\gamma}{\mathrm{E}[1/\tilde{y}]}$.

[8]To simplify the notation, by $\mathrm{E}C$ and $\mathrm{E}P$ we intend the expected present values $\mathrm{E}[PVC]$ and $\mathrm{E}[PVP]$, respectively.

The only difference between the expected present value of the cost to the students and the expected present value of the payment to the government is given by r and ρ. The net social benefit is:

$$\begin{aligned} NS =& NB_{ICL} - NB_{ML} \\ =& (\mathrm{E}P_{ICL} - \mathrm{E}C_{ICL}) - (\mathrm{E}P_{ML} - \mathrm{E}C_{ML}) \\ =& (\mathrm{E}\,[\frac{\gamma\,\tilde{y}}{r}\,(1 - e^{-r\frac{C}{\gamma\tilde{y}}})e^{-rs}\,] - \mathrm{E}\,[\frac{\gamma\,\tilde{y}}{\rho}\,(1 - e^{-\rho\frac{C}{\gamma\tilde{y}}})e^{-\rho s}\,]) \\ &- (\frac{\varphi}{r}\,[1 - e^{-r\frac{C\times\mathrm{E}[1/\tilde{y}]}{\gamma}}\,]e^{-rs} - \frac{\varphi}{\rho}\,[1 - e^{-\rho\frac{C\times\mathrm{E}[1/\tilde{y}]}{\gamma}}\,]e^{-\rho s}). \end{aligned}$$

In order to evaluate which system produces higher welfare we perform some calibrations. When the government's interest rate is lower than the students' time preference rate, our simulation[9] shows that an ICL is the preferred system and the effect of the reform is simply to increase the net social benefit. Females receive lower mean earnings and less uncertain (see Table 1), they benefit more from an ICL than males therefore the net social benefit is higher. The results change when graduates have a time preference rate higher than the government interest rate, in this case a ML produces higher net social welfare.

4 Comparing Mortgage and Income Contingent Loans under Risk Aversion

In this Section we consider individuals who are risk averse and work out their expected utility (represented by equation (1)), under a mortgage loan and an income contingent loan system. We omit the majority of calculations that are shown in more detail in Appendices C. We maintain the assumptions stated in Section 2 and we develop the analysis using a constant relative risk aversion (CRRA) utility function[10]. The CRRA functional form for utility together with the CARA are each quite simple, involve just one parameter, and make analysis in many economic settings quite tractable. The risk aversion measures associated with these forms are also very simple. As a consequence, these two functional forms for utility and the risk preferences they represent have been frequently used in the literature examining expected utility based decisions. There are other more flexible functional forms for utility which allow a wider range of risk preferences to be represented than those available under CARA or CRRA. For example, the hyperbolic absolute risk averse (HARA) family. However these alternative functional forms involve more than one parameter and are less simple to manipulate and use. We do not want to add further complexity to our analysis and so we prefer using a CRRA functional form [11].
Under a mortgage loan, the expected utility is obtained by substituting $R = \varphi$ into equation (1):

$$V_{ML} = \int_{s}^{T+s} e^{-\rho t}\mathrm{E}\left[u\left(\tilde{y} - \varphi\right)\right]dt + \int_{T+s}^{\infty} e^{-\rho t}\mathrm{E}\left[u\left(\tilde{y}\right)\right]dt. \tag{5}$$

To get a closed-form solution for V_{ML}, we use a second order Taylor expansion around the mean

[9]For space reasons we omit these results which are available upon request from the Author.

[10]We developed the analysis also using a constant absolute risk aversion functional form, CARA. The results are available upon request from the Author.

[11]For an extensive discussion of the other functional forms for utility functions see Meyer (2007)

$E[\tilde{y}-\varphi]=1-\varphi$ [12] for utility during the repayment period, and around $E[\tilde{y}]=1$ for the utility after the repayment period:

$$E[u(\tilde{y}-\varphi)] \simeq u(1-\varphi)+\frac{1}{2}u''(1-\varphi)\sigma^2. \tag{6}$$

$$E[u(\tilde{y})] \simeq u(1)+\frac{1}{2}u''(1)\sigma^2. \tag{7}$$

We develop our analysis using a CRRA utility function.

$$u(\tilde{y})=\frac{\tilde{y}^b}{b}.$$

where $b=1-a$ and a is the risk aversion parameter.

After simplifying [13], we get:

$$\begin{aligned} V_{ML_{CRRA}} = \frac{e^{-\rho s}}{\rho}\{&\left(1-e^{-\frac{\rho C}{\varphi}}\right)\left[\frac{(1-\varphi)^b}{b}+\frac{1}{2}(b-1)(1-\varphi)^{b-2}\sigma^2\right] \\ &+e^{-\frac{\rho C}{\varphi}}\left[\frac{1}{b}+\frac{1}{2}(b-1)\sigma^2\right]\}. \end{aligned} \tag{8}$$

Under an income contingent loan we do not know how long people take to repay their education debt. Therefore in the general equation of the expected utility the random earnings appear twice: first in the integral's bounds as random repayment period, second as an argument of the utility function.

$$V_{ICL}=E\left\{\int_s^{\frac{C}{\gamma\tilde{y}}+s} e^{-\rho t}u\left[\tilde{y}(1-\gamma)\right]dt+\int_{\frac{C}{\gamma\tilde{y}}+s}^{\infty} e^{-\rho t}u(\tilde{y})\,dt\right\} \tag{9}$$

Solving the integral we get the following equation:

$$V_{ICL}=\frac{e^{-\rho s}}{\rho}E\left\{\left[1-e^{-\frac{\rho C}{\gamma\tilde{y}}}\right]u\left[\tilde{y}(1-\gamma)\right]+\left[e^{-\frac{\rho C}{\gamma\tilde{y}}}\right]u(\tilde{y})\right\}. \tag{10}$$

To simplify the calculations we define the expression included in the expected value operator as $g(\tilde{y})$. In this way we can apply a second order Taylor expansion of $E[g(\tilde{y})]$ around the mean $E[\tilde{y}]=1$. Then, equation (10) becomes:

$$V_{ICL}=\frac{e^{-\rho s}}{\rho}\left[g(1)+g''(1)\frac{\sigma^2}{2}\right]. \tag{11}$$

The remaining procedure (explained in Appendix C.2) consists of calculating the value of $g(1)$ and $g''(1)$ in general, and with a CRRA utility function in particular. Finally, we substitute the expressions found in equation (11), and we obtain the following result. After simplifying,

[12] See Padula and Pistaferri (2001) and Hartog and Serrano (2003).

[13] See Appendix C.1 for the proof.

the expected utility[14] is:

$$V_{ICL_{CRRA}} = \frac{e^{-\left(s+\frac{C}{\gamma}\right)\rho}}{2b\gamma^2\rho}\{e^{\frac{\rho C}{\gamma}}(1-\gamma)^b\gamma^2[2+(b-1)b\sigma^2] - [(1-\gamma)^b - 1] \cdot [2\gamma^2 + ((b-1)b\gamma^2 + 2(b-1)C\gamma\rho + C^2\rho^2)\sigma^2]\}. \tag{12}$$

We can compare equation (12) and equation (8) through empirical simulations.

4.1 Simulations under Risk Aversion

We perform the simulations according to the different levels of earnings and uncertainty described in Appendix B.2. We calibrate the equations (8) and (12), assuming equal repayment periods, which means absence of hidden subsidies. The results are showed in graphs where we report on the vertical axes the difference between the expected utilities $EU_{ICL} - EU_{ML}$ and on the horizontal axes the parameter that changes.

Looking at Figure 2 we compare the expected utilities under the two systems by gender, allowing first for a variation in the costs of education and then in the individual discount rate. We notice two initial important results: when the cost is low the difference between the two systems is small, although females prefer a ML and males an ICL. For increasing costs, uncertainty matters more: compared to females, standard deviation of males earnings is almost double (see Table 1). We observe that the gap in the strength of preference gets bigger for higher costs. When we assume increasing subjective discount rate, for females the preferences are unchanged, instead for males the utility from an ICL reduces. We remind that the repayment periods, although equal for the two systems, depend on the magnitude of the earnings. Therefore females with lower earnings have longer repayment periods. Moreover, φ is endogenous and depends on both γ and $\mathrm{E}[1/\tilde{y}]$, when the latter is high, the ML utility decreases.

Looking at Figure 3, we consider the effects of family background on the preferences for one system over the other. The top left graph of Figure 3 shows the variation of the expected utilities for increasing costs of education, assuming that the graduates come from families with different earnings in 1980. The effect of low family earnings is not relevant because there are few observations. However it can be used to see what happens when the standard deviation is very high: an ICL is highly preferred for higher costs; and looking at the graph to the top right, when the risk aversion is very large the ICL expected utility drops sharply. This is due, as mentioned above, to the high $\mathrm{E}[1/\tilde{y}]$ and the long repayment period (19 years when family earnings are low compared to 13 years when family earnings are high). When graduates come from families with high earnings, they obtain a low variance wage and this is reflected in a preference for a ML. Graduates from medium family earnings obtain lower earnings on average but more uncertain: therefore an ICL provides higher utility. The same results are confirmed for increasing risk aversion.

Controlling for mother's education (bottom graphs in Figure 3), those whose mother has a degree get the highest earnings and the least uncertain, therefore they prefer a ML. Graduates whose mothers have no qualifications or secondary school qualifications have more volatile

[14] The expected utility with an income contingent loan is equal to the expected utility with a mortgage loan if $\varphi = \gamma$ and the variance of the earnings is zero.

earnings and prefer an ICL, although the first group with more intensity. The same is true for increasing risk aversion, but when it becomes too high the utility from ICL reduces.

Focusing on the subject of degree (see Table 1), the lowest earnings variance is with a Science degree and in fact these graduates are almost indifferent between the two systems (Figure 4). Those with a degree in Art and Humanities have the most uncertain earnings, and they strongly prefer an ICL, with higher utility for increasing cost and risk aversion. In each case, for higher discount rates the preference for an ICL reduces and the opposite happens for higher risk aversion.

We now consider the effects on expected utilities of graduates working in the private and public sector. In Figure 5 the graphs on the left column assume no hidden subsidies, while the graphs on the right column do not. In the latter case, according to Proposition 1, we set $\gamma = \varphi$ and we have $\mathrm{E}(T_{ICL}) > T_{ML}$[15]. In the top graphs of Figure 5 we compare the two loan schemes for increasing costs. When there are no hidden subsidies, as expected, in the private sector graduates prefer an ICL, since they get higher but more uncertain earnings. In the public sector, income is lower but also less uncertain and a ML is more preferred instead. In presence of hidden subsidies an ICL is much more preferred in the private sector. However, the interesting result is that now in the public sector graduates prefer an ICL, with increasing intensity as costs rise. Therefore, the effect of the hidden subsidy is very strong: enough to change the preferences when the level of earnings uncertainty is low. The same behavior is confirmed in the graphs in the middle of Figure 5 where we increase risk aversion. We finally compare the expected utilities under the same system in the two different sectors (graph in the bottom right corner). The difference between ICL in the private sector and ICL in the public sector is negative, and the same happens under a ML, with a slight change for increasing costs. This means that the effect of low uncertainty (typical in the public sector) prevails, and under the same scheme a more stable career gives higher utility.

To draw some conclusion[16], the analysis clearly suggests that without hidden subsidies, when there is very high uncertainty an ICL is always preferred because it is safer than a ML. Excluding extreme situations, the preference for an ICL depends strongly on the combination of level of earnings and its standard deviation. Therefore, if earnings are high and the corresponding standard deviation is below the sample average standard deviation, a ML gives higher utility (e.g. earnings when the mother has a degree). If both earnings and standard deviation are slightly above the respective sample averages (e.g. mother with a secondary school qualification), an ICL is preferred but with small intensity. The effect of risk aversion in general is to increase the preference for an ICL. However, this effect is particularly interesting if combined with high costs and long repayment periods, because for very high risk aversion the utility of an ICL drops drastically. Finally, when we compare a stable career with another which features high earnings uncertainty, in the first case a ML gives always higher utility. However, if we do not rule out the hidden subsidies we observe a sharp increase in the utility under an ICL, sufficient to invert the initial preference for a ML, even for low variance of earnings.

[15]We set a fixed ML installment to £900 pounds, that means a repayment period of around 4 years with the lowest cost, and of 10 years with the highest cost of HE.

[16]The results we have shown are confirmed by several other simulations, where we used different combinations of earnings and standard deviations, according to other information on family background, family composition or type of degree undertaken.

5 Increasing earnings

In this Section we extend our model to incorporate stochastic changes of earnings over time. We make the model more realistic and verify which conditions still hold relative to the case of static earnings. We assume that graduate earnings are no longer affected by a single life time shock, but there is a shock each year throughout the individual working life. To model this assumption we consider the earnings growth rate following a geometric Brownian motion $W(t)$[17]. This means that $y(t)$ satisfies

$$dy(t)/y(t) = \lambda dt + \sigma dW(t). \tag{13}$$

This expression can be interpreted heuristically as expressing the relative, or percentage, increment dy/y in y during an instant of time dt. λ is the deterministic growth rate and σ its standard deviation. Solving[18] the stochastic differential equation (13) we obtain the stochastic earnings:

$$y(t) = y(0)\exp[(\lambda - \frac{1}{2}\sigma^2)t + \sigma W(t)] \tag{14}$$

Equation (14) represents the new earnings we use to compute the expected utilities under the two loan schemes. Since it is not straightforward to obtain an algebraic solution for the expected utilities under an ICL we adopt a numerical method. We consider a discrete form of equation (14) because it is more relevant to our problem. The method is reported in detail in the Appendix D.1. Briefly, we generate many earnings paths of the same length (equal to a working life period of 40 years), and we use them to compute the expected utilities. Each earnings path produces one level of utility, therefore we average over the number of paths created. Ultimately we get the average expected utilities under an ICL (A_{ICL}) and ML (A_{ML}). What is important to stress under this new approach is that two new parameters enter the model. Looking at the equation (14), they are the initial earnings (y_0) and its deterministic growth rate (λ). Moreover, σ is the volatility of the Brownian motion and represents the maximum variation of the earnings in the interval t (for us one year). As we did in the first part of this work we use real data to calibrate the model and simulate different scenarios, in the Appendix D.2 are described the new settings.

5.1 Simulations

The results of the simulations are showed in Tables 2, where we report the value of the difference between the average expected utilities under the two funding systems: $A_{ICL} - A_{ML}$. Each time we change one parameter keeping the others equal, and we repeat the simulation for low initial earnings and high initial earnings. Looking at Panel A of Table 2, we notice a first important effect: the preference for one system over the other depends strongly on the level of the initial earnings. For low starting earnings an ICL is the favored system, but for high starting earnings the utility under an ICL declines sharply and a ML can be preferred. Another important result is that the level of uncertainty matters less than in the case with static earnings, although the direction of the effect is the same. We observe, in fact, that for higher earnings volatility the utility of an ICL increases but at a very slow rate. When the earnings are high the effect of uncertainty is more evident, in fact for $\rho = 8\%$ the initial preference for a ML is replaced by a

[17] $W(t)$ is Normally distributed with $\mathrm{E}(W(t)) = 0$ and $Var(W(t)) = t$.

[18] The solution is standard and more details can be found in Diacu (2000) and Yor (2001).

preference for an ICL.

For a higher subjective discount rate, we notice a reduction of the gap between the two systems if the earnings are low. For high earnings the preference for an ICL is increasing but the two systems give almost equal utility. In Panel B of Table 2, we observe that when the ML repayment period is very long, an ICL is still preferred for low earnings, but not for high earnings. Conversely, short ML repayment periods always make an ICL preferable. In other words we can also say that the size of the ML installments can change the preference: higher installments imply that the utility of a ML reduces sharply.

The effect of increasing risk aversion is to strengthen the preferences for the desired system. Low income earners prefer an ICL, and becoming more risk averse increase their utility under this system. Instead for high income earners the effect of risk aversion is correlated to the level of uncertainty. If σ is low, they prefer a ML even at high risk aversion. If the level of σ is high the two systems are more or less equal, although for high risk aversion a ML becomes the favourite.

5.2 Policy Implications

We perform some further simulations using the feature of the English HE reform to indicate some practical policy implication from our model.

In the top panel of Table 3 we notice that if the level of the initial earnings is low, an ICL is always preferred and increasingly when the costs rise. When the initial earnings are low the effect is reversed and a ML is the preferred system.

So far we have assumed that individuals will always participate in HE irrespective of the loan scheme but HE participation[19] may itself depend on the parameters of the loan schemes available. So we now consider the level of participation in HE under the two systems.

We assume a HE education wage premium of around 30%, consistent with most of the OLS estimations of the college premium. We model this assuming constant earnings for those not going to college, which are 30% lower than the initial earnings of those going to college. We compute the total present values of the net earnings under a ML and an ICL and we compare them with the total present value of the non schooling earnings. We consider the same parameters as in the previous simulations.

Observing the bottom panel of Table 3, when the initial earnings are low and we have low σ the participation is almost the same under the 2 systems. Instead when σ increases, participation under an ICL is higher relative to a ML. Keeping σ constant, e.g. 5%, and assuming the pre-reform cost of education, the participation in HE with an ICL is 7.6% higher than under a ML. Assuming the post-reform cost of education, we observe a bigger effect of an ICL on participation, around 26% higher than a ML. When we consider a high level of earnings the difference between the two systems is very small, with only a slightly larger participation rate under a ML.

6 Conclusion

In this work we presented a theoretical model which combined the riskiness of the investment in HE, due to the uncertainty of its outcomes, and two loan-based systems of funding higher

[19] By participation we intend individuals that went to HE.

education. We assumed that risk averse and risk neutral individuals receive a loan from the government to finance their education. We derived and compared, under an income contingent loan and a mortgage loan schemes, the lifetime expected utilities of graduates who receive a wage affected by a single stochastic shock. We have explicitly dealt with the hidden subsidies, which are the main drawback of an ICL, and when they are ruled out risk neutral individuals always prefer a ML. The findings for risk averse individuals are obtained by calibrating our model using real data on graduate earnings and their volatility. Our main results is that for high earnings uncertainty and also for increasing risk aversion, an ICL is the preferred system. We evaluated our model without hidden subsidies and under different possible scenarios. We found that graduates from low educated parental background, males over females, graduates working in the private sector prefer an ICL. While for graduates with less dynamic careers, such as in the public sector, a ML is the preferred system.

In the second part of the work, we changed the assumptions on the earnings, allowing for a stochastic growth across the entire working life. We compared the expected utilities under the two loan schemes using a numerical method. We found that the preference for one system over the other is driven by the level of the initial earnings and the size of the ML installment. In general, for low initial earnings an ICL is the preferred system. Finally, we showed how our model can offer interesting policy implications for the debate on student loans. We have considered the features of the UK higher education reform which has switched the funding system from a ML to an ICL and increased the university fees. Our model suggested that an ICL is the preferred system and increases participation in HE compared to a ML funding system.

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A Appendix: Proofs

A.1 Proofs of Proposition 1

We know that $T_{ICL} = \frac{C}{\gamma \tilde{y}}$ and $T_{ML} = \frac{c}{\varphi}$, and given the assumption $\mathrm{E}[\tilde{y}] = 1$ we compute the expected value of the repayment period under an ICL.

$$\mathrm{E}(T_{ICL}) = \mathrm{E}(\frac{C}{\gamma \tilde{y}}) = \frac{C}{\gamma} \times \mathrm{E}(\frac{1}{\tilde{y}})$$

by the Jensen's inequality we know that

$$\frac{C}{\gamma} \times \mathrm{E}(\frac{1}{\tilde{y}}) > \frac{C}{\gamma \mathrm{E}(\tilde{y})}$$

that implies

$$\mathrm{E}(\frac{1}{\tilde{y}}) > 1.$$

Given this result it is straightforward to prove the point (a):
if $\gamma = \varphi$ then

$$\frac{C}{\gamma} \times \mathrm{E}(\frac{1}{\tilde{y}}) > \frac{C}{\varphi} \rightarrow \mathrm{E}(T_{ICL}) > T_{ML}.$$

Point (b)
We assume $\mathrm{E}(T_{ICL}) = T_{ML}$ that means

$$\frac{C}{\gamma} \times \mathrm{E}(\frac{1}{\tilde{y}}) = \frac{C}{\varphi}$$

we get γ:

$$\gamma = \varphi \times \mathrm{E}(\frac{1}{\tilde{y}}) \quad \Longrightarrow \gamma > \varphi$$

since $\mathrm{E}(\frac{1}{\tilde{y}}) > 1$.

A.2 Proof of Proposition 2

Under risk neutrality equation (1) becomes

$$V = \mathrm{E}(\int_s^{\infty} \mathrm{E}^{-\rho t} \tilde{y}\ dt) - \mathrm{E}(\int_s^{T+s} e^{-\rho t} R\ dt) \tag{15}$$

So we can compare only the expected costs. Under a ML the present value of the cost of size C is:

$$\begin{aligned} PVC_{ML} &= \int_s^{T+s} \varphi\, e^{-\rho t}\, dt \\ &= e^{-\rho s}\, \frac{\varphi}{\rho}\, [1 - e^{-\rho \frac{C}{\varphi}}]\,. \end{aligned} \tag{16}$$

Under an ICL the present value of the cost of size C is:

$$
\begin{aligned}
PVC_{ICL} &= \int_{s}^{\tilde{T}+s} \tilde{y}\,\gamma\, e^{-\rho t}\, dt \\
&= e^{-\rho s}\,\frac{\gamma\,\tilde{y}}{\rho}\left[1 - e^{-\rho\frac{C}{\gamma\tilde{y}}}\right] \quad .
\end{aligned}
\tag{17}
$$

Knowing that $\mathrm{E}(\tilde{y}) = 1$, we take the expected value of both the equations above.

$$
\mathrm{E}(PVC_{ML}) = \frac{\varphi}{\rho}\,[1 - e^{-\rho\frac{C}{\varphi \mathrm{E}(\tilde{y})}}]e^{-\rho s} \tag{18}
$$

$$
\mathrm{E}(PVC_{ICL}) = \mathrm{E}\,\big[\frac{\gamma\,\tilde{y}}{\rho}\,(1 - e^{-\rho\frac{C}{\gamma\tilde{y}}})e^{-\rho s}\,\big] \tag{19}
$$

Case $\gamma = \varphi$

Under this condition we have $\mathrm{E}[T_{ICL}] > T_{ML}$. We can easily observe that the expected values of the costs can be written:

$$
\mathrm{E}(PVC_{ML}) = f[\mathrm{E}(\tilde{y})]
$$

$$
\mathrm{E}(PVC_{ICL}) = \mathrm{E}f(\tilde{y})
$$

Since $f(\tilde{y}) = \frac{\gamma\,\tilde{y}}{\rho}\left(1 - e^{-\frac{\rho\,C}{\gamma\,\tilde{y}}}\right)e^{-\rho s}$ is a concave function[20]by the Jensen's inequality we obtain that the expected costs under an ICL are lower than the expected costs under a ML: $\mathrm{E}(PVC_{ICL}) < \mathrm{E}(PVC_{ML})$. According to equation (15) the expected utility under an ICL is higher than the expected utility under a ML.

Case $\mathrm{E}[T_{ICL}] = T_{ML}$

To verify which costs are higher we have to substitute $\varphi = \frac{\gamma}{\mathrm{E}(1/\tilde{y})}$ in the equation (18) and then compare the two equations of the present value of the costs under the two systems:

$$
\mathrm{E}(PVC_{ML}) = \frac{\varphi}{\rho}\,[1 - e^{-\rho\frac{C}{\frac{\gamma}{\mathrm{E}(1/\tilde{y})}\mathrm{E}(\tilde{y})}}]e^{-\rho s} \tag{20}
$$

$$
\mathrm{E}(PVC_{ICL}) = \mathrm{E}\,\big[\frac{\gamma\,\tilde{y}}{\rho}\,(1 - e^{-\rho\frac{C}{\gamma\tilde{y}}})e^{-\rho s}\,\big] \tag{21}
$$

As we can see the comparison is not straightforward, therefore we adopt a numerical solution using the real data on graduate earnings provided in our dataset from BCS70.

In order to be consistent with the assumption $\mathrm{E}(\tilde{y}) = 1$, we first standardize the annual gross earnings of the graduates. We call w_i the earnings in the sample and divide each of the them by the sample mean, and we call this new variable z.

$$
z_i = \frac{w_i}{\frac{1}{n}\sum_{i=1}^{n} w_i} \quad for\;\; i = 1...n
$$

[20] $f''(\tilde{y}) = -\frac{\rho C^2 e^{-\left(s+\frac{C}{\gamma\tilde{y}}\right)\rho}}{\gamma\tilde{y}^3}$. It is reasonable to assume that γ, ρ and C are all greater or equal than zero. Therefore, the second derivative of $f(\tilde{y})$ is always negative when the shock on earnings is positive: $f''(\tilde{y}) < 0, \quad \forall\,\tilde{y} > 0$.

then

$$\overline{z} = \frac{1}{n}\sum_{i=1}^{n} z_i = 1$$

and we use the sample analogue $\overline{z}$ to estimate $\mathrm{E}(\tilde{y}) = 1$. Instead, to estimate $\mathrm{E}(\frac{1}{\tilde{y}})$ we generate its sample analogue

$$\overline{h} = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{z_i} \quad .$$

In our sample $\overline{h} > \frac{1}{\overline{z}}$, then the Jensen's inequality holds.

B Appendix: Data and Calibrations

In this Appendix we illustrate the BCS70 dataset used as a basis for calibrating the theoretical model. We describe graduate earnings and their standard deviation, in different environments, in order to get an idea of the wage uncertainty. An important assumption is the absence of selection bias, although we know that it could matter even for variance comparisons (Chen, 2008). However here we are more interested in observing how the theoretical model works under different potentially real situations[21]. We assume that the college premium is around 30%, consistent with most of the OLS estimations of the college premium, and the loan scheme does not affect the wage distribution.

B.1 Data set BCS1970

Our statistics are based on the 1970 British Cohort Study (BCS70), that takes as its subjects around 17,000 British births in the week 5-11 April 1970. Subsequently, full sample surveys took place at ages 5, 10, 16, 26 and 30. BCS70 highlights all aspects of the health, educational and social development of its subjects as they passed through childhood and adolescence. In later sweeps, the information collected covers their transitions to adult life, including leaving full-time education, entering the labour market, setting up independent homes, forming partnerships and becoming parents. (Bynner, Butler et al., 2002). The initial sample follow-up in 1999-00 consists of 11261 respondents aged 30. The smaller sample size in the 1999-00 survey relative to the original survey in 1970 depends on sample attrition due to nonresponse and it cannot be avoided[22]. The lowest response rate in the BCS70 study was registered in the postal survey conducted in 1996 at age 26, the loss of observations was mainly due to a postal strike. However in the previous surveys, above all those based on interviews to the parents of the cohort's members, the rate of non response was quite high[23].

[21] The presence of selection bias is potentially an issue of what we are aware, however there is little evidence in the literature concerning the selection into subjects and into job sectors.

[22] In general, attrition due to non-response is low in the non-adult sweeps (1-3) and increases at the adult sweeps (4-5). For example, the response rates of the sweep 0 observed sample is over 86% at sweeps 1, 2 and 3 falling to around 73% at sweeps 4 and 5 (1970 British Cohort Study Technical Report, Calderwood et al. 2004).

[23] It should be noted that the reason for non-participation at a later sweep may be because the cohort member has died or permanently emigrated. It is, for example, also possible for data to be missing for one part of the schedule especially as, during the years of childhood, data were obtained from different sources (parents, teachers and medical personnel)(1970 British Cohort Study Technical Report (2004)).

In general, the age 30 survey (1999-2000) was the first systematic attempt with widespread coverage to collect qualification and earnings data. It had a high response rate and it involved face to face interviews. For the purposes of our work, we merge the sweeps 1999-2000, 1980 and 1986. The last two sweeps are used because they provide information on family background: that is, family earnings and parental education. We have to stress the point that BCS is the only dataset that has family background, and it seems useful for looking at earnings variance in graduation.

In our sample we include observations if: respondents have a NVQ 4 equivalent qualification in 2000[24]; they are in the labour market and earn a positive wage after graduation[25], expressed in 2000 prices. In particular, we consider those that got a degree from 1987 to 2000 and start working not earlier than the same year of graduation. This implies that the longest working period is 13 years, but we only consider the earnings in 1999-2000 and for full time or part time employees[26]. According to these criteria in the final sample there are 1177 respondents.

B.2 Descriptive Statistics

In Table 1 we show some descriptive statistics of the average annual gross wage and its standard deviation according to the individual characteristics, family background, degree subjects and job sector.
The average earnings in the sample are around £24000 with a standard deviation of £18300. Male average earnings are around 40% higher than female earnings, but also more than twice their variance

It is useful to consider a breakdown of data from family background because it is one of the determinant of participation in HE and because family earnings determine how much the individuals are allowed to borrow in the English loan schemes. We consider then the family earnings of the cohort members in 1980, when they are 10 years old. We define "low" family earnings below £99 per week in 1980 prices; "medium" family earnings between £100 and £200 per week; high family earnings above £200 per week. Unfortunately, we discount the graduates from low earnings families because they just are 2% of the sample. The data for medium and high earnings family look more reasonable and with a relatively low uncertainty compared to the graduate average. Observing the graduate earnings given the mother qualifications in 1980, those with a graduate mother get the highest earnings, but the most uncertain is when the mother hold an 'O level' (secondary school qualification).

In Table 1 we consider three degree subjects, the earnings are above the average in all the cases, and quite similar to each other. However, those that took a degree in sciences (around 25% of the sample) have the lowest standard deviation. Finally, looking at the job sectors: 62% of the graduates work in the private sector and earn around 30% more than those in the public sector. However, in the latter the age earnings profile is flat and this is reflected by a very low level of uncertainty.

[24]The variable has been generated according to the UK national qualifications framework, NVQ equivalent level 4 includes academic qualifications (Degree and HE Diploma), vocational qualifications (BTEC Higher Certificate/ Diploma, HNC/HND) and occupational qualifications (NVQ level 4, Professional degree level qualifications, Nursing/paramedic, Other teacher training qualification, City & Guilds Part 4, RSA Higher Diploma).

[25]We exclude those working before and during education because this is a specific assumption in the theoretical model.

[26]This restriction allow us to clean from many inconsistencies in the earnings, and it is based on work undertaken by Dearden et al. (2008).

B.3 Calibration Settings

After the British Higher Education reform the annual cost of education was set up to a max of £3000 pounds, while before the reform the cost was £1150 a year[27]. Assuming a 3-year degree, we set two levels of total cost: £3450 and £9000.
In all the computations we set the following parameters:

- risk aversion: $a = \{0.25, \quad 0.5, \quad 0.75, \quad 1.5\}$, following the literature (Olson, White and Shefrin, Weiss);
- subjective discount factor: $\rho = \{0.08, \quad 0.15, \quad 0.3\}$, following the literature;
- cost of education: $C = \{£3450, \quad £9000\}$, before and after the English Higher Education reform;
- ICL repayment rate: $\gamma = \{0.02, \quad 0.09, \quad 0.2\}$, where 0.09 is the current rate fixed by the English reform;

In the our analysis when we change one parameter we keep the others constant at these levels: ICL repayment rates 9% (for English relevance), subjective discount rate 8%, cost £9000 (English current cost for 3-year degree), risk aversion 0.5. We do not consider every conceivable combination of parameters. However our qualitative results on the effect of any parameter is not sensitive to the assumed values of the other parameters.

C Appendix: Expected Utilities

C.1 Expected Utility with a Mortgage Loan

The Taylor approximation in equation (6) is the following

$$\begin{aligned} \mathrm{E}[u(\tilde{y}-\varphi)] &= \mathrm{E}\left\{u\left(1-\varphi\right)+u^{'}\left(1-\varphi\right)\left(\tilde{y}-1\right)+\frac{1}{2}u^{''}\left(1-\varphi\right)\left(\tilde{y}-1\right)^{2}\right\} \\ &= u\left(1-\varphi\right)+u^{'}\left(1-\varphi\right)\mathrm{E}(\tilde{y}-1)+\frac{1}{2}u^{''}\left(1-\varphi\right)\mathrm{E}(\tilde{y}-1)^{2} \\ &= u\left(1-\varphi\right)+\frac{1}{2}u^{''}\left(1-\varphi\right)\sigma^{2}. \end{aligned} \tag{22}$$

Plugging the equations (6) and (7) in the equation (5), substituting $T = C/\varphi$ and solving the integral, we obtain:

$$\begin{aligned} V_{ML} = &\frac{e^{-\rho s}}{\rho}\left(1-e^{-\frac{\rho C}{\varphi}}\right)\left[u\left(1-\varphi\right)+\frac{1}{2}u^{''}\left(1-\varphi\right)\sigma_s^2\right] \\ &+\frac{e^{-\rho s}}{\rho}e^{-\frac{\rho C}{\varphi}}\left[u\left(1\right)+\frac{1}{2}u^{''}\left(1\right)\sigma_s^2\right]. \end{aligned} \tag{23}$$

Finally, substituting a CRRA utility function in equation(23) and simplifying we get equation (8).

[27] In practice all institutions have charged the maximum.

C.2 Expected Utility with an Income Contingent Loan

In Section (4) we defined a new function $g(\tilde{y})$ as:

$$g(\tilde{y}) = \left[1 - e^{-\frac{\rho C}{\gamma \tilde{y}}}\right] u\left[\tilde{y}\left(1-\gamma\right)\right] + \left[e^{-\frac{\rho C}{\gamma \tilde{y}}}\right] u\left(\tilde{y}\right) \tag{24}$$

We rewrite the equation (10)

$$V_{ICL} = \frac{e^{-\rho s}}{\rho} \mathrm{E}[g(\tilde{y})] \tag{25}$$

and we apply a second order Taylor expansion to $\mathrm{E}[g(\tilde{y})]$, around the mean $\mathrm{E}\left[\tilde{y}\right] = 1$, then:

$$\begin{aligned} \mathrm{E}[g(\tilde{y})] &= \mathrm{E}\left\{ g(1) + g'(1)(\tilde{y}-1) + g''(1)\frac{(\tilde{y}-1)^2}{2} \right\} \\ &= g(1) + g'(1)\mathrm{E}(\tilde{y}-1) + \frac{g''(1)}{2}\mathrm{E}\left(\tilde{y}-1\right)^2 \\ &= g(1) + g''(1)\frac{\sigma_s^2}{2}. \end{aligned} \tag{26}$$

The equation (25) becomes

$$V_{ICL} = \frac{e^{-\rho s}}{\rho}\left[g(1) + g''(1)\frac{\sigma_s^2}{2}\right] \tag{27}$$

From now on we follow this procedure:

1. we work out the value of g(1), in general and with a CRRA utility function;
2. we work out the first derivative and the second derivative of g($\tilde{y}$), both in general and with a CRRA utility function;
3. we calculate $g'(1)$ and $g''(1)$ using a CRRA utility function;
4. we substitute the equations of $g(1)$ and $g''(1)$, using a CRRA utility function, in the equation (27) and we obtain equations (34) and (12).

- **Value of g(1)**

  In general,

$$g(1) = \left[1 - e^{-\frac{\rho C}{\gamma}}\right] u\left[(1-\gamma)\right] + \left[e^{-\frac{\rho C}{\gamma}}\right] u\left(1\right) \tag{28}$$

Using a CRRA utility function we have

$$g(1)_{CRRA} = \frac{1}{b}[-e^{-\frac{\rho C}{\gamma}}((1-\gamma)^b - 1) + (1-\gamma)^b]. \tag{29}$$

- **Value of g'($\tilde{y}$)**

In general,

$$g'(\tilde{y}) = u'[\tilde{y}(1-\gamma)](1-\gamma)\left[1-e^{-\frac{\rho C}{\gamma\tilde{y}}}\right] + u[\tilde{y}(1-\gamma)]\left[\frac{-\rho C e^{-\frac{\rho C}{\gamma\tilde{y}}}}{\gamma\tilde{y}^2}\right]$$
$$+ u'(\tilde{y})[e^{-\frac{\rho C}{\gamma\tilde{y}}}] + u(\tilde{y})\left[\frac{\rho C e^{-\frac{\rho C}{\gamma\tilde{y}}}}{\gamma\tilde{y}^2}\right] \tag{30}$$

using a CRRA utility function:

$$g'(\tilde{y})_{CRRA} = (\tilde{y}(1-\gamma))^{b-1}(1-\gamma)\left[1-e^{-\frac{\rho C}{\gamma\tilde{y}}}\right] + (\tilde{y}(1-\gamma))^{b}\left[\frac{-\rho C e^{-\frac{\rho C}{\gamma\tilde{y}}}}{b\gamma\tilde{y}^2}\right]$$
$$+ \tilde{y}^{b-1}[e^{-\frac{\rho C}{\gamma\tilde{y}}}] + \left[\frac{\tilde{y}^{b-2}\rho C e^{-\frac{\rho C}{\gamma\tilde{y}}}}{b\gamma}\right]. \tag{31}$$

- **Value of g"($\tilde{y}$)**

$$g''(\tilde{y}) = \frac{e^{-\frac{\rho C}{\gamma\tilde{y}}}\rho C(2\gamma\tilde{y}-\rho C)}{\tilde{y}^4\gamma^2}u[\tilde{y}(1-\gamma)] + \frac{e^{-\frac{\rho C}{\gamma\tilde{y}}}\rho C(-2\gamma\tilde{y}+\rho C)}{\tilde{y}^4\gamma^2}u(\tilde{y})$$
$$- \frac{2e^{-\frac{\rho C}{\gamma\tilde{y}}}\rho C(1-\gamma)}{\tilde{y}^2\gamma}u'[\tilde{y}(1-\gamma)] + \frac{2e^{-\frac{\rho C}{\gamma\tilde{y}}}\rho C}{\tilde{y}^2\gamma}u'(\tilde{y})$$
$$+ \left[1-e^{-\frac{\rho C}{\gamma\tilde{y}}}\right](1-\gamma)^2 u''[\tilde{y}(1-\gamma)] + \left[e^{-\frac{\rho C}{\gamma\tilde{y}}}\right]u''(\tilde{y}). \tag{32}$$

Now we work out $g''(\tilde{y})$ using a a CRRA and evaluating in $\tilde{y} = 1$

$$g''(1)_{CRRA} = \frac{1}{b\gamma^2}\{e^{-\frac{\rho C}{\gamma}}[(b-1)b\gamma^2[1+(e^{\frac{\rho C}{\gamma\tilde{y}}}-1)(1-\gamma)^b]$$
$$+ 2\rho C(b-1)\gamma(1-(1-\gamma)^b) + C^2\rho^2(1-(1-\gamma)^b)]\}. \tag{33}$$

- **Results**

Substituting $g(1)$ and $g''(1)$ in equation (27) we get the general expected utility under an income contingent loan:

$$V_{ICL} = \left[1-e^{-\frac{\rho C}{\gamma}}\right]u\left[(1-\gamma)\right] + \left[e^{-\frac{\rho C}{\gamma}}\right]u\left(1\right)$$
$$+ [\frac{e^{-\frac{\rho C}{\gamma}}\rho C(2\gamma-\rho c)}{\gamma^2}u[1-\gamma] + \frac{e^{-\frac{\rho C}{\gamma}}\rho C(-2\gamma+\rho c)}{\gamma^2}u(1)$$
$$- \frac{2e^{-\frac{\rho C}{\gamma}}\rho C(1-\gamma)}{\gamma}u'[1-\gamma] + \frac{2e^{-\frac{\rho C}{\gamma}}\rho C}{\gamma}u'(1)$$
$$+ \left[1-e^{-\frac{\rho C}{\gamma}}\right](1-\gamma)^2 u''[1-\gamma] + \left[e^{-\frac{\rho C}{\gamma}}\right]u''(1)]\frac{\sigma_s^2}{2}. \tag{34}$$

Substituting in equation (27) the equations for $g(1)$ and $g''(1)$ with a CRRA utility function, we obtain equation (12).

D Appendix: Increasing Earnings

D.1 Numerical Method - Brownian Motion

1. We generate a path of annual earnings for an individual working life. Since the problem requires a discrete solution, we apply the Euler-Maruyama method that takes the form

$$y_j = y_{j-1} + y_{j-1}\lambda\Delta t + y_{j-1}\sigma(W(\tau_j) - W(\tau_{j-1})). \tag{35}$$

   To generate the increments $W(\tau_j) - W(\tau_{j-1})$ we compute discretized Brownian motion paths, where $W(t)$ is specified at discrete t values. As explained in Higham (2001) we first discretize the interval $[0, I]$. We set $dt = I/N$ for some positive integer N, and let W_j denote $W(t_j)$ with $t_j = jdt$. According to the properties of the standard Brownian motion $W(0) = 0$ and

$$W_j = W_{j-1} + dW_j \tag{36}$$

   where dW_j is an independent random variable of the form $\sqrt{dt}N(0,1)$. The discretized brownian motion path is a 1-by-N array, where each element is given by the cumulative sum in equation (36). To generate equation (35), we define $\Delta t = I/L$ for some positive integer L, and $\tau_j = \Delta t$. As in Higham (2001) we choose the stepsize Δt for the numerical method to be an integer multiple $R \geq 1$ of the Brownian motion increment dt: $\Delta t = Rdt$. Finally, we get the increment in equation (35) as cumulative sum:

$$W(\tau_j) - W_{(\tau_{j-1})} = W(jRdt) - W((j-1)Rdt) = \sum_{h=jR-R+1}^{jR} dW_h. \tag{37}$$

   The Brownian motion of equation (36) is produced setting $I = 1$ and $N = 160$ in order to have a small value of dt. Using a random number generator we produce 160 "pseudorandom" numbers from the N(0,1) distribution. The increments of equation (37) are computed setting $R = 4$, in order to have 40 annual earnings.

2. Income contingent loan. We work out the yearly repayments as fixed percentage of the stochastic earnings generated. If the earnings are higher than £15000 the payments are positive, otherwise they are zero. We then built a vector whose elements are the cumulative sum of the repayments, in order to see the amount of loan repaid. To obtain the repayment period, we observe the years in which the cumulative sum of the payments is equal[28] to the cost of education. We work out the individual utility as discounted sum of the net earnings during and after the repayment period, up to the end of the working life. We use a CRRA utility function.

3. Mortgage loan. We set the fixed repayment period as the ratio between the cost of education and the annual installment. The individual utility is given by the discounted

[28]Since it is almost impossible to get a value equal to the cost, when the repayment is slightly greater than it we assume the debt has been paid off.

sum of the net earnings during and after the repayment period. We use a CRRA utility function. However, it can happen that the annual earnings are lower than the installment, in a usual mortgage loan the individual repays in the subsequent years at a higher interest rate. Here to highlight the loss of utility in the case of no repayment in one year, we compute the level of the utility for that year as a negative percentage[29] of the annual earnings.

4. From steps (2) and (3) we obtain a single value for the utility for an individual earnings path generated in point (1). We generalize our method generating a high number of earnings paths (1000) and for each path we compute a level of utility. We then work out the average utility under both financing scheme and the difference of the average in order to compare the two systems.

5. We let the various parameters change and we repeat steps (1) to (4), observing the trend of the difference of the average utility under the two funding schemes.

D.2 Setting the new model

Under this new stochastic framework the only variable that we fix using actual data is the initial earnings. In our BCS70 dataset, we generate a new variable for initial earnings, using graduate earnings within four years of graduation. In Table 1 we give an idea of the initial wage distribution reporting some percentiles. In our simulation we use £8,600, which is the mean of the fifth percentile and we define it as low initial earnings. We consider as high initial earnings the median value of £23,987, since all the simulations' results using a bigger value are similar. Uncertainty now affects the growth of earnings over years, we choose three levels of σ (0.02, 0.05, 0.15) in order to have different intensities of the effect of the stochastic shock on earnings. For example, $\sigma = 0.05$ means that the maximum annual variation of the earnings can be 5% around the trend growth, which occurs if $\sigma = 0$.

In our dataset we do not have information on the growth rate of the earnings because we observe a cross section in 1999-00. If we had merged this with the sweep of 1996 we would have lost a lot of observations, since that survey was conducted through mail questionnaires and many people did not respond. Therefore, we would have ended up with just a few observations. We therefore simply set three values for the deterministic growth rate ($\lambda = 0.5, \quad 1, \quad 1.5$). Assuming no uncertainty, for example a $\lambda = 0.5$ corresponds to a increase of 1.2% per year of initial earnings over 35 years. If $\lambda = 1$ the increase of the initial earnings is 2.4% p.a.

Extension from the static earnings model would be simple if earnings never fell below a threshold - effectively moving the bankruptcy constraint from 0 to £15,000. The £15,000 threshold is really just an institutional detail of the English system - there for equity reasons. We simulate with different thresholds: first zero threshold in order to have a similar case as that analyzed in the previous section, then a threshold of £10,000 and finally a threshold of £15,000 as in the English reform. This means that if in any year the earnings are below the threshold no payment is due. We perform the simulations setting first an equal repayment period[30] between the 2 systems. Then we fix the ML installment to £1000 for a cost of £9000; and to £1500 and £2400 for a cost of £12000. In this way we get three fixed repayment periods

[29] We set this percentage equal to the average-low interest rate for a typical mortgage loan e.g. around 5%.

[30] Since the earnings are stochastic, we first get the repayment period under an ICL and then set this value also for a ML.

of 10, 8, 5 years respectively. The costs are chosen according the English Reform, that is £3000 per year for a 3-year degree. We finally set the other parameters as stated in Appendix B.3.

Table 1: Graduates Earnings and Standard deviations - BCS1970

	mean	**sd**	%
Total sample	24,023	18,369	100
by Gender			
male	27,898	22,577	52.93
female	19,666	10,407	47.07
by Family income in 1980			
low	32,384	56,744	2.12
medium	23,053	16,882	64.49
high	25,355	13,182	21.16
by Mother Qualifications in 1980			
no qualifications	22,306	18,957	30.08
O level	25,773	22,908	22.51
degree	27,149	15,615	6.12
by Degree Subjects			
Sciences	26,782	16,828	24.81
Social Sciences	25,858	21,385	12.40
Art and humanity	26,526	25,277	15.72
by Job Sector			
private	26,434	21,703	62.53
public	20,357	9,911	30.42
Initial earnings			
low (5th perc)	8,640	2,652	
median	23,987	13,147	
high (95th perc)	53,002	20,505	

Value in thousands UK sterling at 2000 prices
Sample size 1177.

Table 2: Multiple Stochastic Shocks on Earnings - $AU_{ICL} - AU_{ML}$

Panel A: ρ *and* σ *changing*

	$\varphi = £1000$	$C = £9000$	$T_{ML} = 9$
	$\gamma = 9\%$	$\lambda = 1\%$	$ra = 0.5$
Low Initial Earnings			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$\rho = 8\%$	67.2721	67.2994	67.4311
$\rho = 15\%$	51.5189	51.5387	51.6331
$\rho = 30\%$	32.7445	32.7559	32.8088
High Initial Earnings			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$\rho = 8\%$	-0.3704	-0.2421	0.0227
$\rho = 15\%$	0.7800	0.8651	1.1011
$\rho = 30\%$	1.4404	1.4808	1.6262

Panel B: φ and σ changing

	$C = £9000$ $\rho = 8\%$		
	$\gamma = 9\%$ $\lambda = 1\%$ $ra = 0.5$		
Low Initial Earnings			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$\varphi = £500$ $T_{ML} = 18$	48.8080	48.8332	48.9630
$\varphi = £1000$ $T_{ML} = 9$	67.2721	67.2994	67.4311
$\varphi = £3000$ $T_{ML} = 3$	90.6606	90.6905	90.8231
High Initial Earnings			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$\varphi = £500$ $T_{ML} = 18$	-10.9630	-10.8355	-10.5687
$\varphi = £1000$ $T_{ML} = 9$	-0.3704	-0.2421	0.0227
$\varphi = £3000$ $T_{ML} = 3$	10.9286	11.0558	11.3084

Panel C: ra and σ changing

	$\varphi = £1000$	$C = £9000$	$T_{ML} = 9$
	$\gamma = 9\%$ $\lambda = 1\%$ $\rho = 8\%$		
Low Initial Earnings			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$ra = 0.25$	6.9826	6.9870	7.0090
$ra = 0.5$	67.2721	67.2994	67.4311
$ra = 1.2$	38271.00	38265.00	38242.00
High Initial Earnings			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$ra = 0.25$	-0.0224	-0.0121	0.0106
$ra = 0.5$	-0.3704	-0.2421	0.0227
$ra = 1.2$	-743.0753	-594.7867	-340.0252

Table 3: Policy Implications - Effects English Reform

$AU_{ICL} - AU_{ML}$

	$\varphi = £1000$ $T_{ML} = 9$		
	$\gamma = 9\%$ $\lambda = 1\%$ $\rho = 8\%$		
Low Initial Earnings			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$cost - pre$	2244	2268	2278
$cost - post$	6246	6101	6022
High Initial Earnings			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$cost - pre$	-539.46	-562.13	-615.31
$cost - post$	-411.55	-311.82	-304.39

Participation in HE

% Difference of individuals under ICL-ML

	$\varphi = £1000$ $T_{ML} = 9$		
	$\gamma = 9\%$ $\lambda = 1\%$ $\rho = 8\%$		
Low Initial Earnings			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$cost - pre$	0	7.6	8.5
$cost - post$	0.3	26.0	23.0
High Initial Earnings			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$cost - pre$	0	-0.8	-0.5
$cost - post$	0	-0.1	-0.5

Figure 1: $EC_{ML} - EC_{ICL}$ - Expected costs and Risk Neutrality

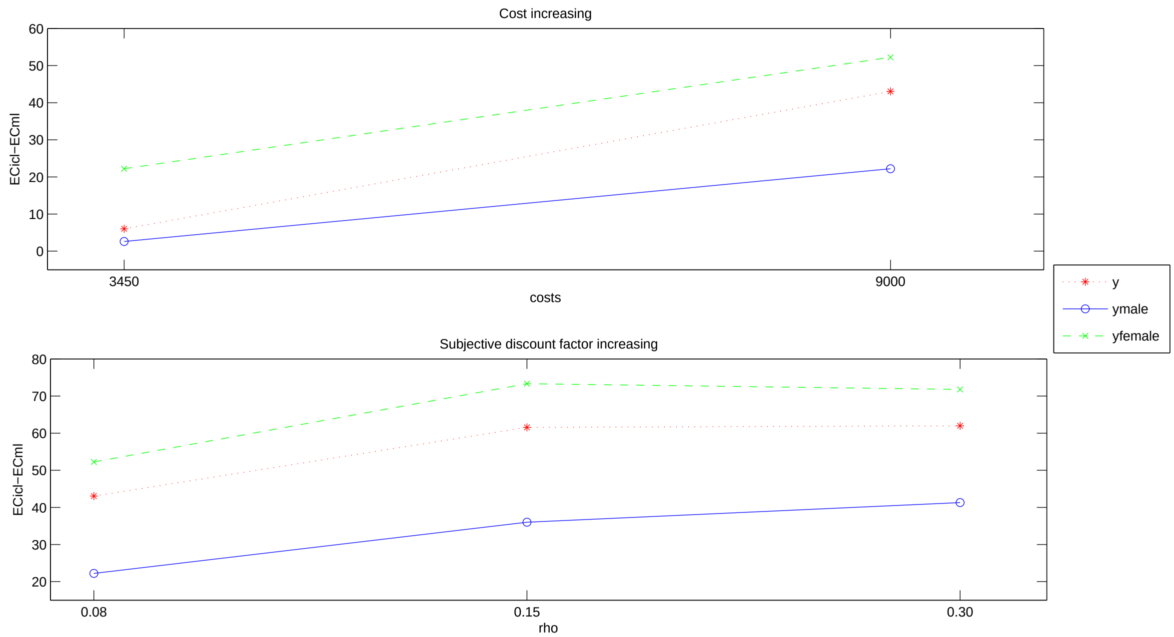

Figure 2: $EU_{ICL} - EU_{ML}$ - Individual Characteristics - Gender

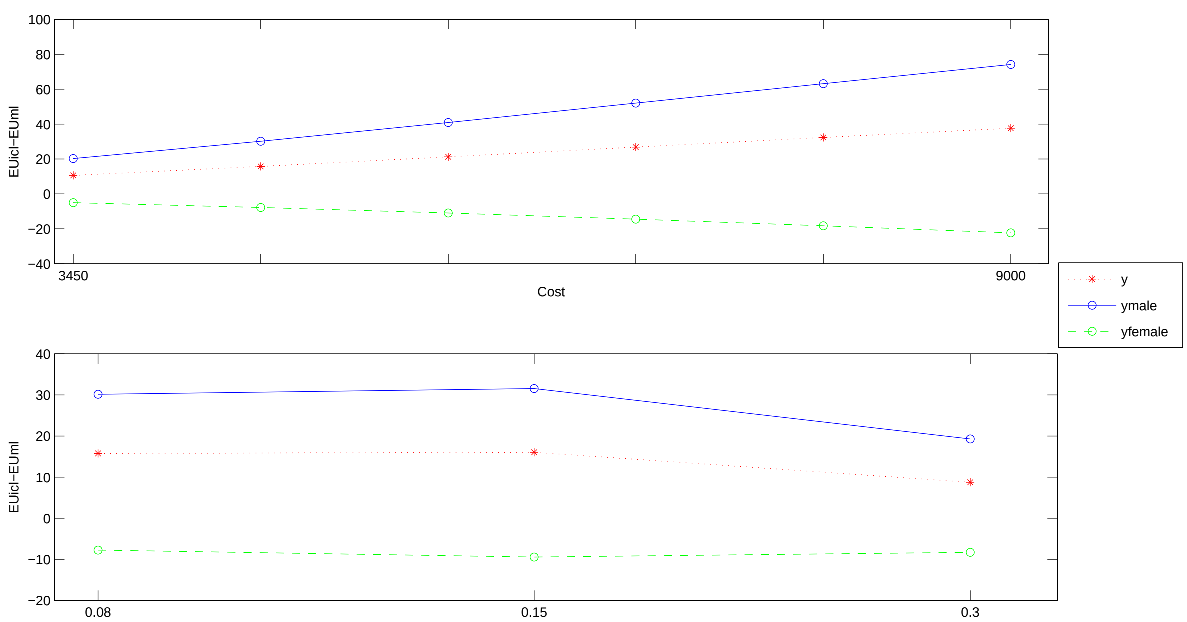

Figure 3: $EU_{ICL} - EU_{ML}$ - Family Background

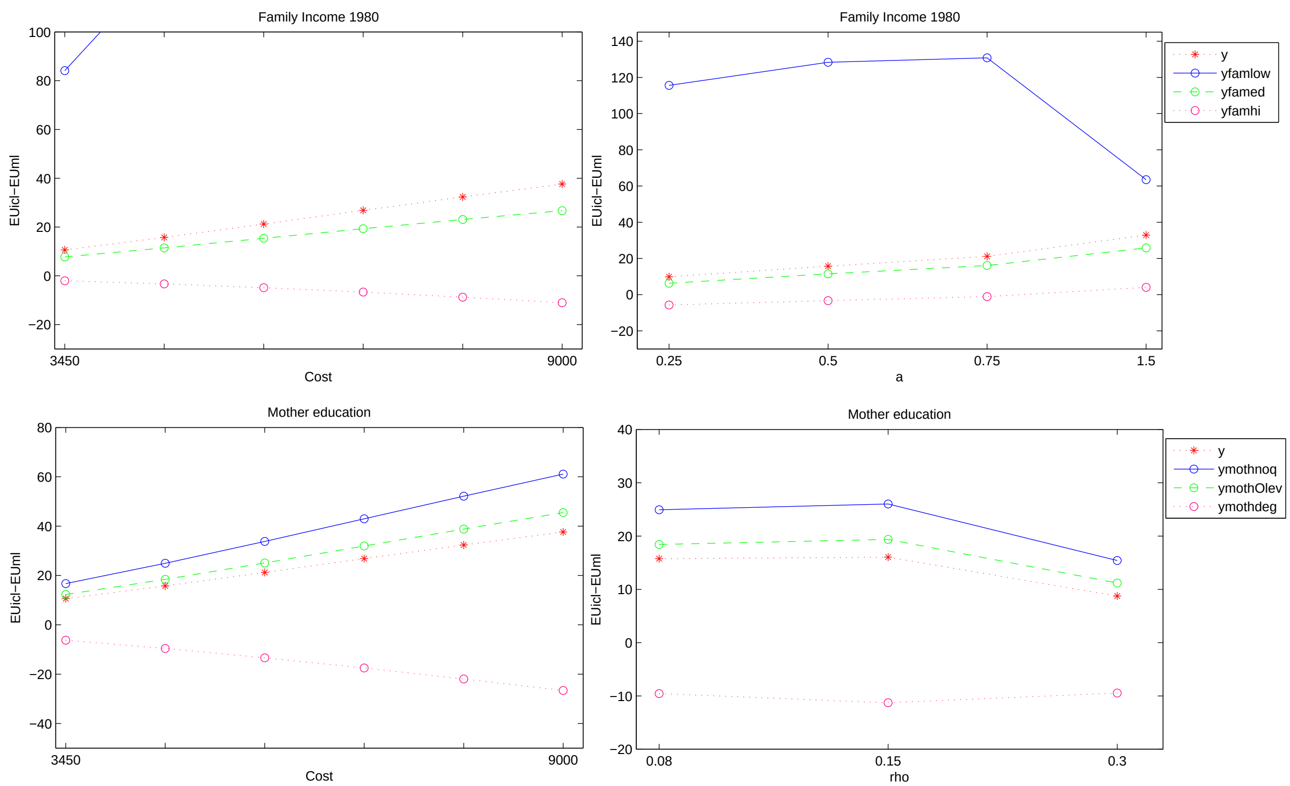

Figure 4: $EU_{ICL} - EU_{ML}$ - Degree Subjects

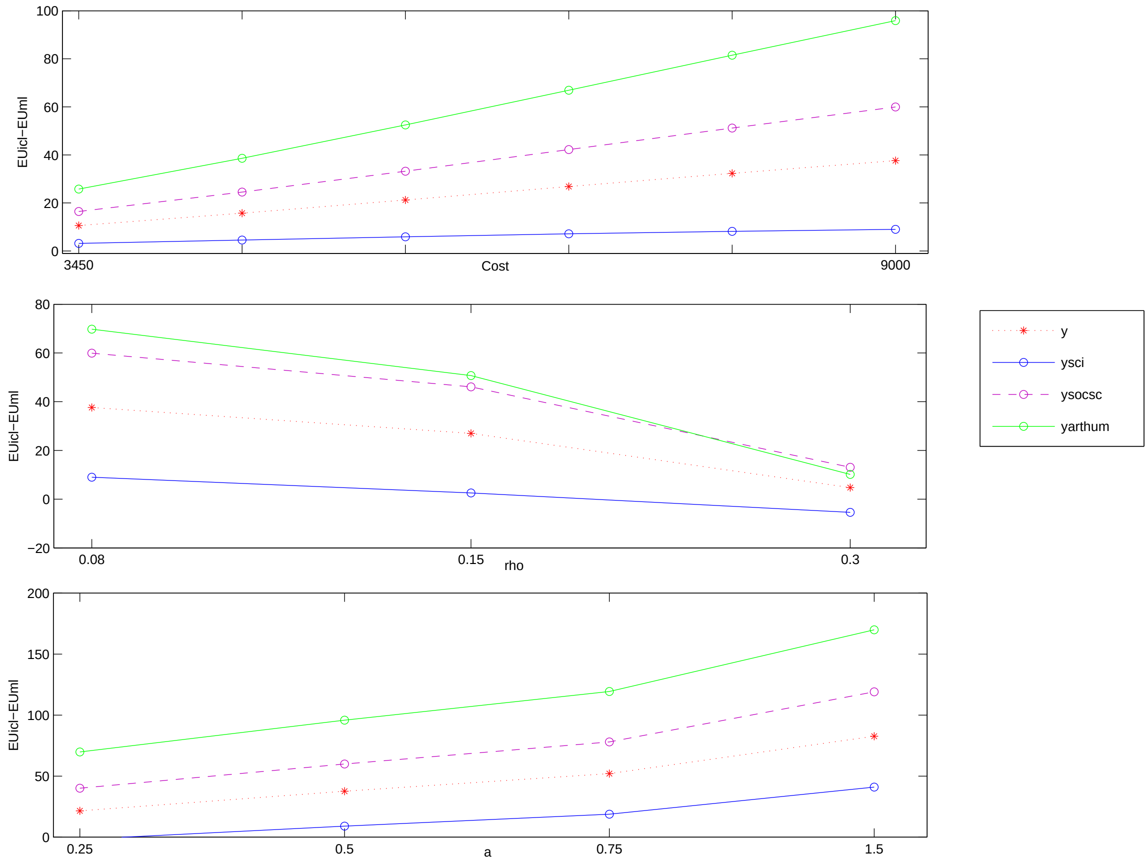

Figure 5: $EU_{ICL} - EU_{ML}$ - Public versus Private sector