5 Antennas

5.1 INTRODUCTION

In Chapter 1 we saw that electromagnetic waves can propagate through a uniform dielectric medium whose boundaries are so far away that they can be ignored. The subsequent chapters examined the effects of boundaries on wave propagation and, in particular, the properties of waves guided by transmission lines and waveguides. In this chapter we consider how waves in free space can be excited by waves in waveguides and vice versa. Essentially this is a problem of matching between the two media of propagation and a device which performs that function is called an antenna. Antennas are familiar objects in the modern world with its multitude of radio and television aerials (the old word for antennas) and the growing number of dishes for receiving satellite transmissions.

5.2 MAGNETIC VECTOR POTENTIAL

Before proceeding to the theory of antennas we require one new idea: the magnetic vector potential. In elementary textbooks (Carter, 1986) it is shown that the electric field can be calculated from the electrostatic potential using

$$\boldsymbol{E} = -\nabla V \tag{5.1}$$

where

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z} \tag{5.2}$$

in rectangular Cartesian coordinates. Equation (5.1) can also be interpreted in other coordinate systems as we shall see. Now (5.1) was derived from the theory of electrostatics but it cannot be correct for problems involving electromagnetic waves because it neglects the part of the electric field produced by a changing magnetic field. To get round this problem we define a new vector $\boldsymbol{A}$, known as the magnetic vector potential, by

$$B = \nabla \wedge A. \tag{5.3}$$

It may be recalled that the magnetic scalar potential defined by an equation analagous to (5.1) is of limited value because it is not a single-valued function (Carter, 1986, p. 56). Substituting (5.3) into (1.8) gives

$$\nabla \wedge E = -\frac{\partial}{\partial t}(\nabla \wedge A) \tag{5.4}$$

so that

$$\nabla \wedge \left(E + \frac{\partial A}{\partial t}\right) = 0. \tag{5.5}$$

Now suppose that the quantity in the brackets is equal to minus the gradient of the electrostatic potential V. Then

$$E = -\nabla V - \frac{\partial A}{\partial t}. \tag{5.6}$$

This equation, therefore, replaces (5.1) and reduces to it when the problem is a static one. The substitution of ∇V is justified because it can be shown that

$$\nabla \wedge (\nabla V) \equiv 0. \tag{5.7}$$

When (5.3) is substituted into (1.7) the result is

$$\nabla \wedge (\nabla \wedge A) = \mu_0 J + \mu_0 \frac{\partial D}{\partial t}, \tag{5.8}$$

where, for simplicity, it has been assumed that only fields in free space are to be considered. It can be shown that the left-hand side of this equation can be rewritten

$$\nabla \wedge (\nabla \wedge A) \equiv \nabla(\nabla \cdot A) - \nabla^2 A. \tag{5.9}$$

In fact A is not completely defined by (5.3) and we are free to impose the additional condition

$$\nabla \cdot A = -\varepsilon_0 \mu_0 \frac{\partial V}{\partial t}. \tag{5.10}$$

This is known as the Lorentz condition. It can be shown to be completely consistent with Maxwell's equations and it has the effect of making A depend solely on the distribution of currents as we shall see.

Substitution into (5.8) from (5.6), (5.9) and (5.10) gives

$$\nabla^2 A - \varepsilon_0 \mu_0 \frac{\partial^2 A}{\partial t^2} = -\mu_0 J. \tag{5.11}$$

This is a wave equation relating the components of A to the corresponding components of the source current.

Similarly, taking the divergence of (5.6) we have

$$\nabla \cdot E = -\nabla^2 V - \frac{\partial}{\partial t}(\nabla \cdot A) = \varrho/\varepsilon_0$$

so that

$$\nabla^2 V - \varepsilon_0 \mu_0 \frac{\partial^2 V}{\partial t^2} = -\varrho/\varepsilon_0. \tag{5.12}$$

Comparison of (5.11) and (5.12) shows that A bears a relationship to current sources very similar to that borne by V to charge sources.

5.3 RETARDED POTENTIALS

Equations (5.11) and (5.12) express the idea that if a current or a charge is changing with time the effect travels outwards with the speed of light. Thus if we are examining the electrostatic potential at a point P distant r from a charge Q the potential at time t will depend upon the value of Q at the earlier time $(t - r/c)$. This value of the charge is referred to as the retarded value and is denoted by $[Q]$. Using this notation the solution of (5.12) can be written

$$V = \frac{1}{4\pi\varepsilon_0}\frac{[Q]}{r} \tag{5.13}$$

for a point charge. For a distribution of charge this equation must be integrated over the space containing the charge to give

$$V = \frac{1}{4\pi\varepsilon_0}\iiint \frac{[\varrho]}{r}\,dv, \tag{5.14}$$

where the distance r ranges over the volume v as the integration is carried out.

By analogy the magnetic vector potential is given by

$$A = \frac{\mu_0}{4\pi}\iiint \frac{[J]}{r}\,dv. \tag{5.15}$$

These ideas are put to use in the next section.

5.4 SMALL ELECTRIC DIPOLE

Figure 5.1 shows one possible way of launching waves from the end of a coaxial line. The outer conductor of the line is connected to the edge of a hole in a large conducting plane whilst the end of the inner conductor projects a short distance into the space beyond the plane, where it is

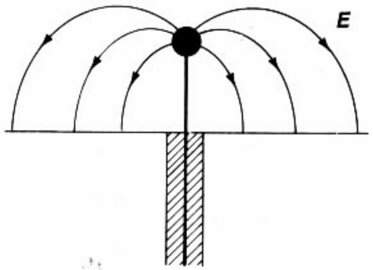

Fig. 5.1 The electric field of a monopole antenna formed by allowing the central conductor of a coaxial line to project through a conducting plane.

terminated by a small sphere. In this context 'short' means 'short compared with the free-space wavelength'. The end of the line is a capacitor whose electric field pattern is somewhat as shown in Fig. 5.1. If is assumed that all the charges accumulate on the spherical end then the current in the connecting wire can be considered to be uniform along its length and varying sinusoidally with time.

To analyse the antenna shown in Fig. 5.1 we employ the method of images and replace the lower half of the diagram by an identical antenna with opposite polarity as shown in Fig. 5.2. This arrangement with its pair of positive and negative charges is known as a dipole. The figure shows the electric and magnetic fields around the dipole at a moment when the polarities of the charges and the current in the wire are as shown. Examination of the directions of $\boldsymbol{E}$ and $\boldsymbol{H}$ shows that the Poynting vector is directed outwards. In the next quarter cycle, however, the direction of $\boldsymbol{H}$ is reversed and the Poynting vector points inwards suggesting that the field around the dipole stores energy but does not radiate it. We therefore have to show that the dipole does indeed radiate.

The field pattern around the dipole is cylindrically symmetrical. At large distances from it the length of the dipole is insignificant and the problem has spherical symmetry. It is best, therefore, to examine the problem in the spherical polar co-ordinate system shown in Fig. 5.3 (see Appendix B).

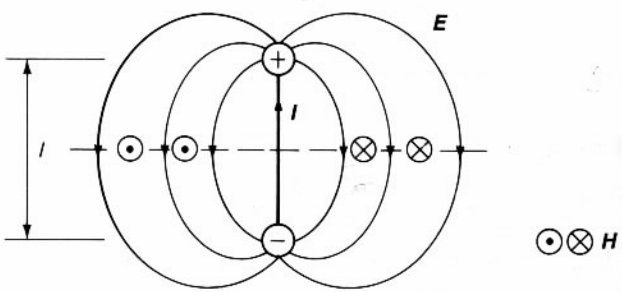

Fig. 5.2 The electric field around an alternating electric dipole.

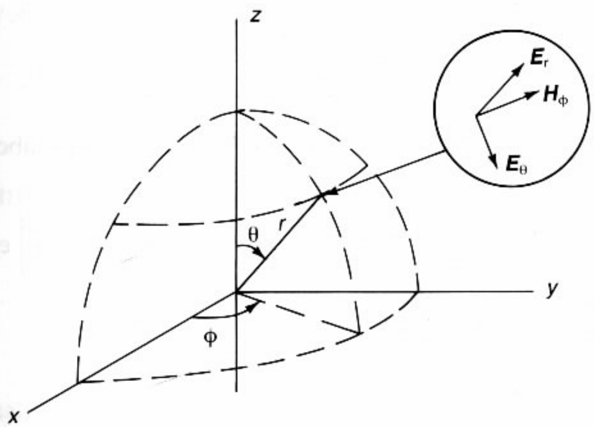

Fig. 5.3 Spherical polar coordinates.

From the symmetry of the problem we expect that the electric field will have r and θ components and the magnetic field only a ϕ component as shown in the inset to Fig. 5.3.

The dipole is represented by a pair of oscillating charges

$$q = \pm q_0 e^{j\omega t} \tag{5.16}$$

located at $\pm l/2$ as shown in Fig. 5.4. The potential at P is then

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_0 e^{j(\omega t - k_0 r_1)}}{r_1} - \frac{q_0 e^{j(\omega t - k_0 r_2)}}{r_2}\right], \tag{5.17}$$

where r_1 and r_2 are the distances from the two charges to P. If $r \gg l$ we can write

$$r_1 = r - \tfrac{1}{2}l\cos\theta = r - \delta r$$

and

$$r_2 = r + \tfrac{1}{2}l\cos\theta = r + \delta r \tag{5.18}$$

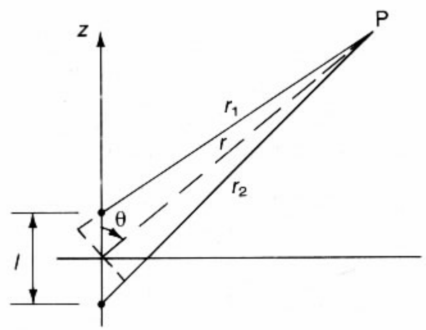

Fig. 5.4 Geometry of the radiation from a small electric dipole antenna.

so that

$$V = \frac{q_0}{4\pi\varepsilon_0 r}\left[\frac{e^{jk_0\delta r}}{(1-\delta r/r)} - \frac{e^{-jk_0\delta r}}{(1+\delta r/r)}\right] e^{j(\omega t - k_0 r)}. \tag{5.19}$$

Expanding the terms as power series and neglecting terms above the first order in $(\delta r/r)$ we obtain

$$V \simeq \frac{q_0}{4\pi\varepsilon_0 r}\left[\left(1+\frac{\delta r}{r}\right)\left(1+jk_0\delta r\right) - \left(1-\frac{\delta r}{r}\right)\left(1-jk_0\delta r\right)\right] e^{j(\omega t - k_0 r)}$$

$$\simeq \frac{q_0 l\cos\theta}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + j\frac{k_0}{r}\right) e^{j(\omega t - k_0 r)} \tag{5.20}$$

which can be written

$$V \simeq \frac{[p]\cos\theta}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + j\frac{k_0}{r}\right), \tag{5.21}$$

where $p = q_0 l$ is the electric dipole moment. The first term in (5.21) falls off more rapidly with r than does the second one. It represents the field of an electrostatic dipole. The second term arises because of the difference between the propagation times from the two charges to the point P.

The magnetic vector potential of the dipole is from (5.15)

$$\boldsymbol{A} = \frac{j\omega\mu_0[\boldsymbol{p}]}{4\pi r} \tag{5.22}$$

because

$$\iiint \boldsymbol{J}\,dv = I\boldsymbol{l} = \frac{dq}{dt}\boldsymbol{l} = j\omega\boldsymbol{p}. \tag{5.23}$$

The electric field of the dipole is obtained by substituting the electric and magnetic potentials from (5.21) and (5.22) into (5.6). It is convenient to consider separately the contributions from the two potentials. In spherical polar coordinates

$$\nabla V = \hat{\boldsymbol{r}}\frac{\partial V}{\partial r} + \frac{\hat{\boldsymbol{\theta}}}{r}\frac{\partial V}{\partial \theta} + \frac{\hat{\boldsymbol{\phi}}}{r\sin\theta}\frac{\partial V}{\partial \phi}, \tag{5.24}$$

(see Appendix B) where $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\phi}}$ are unit vectors in the three co-ordinate directions. Thus the electric potential contributes the field components

$$E_r = \frac{[p]\cos\theta}{4\pi\varepsilon_0}\left(\frac{2}{r^3} + \frac{2jk_0}{r^2} - \frac{k_0^2}{r}\right)$$

$$E_\theta = \frac{[p]\sin\theta}{4\pi\varepsilon_0}\left(\frac{1}{r^3} + j\frac{k_0}{r^2}\right)$$

$$E_\phi = 0. \tag{5.25}$$

The vector $\boldsymbol{A}$ is in the z direction so it has components

$$A_r = \boldsymbol{A}\cos\theta$$

$$A_\theta = -\boldsymbol{A}\sin\theta$$

$$A_\phi = 0, \tag{5.26}$$

giving contributions to the electric field from the magnetic potential

$$E_r = \frac{k_0^2[p]}{4\pi\varepsilon_0 r}\cos\theta$$

$$E_\theta = -\frac{k_0^2[p]}{4\pi\varepsilon_0 r}\sin\theta \tag{5.27}$$

so that, finally

$$E_r = \frac{[p]\cos\theta}{2\pi\varepsilon_0}\left(\frac{1}{r^3} + j\frac{k_0}{r^2}\right)$$

$$E_\theta = \frac{[p]\sin\theta}{4\pi\varepsilon_0}\left(\frac{1}{r^3} + j\frac{k_0}{r^2} - \frac{k_0^2}{r}\right). \tag{5.28}$$

The magnetic field is obtained from (5.3) and (5.22) making use of the expression for the curl of $\boldsymbol{A}$ in spherical polar co-ordinates (see Appendix B)

$$\nabla\wedge\boldsymbol{A} = \frac{\hat{\boldsymbol{r}}}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right] + \frac{\hat{\boldsymbol{\theta}}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \frac{\hat{\boldsymbol{\phi}}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]. \tag{5.29}$$

Fortunately most of the terms in this fearsome expression are zero because $\boldsymbol{A}$ has only r and θ components and these only vary with θ. Thus the only component of $\boldsymbol{H}$ is

$$H_\phi = \frac{1}{\mu_0 r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

$$= j\frac{\omega[p]\sin\theta}{4\pi}\left[\frac{1}{r^2} + j\frac{k_0}{r}\right]. \tag{5.30}$$

We recall that $j\omega p = Il$.

The complete expressions for the field of the dipole are rather unwieldy so it is useful to consider them in three parts. We note that some of the terms fall off with increasing r much faster than others. The terms which fall off fastest are

$$E_r = \frac{[p]\cos\theta}{2\pi\varepsilon_0 r^3}$$

$$E_\theta = \frac{[p]\sin\theta}{4\pi\varepsilon_0 r^3}. \qquad (5.31)$$

These correspond to the electrostatic field of the dipole and represent a capacitive storage of energy.

The next terms are

$$E_r = \mathrm{j}\frac{k_0[p]\cos\theta}{2\pi\varepsilon_0 r^2}$$

$$E_\theta = \mathrm{j}\frac{k_0[p]\sin\theta}{4\pi\varepsilon_0 r^2}$$

$$H_\phi = \mathrm{j}\,\frac{\omega[p]\sin\theta}{4\pi r^2}. \qquad (5.32)$$

The magnetic field is just that given by the Biot–Savart law for a current element (Carter, 1986, p. 53), so this term also represents a quasi-static field. If the direction of the Poynting vector for the fields in (5.32) is examined it is found that the energy associated with them is circulating close to the dipole. These terms are known as the induction field. Together with the electrostatic field they form the near field of the dipole.

The remaining terms are the far field

$$E_\theta = \frac{-k_0^2[p]\sin\theta}{4\pi\varepsilon_0 r}$$

$$H_\phi = \frac{-\omega k_0[p]\sin\theta}{4\pi r}. \qquad (5.33)$$

These are in phase with each other and the Poynting vector is directed radially outwards indicating that they represent radiation of power by the dipole. The wave impedance is

$$Z_0 = \frac{E_\theta}{H_\phi} = \frac{k_0}{\omega\varepsilon_0} = \sqrt{\left(\frac{\mu_0}{\varepsilon_0}\right)} \qquad (5.34)$$

exactly as for a plane TEM wave. In fact such a plane wave can be thought of as the limiting case of a spherical wave at large distances from the antenna.

The time average value of the Poynting vector is

$$S_r = \frac{1}{r}\left(\frac{p}{4\pi r}\right)^2 Z_0\omega^2 k_0^2 \sin^2\theta. \qquad (5.35)$$

To find the radiated power we integrate this over the surface of a sphere. The annular element of area shown in Fig. 5.5 has radius $r\sin\theta$ and width $r\,d\theta$. Its area is therefore $2\pi r^2 \sin\theta\, d\theta$. The power flow is given by

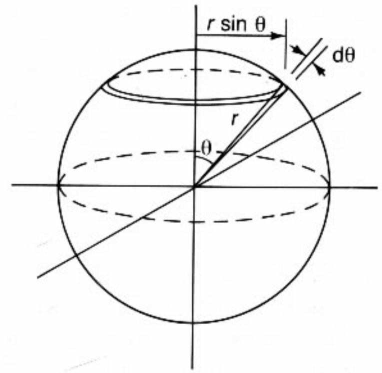

Fig. 5.5 Geometry for integrating the Poynting vector of a small dipole radiator over a sphere.

$$P = \int_0^\pi S_r\, 2\pi r^2 \sin\theta\, d\theta$$

$$= \frac{\omega^2 k_0^2 p^2 Z_0}{12\pi} = \frac{k_0^2 I^2 l^2 Z_0}{12\pi}. \qquad (5.36)$$

This expression is independent of the radius of the sphere as we would expect since the total power flow is the same through any closed surface surrounding the dipole. Equation (5.36) can be expressed in a slightly different form by recalling that $k_0 = 2\pi/\lambda$

$$P = 40\pi^2 I^2 (l/\lambda)^2. \qquad (5.37)$$

The input resistance of the dipole is, therefore,

$$R_r = \frac{2P}{I^2} = 80\pi^2 (l/\lambda)^2. \qquad (5.38)$$

This is known as the radiation resistance of the dipole, it can be seen that it increases with length. If we assume that $(l/\lambda) = 0.1$ which is about the maximum value for which the theory is valid then $R_r = 7.9\,\Omega$. This figure presents two problems. First it is not well matched to typical transmission-line impedances and, second, it means that a transmitter must supply large currents at low voltage giving rise to large ohmic losses in the connecting cables. The near field of the antenna stores electromagnetic energy and, therefore, contributes a reactive component to the input impedance (see Jordan and Balmain, 1968).

If the power radiated from the dipole were distributed uniformly over the surface of a sphere of radius r then the magnitude of the Poynting vector would be

$$S_i = \frac{1}{3}\left(\frac{p}{4\pi r}\right)^2 Z_0\omega^2 k_0^2. \qquad (5.39)$$

This is the power density radiated by an isotropic ('same in all directions') antenna. The directivity of an antenna is defined by comparing the magnitude of the Poynting vector in a given direction with the magnitude of the Poynting vector of an isotropic antenna (with the same input power) at the same position in space. Thus the directivity is given by

$$D(\theta, \phi) = \frac{S_r(r, \theta, \phi)}{S_i(r)}. \tag{5.40}$$

The directivity of a small dipole is 1.5 from (5.35) and (5.39). This may also be expressed in decibels as 1.76 dBi where the symbol dBi indicates that the reference signal is that of an isotropic radiator. Alternatively directivity may be defined relative to the radiation pattern of a standard antenna with the same input power.

Not all the power input to an antenna is radiated. Some of it is dissipated in the antenna. The ratio of the radiated power to the input power is the radiation efficiency of the antenna

$$\eta = \frac{P_r}{P_{in}}. \tag{5.41}$$

The gain of an antenna relates its radiation pattern to the input power. Thus

$$G(\theta, \phi) = \eta D(\theta, \phi). \tag{5.42}$$

The difference between the gain and the directivity of an antenna is that the former takes account of losses within the antenna whilst the latter does not. The word 'gain' is sometimes used, loosely, in place of 'directivity'. Although the gain and directivity of an antenna can be specified in any direction the usual practice is to quote the figures corresponding to the direction in which the power density is maximum.

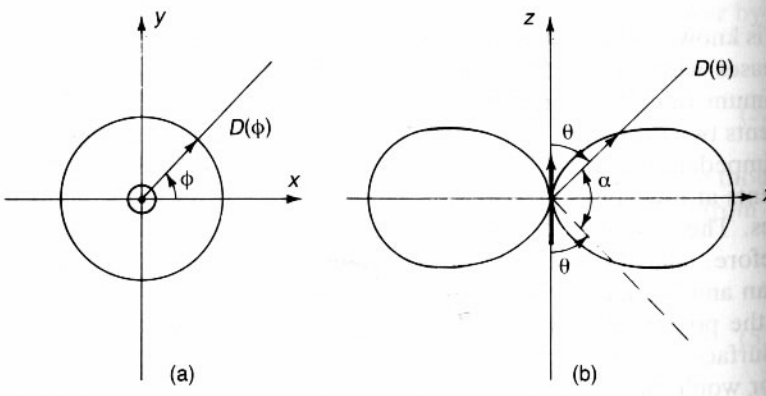

Fig. 5.6 Polar radiation diagrams for a small electric dipole antenna: (a) in a plane perpendicular to the dipole and (b) in a plane containing the dipole.

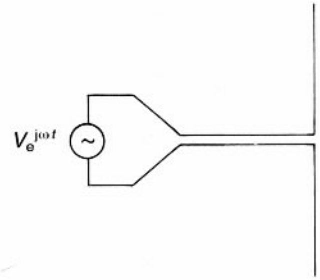

Fig. 5.7 Arrangement of a centre-fed dipole antenna.

The directional properties of antennas are usually displayed as polar plots of the directivity. Figure 5.6 shows the polar plots for a small dipole in planes perpendicular to and parallel to the dipole. A useful measure of the directional properties of an antenna is the angle at which the power density is half the maximum value, that is

$$S_r(\theta) = \tfrac{1}{2}(S_r)_{max}. \tag{5.43}$$

For a small dipole $S_r(\theta) \propto \sin^2\theta$ so that, from (5.43), $\theta = 45°$. The included angle (α in Fig. 5.5(b)) is known as the half-power beamwidth. For a small dipole it is 90°.

So far we have assumed that the dipole is fed by a coaxial cable as shown in Fig. 5.1. A very common alternative is the centre-fed dipole shown in Fig. 5.7 in which the feeder is a two-wire line.

5.5 THE RECIPROCITY THEOREM

So far we have considered antennas as radiating elements. They can, equally well, act as receivers of electromagnetic radiation. The properties of an antenna as a transmitter and a receiver are linked by the reciprocity theorem which states that

> In a linear system the response at a point to a stimulus at another point is unchanged when the stimulus and response are exchanged.
>
> (Carter and Richardson, 1972)

An example of a linear system is that comprising a pair of antennas as shown in Fig. 5.8(a). Provided that the antennas and the medium surrounding them are linear, passive and isotropic then the system shown in the figure can be represented by the matrix equation

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}. \tag{5.44}$$

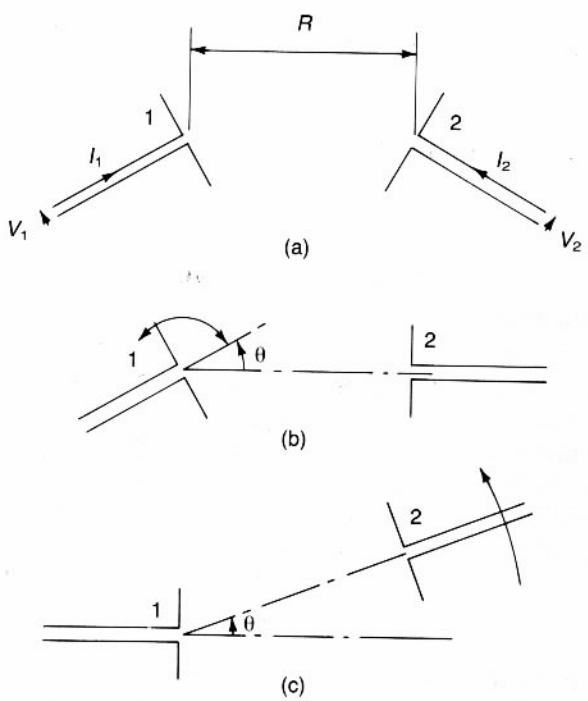

Fig. 5.8 Communication between a pair of antennas: (a) general arrangement, (b) when antenna 1 is rotated and (c) when antenna 2 is rotated.

The reciprocity theorem implies that the relationship between V_2 and I_1 is the same as that between V_1 and I_2 so that $Y_{12} = Y_{21}$. Thus an alternative statement of the reciprocity theorem is

> The impedance and admittance matrices of passive linear systems are symmetrical.

As applied to the pair of antennas shown in Fig. 5.8(a) the effect of the theorem is that the behaviour of the system is unchanged when the transmitting and receiving antennas are exchanged. Now the directivity of antenna 1 can be measured either by keeping antenna 2 fixed and rotating antenna 1 or by keeping antenna 1 fixed and moving antenna 2 as shown in Figs. 5.8(b) and (c). Thus the directivity of an antenna is the same whether it is transmitting or receiving.

When an antenna is acting as a receiver a useful measure of its effectiveness is the area from which it gathers power. The effective aperture is defined by

$$A_e = \frac{\text{Power absorbed by load}}{\text{Power density in incident wave}}. \tag{5.45}$$

To find the relationship between the effective aperture and the gain of an antenna we consider a pair of antennas as shown in Fig. 5.8(a). We shall assume that the separation between the antennas is large enough for the wave at the receiving antenna to be effectively that of a uniform plane wave. The power density is then

$$S = g_1 \frac{P_{in}}{4\pi R^2}, \tag{5.46}$$

where P_{in} is the input power and g_1 the gain of antenna 1. Then, from (5.45) the power absorbed by the load is

$$P_L = A_{e2} S = \frac{A_{e2} g_1}{4\pi R^2} P_{in}. \tag{5.47}$$

Now the reciprocity theorem tells us that the relationship between the input power and the load power must be the same if the roles of the antennas are exchanged so

$$P_L = \frac{A_{e1} g_2}{4\pi R^2} P_{in} \tag{5.48}$$

and therefore

$$\frac{A_{e1}}{A_{e2}} = \frac{g_1}{g_2} \tag{5.49}$$

so that the effective aperture of an antenna is proportional to the gain. The constant of proportionality may be found by considering a small lossless dipole acting as a receiving antenna as shown in Fig. 5.9(a). The voltage induced in the antenna is $E_i l$. The equivalent circuit of the dipole connected to the load is shown in Fig. 5.9(b). The maximum power theorem requires $Z_L = R_r$ for maximum power transfer to the load. The load power is then

$$P_L = \frac{1}{2}\left(\frac{V}{2}\right)^2 \frac{1}{R_r} = \frac{E_i^2 l^2}{8R_r}. \tag{5.50}$$

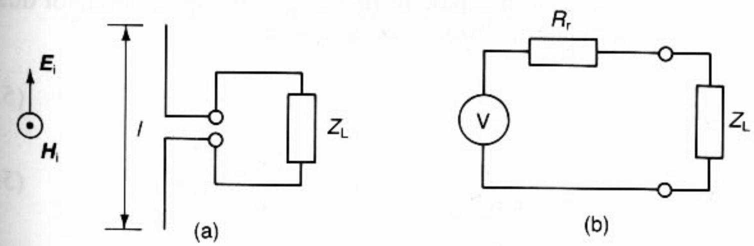

Fig. 5.9 Dipole receiving antenna: (a) general arrangement and (b) equivalent circuit.

But the power density in the incident wave is

$$S = \frac{E_i^2}{2Z_0} \tag{5.51}$$

so, from (5.45)

$$A_e = \frac{Z_0}{4R_r} l^2. \tag{5.52}$$

Substituting for the radiation resistance of the dipole from (5.38) we have

$$A_e = \frac{\lambda^2}{4\pi}\frac{3}{2}. \tag{5.53}$$

Therefore, since the gain of a small dipole is 1.5, we conclude that, for any antenna

$$A_e = \frac{\lambda^2}{4\pi} g. \tag{5.54}$$

5.6 SMALL MAGNETIC DIPOLE

The symmetry of Maxwell's equations in free space means that, if the fields $\boldsymbol{E}(r, t)$ and $\boldsymbol{H}(r, t)$ are a solution of them, then the fields

$$\boldsymbol{E}'(r, t) = -\frac{1}{\varepsilon_0}\boldsymbol{H}(r, t)$$

and

$$\boldsymbol{H}'(r, t) = \frac{1}{\mu_0}\boldsymbol{E}(r, t) \tag{5.55}$$

are also a solution. This is easily demonstrated by substituting (5.55) into (1.7) and (1.8). The fields given by (5.55) are called the duals of the original fields and their existence demonstrates the principle of duality. This principle allows us to deduce the solution to one problem from that for its dual.

The dual of the electric dipole considered in Section 5.4 is the magnetic dipole. The fields around it can be derived by applying principle of duality to the fields of the electric dipole with the results

$$H_r = \frac{[p]}{2\pi\varepsilon_0\mu_0}\left(\frac{1}{r^3} + \mathrm{j}\frac{k_0}{r^2}\right)\cos\theta \tag{5.56}$$

$$H_\theta = \frac{[p]}{4\pi\varepsilon_0\mu_0}\left(\frac{1}{r^3} + \mathrm{j}\frac{k_0}{r^2} - \frac{k_0^2}{r}\right)\sin\theta \tag{5.57}$$

$$E_\phi = \frac{-[p]}{4\pi\varepsilon_0}\left(\frac{\mathrm{j}\omega}{r^2} - \frac{\omega k_0}{r}\right)\sin\theta. \tag{5.58}$$

The near field of this dipole is

$$H_r = \frac{[p]\cos\theta}{2\pi\varepsilon_0\mu_0 r^3} \tag{5.59}$$

$$H_\theta = \frac{[p]\sin\theta}{4\pi\varepsilon_0\mu_0 r^3} \tag{5.60}$$

which is just the field of a magnetic dipole whose dipole moment is

$$[j] = \frac{[p]}{\varepsilon_0\mu_0} \tag{5.61}$$

as is shown in texts on magnetostatics (Bleaney and Bleaney, 1976). A magnetic dipole is realised as a small current loop as shown in Fig. 5.10 with

$$j = \pi a^2 I. \tag{5.62}$$

The loop must be small enough for there to be no appreciable phase difference between the currents in different parts of the loop. The far field of a magnetic dipole antenna is therefore

$$H_\theta = -\frac{[j]k_0^2\sin\theta}{4\pi r} \tag{5.63}$$

$$E_\phi = \frac{\mu_0[j]\omega k_0\sin\theta}{4\pi r}. \tag{5.64}$$

The reversal in the sign of one of these equations compared with (5.33) ensures that the direction of the Poynting vector is outwards. By analogy with (5.36) the total radiated power is

$$P = \frac{1}{12\pi}(Z_0 k_0^4 j^2), \tag{5.65}$$

where j is given by (5.62). If the dipole has the form of the current loop shown in Fig. 5.8 then the radiation resistance is

$$R_r = \frac{8\pi Z_0}{3}\left(\frac{\pi a}{\lambda}\right)^4. \tag{5.66}$$

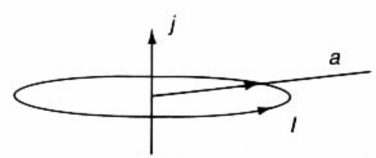

Fig. 5.10 Small magnetic dipole antenna.

Comparing this expression with (5.38) we see that the radiation resistance of a magnetic dipole varies much more rapidly with frequency than that of an electric dipole. If the diameter of the loop is 0.1λ then the radiation resistance is $1.92\,\Omega$. This is an inconveniently small impedance but it can easily be increased by using a coil of several turns. For N turns the radiation resistance increases as N^2. A further increase can be obtained by winding the coil on a ferrite rod. This is the kind of antenna usually used in portable radios.

5.7 HALF-WAVE DIPOLE

The inconveniently small radiation resistance of an electric dipole can be increased by making it longer so that it resonates. If each arm of the dipole is a quarter wavelength long then there is a standing wave and the current varies sinusoidally along it as shown in Fig. 5.11. When this antenna is compared with that shown in Fig. 5.2 it is seen that the charges accumulate along the length of the dipole rather than being concentrated at its ends. This antenna can be analysed by assuming that it is made up of a large number of small dipoles as shown in Fig. 5.11(a). At large distances from the antenna the path distance is effectively the same for all elements as far as the signal amplitude is concerned. For the element shown the phase is

$$\exp \mathrm{j}[\omega t - k_0(R - x \sin\theta)] \tag{5.67}$$

and the dipole moment is

$$p = -\mathrm{j}\frac{I\,\mathrm{d}x}{\omega} \cos k_0 x. \tag{5.68}$$

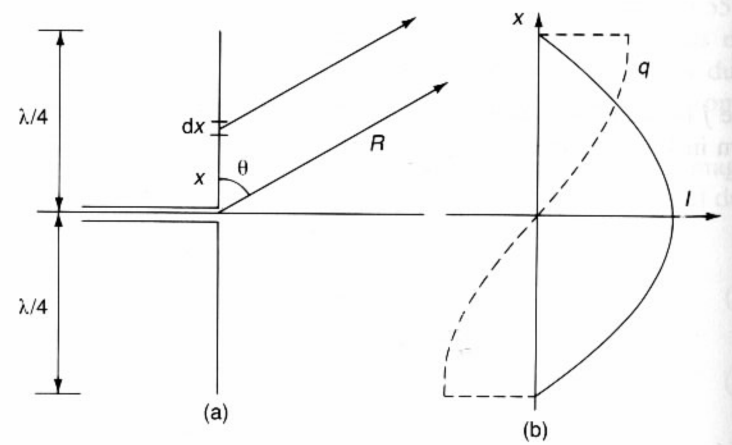

Fig. 5.11 Half-wave dipole antenna: (a) geometry and (b) current and charge distributions assumed.

The variation cannot be exactly sinusoidal because of the radiation of energy from the antenna but careful experiments have shown that the departure from the distribution assumed is insignificant. The electric field at large distances from the antenna is

$$E_\theta = -\mathrm{j}\frac{k_0^2 \sin\theta\, I}{4\pi\varepsilon_0\omega R}\int_{-\lambda/4}^{\lambda/4} \cos k_0 x\, \mathrm{e}^{\mathrm{j}[\omega t - k(R - x\sin\theta)]}\,\mathrm{d}x. \tag{5.69}$$

This expression can be integrated to give

$$E_\theta = \frac{\mathrm{j}Z_0}{2\pi R}\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}[I]. \tag{5.70}$$

This field pattern is very like that of the short dipole. It has cylindrical symmetry, maximum field in the plane normal to the antenna and zero radiation along the axis of the antenna. The half-power beam width is 78° and the maximum directivity is 1.64, figures which do not differ markedly from those for a short dipole. The big difference is in the radiation resistance which is $73\,\Omega$ for a half-wave dipole. This is much more practical and half-wave dipoles are in common use for television receiving antennas. Incidentally this figure explains why the characteristic impedance of the cable used for television aerial downleads is $75\,\Omega$ rather than the usual $50\,\Omega$.

An arrangement which is sometimes used is a monopole antenna combined with a ground plane along the lines of Fig. 5.1. This has a radiation resistance which is just half of that of the corresponding dipole and has double the directivity because the radiation is concentrated over one hemisphere. Short monopoles are used for car radio aerials using the bodywork of the car as the ground plane. Quarter-wave monopoles are used for VHF radio transmitters with the conductivity of the earth supplying the ground plane.

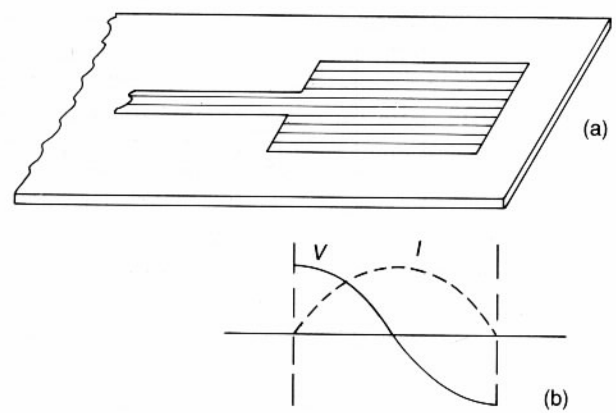

Fig. 5.12 Half-wave dipole microstrip antenna.

Dipole antennas can be realized in microstrip. Figure 5.12 shows one possible arrangement. A stripline of normal characteristic impedance feeds a much wider section whose impedance is therefore much lower. Because of the big difference in the impedances the feed line looks very much like an open circuit to the wider line which then acts as a resonator (see Chapter 7 for a discussion of resonators). The standing wave excited has sinusoidal voltage and current distributions as shown and the section of line therefore acts as a dipole antenna. Further information about microstrip antennas is given by Rudge *et al.* (1982–3).

5.8 DIPOLE ARRAYS

The single dipole antennas discussed so far have the disadvantage that they are not strongly directional. For many purposes it is desirable to radiate the greater part of the power in a narrow beam or to have a receiving antenna with a large effective aperture. This increase in directivity can be achieved by using more than one dipole. The number of possible arrangements is very large so here we shall concentrate on illustrating the basic principles involved.

Figure 5.13 shows two half-wave dipoles arranged parallel to each other and d apart. Each dipole radiates uniformly in all directions in the plane of the paper. We will assume that the dipoles are driven with currents of equal amplitude and relative phase α. Then the phase difference between the signals from the two dipoles at a distant point in the plane is

$$\psi = k_0 d \cos\phi + \alpha. \tag{5.71}$$

The total field intensity at that point is

$$\begin{aligned} \boldsymbol{E} &= \boldsymbol{E}_0 \exp \mathrm{j}\psi/2 + \boldsymbol{E}_0 \exp -\mathrm{j}\psi/2 \\ &= 2\boldsymbol{E}_0 \cos \psi/2, \end{aligned} \tag{5.72}$$

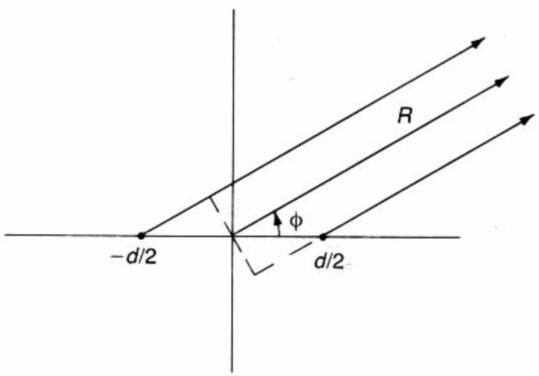

Fig. 5.13 Geometry of an antenna comprising a pair of parallel half-wave dipoles.

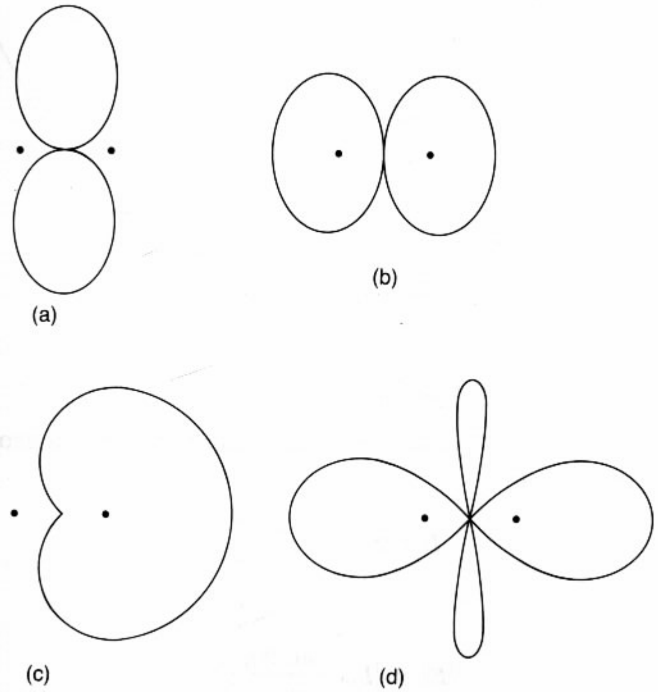

Fig. 5.14 Radiation patterns of antennas comprising pairs of half-wave dipoles having the following spacings and phases: (a) $\lambda/2$, zero, (b) $\lambda/2$, 180°, (c) $\lambda/4$, −90° and (d) λ, zero (after Jordan and Balmain, 1968).

where $\boldsymbol{E}_0$ is the field strength due to one dipole on its own. Different radiation patterns can be produced by various choices of the separation between the dipoles and of the phase difference between them. Figure 5.14 shows a number of examples. These show how the direction of the maximum directivity can be steered by changing the relative phase of the signals fed to the dipoles. Figure 5.14(c) is interesting because it shows a radiation pattern which does not have a backward lobe. Figure 5.14(d) shows the presence of sidelobes in addition to the main lobes.

The cardioid radiation pattern of Fig. 5.14(c) is given by

$$\boldsymbol{E} = 2\boldsymbol{E}_0 \cos[(\cos\phi - 1)\pi/4]. \tag{5.73}$$

This has a maximum value of 2 in the $\phi = 0$ direction because the signals from the two dipoles are in phase. The maximum directivity of the antenna is thus four times that of one dipole that is 6.56 if half wave dipoles are used.

By adding more dipoles we can get greater directivity. The field at a remote point due to the array of N dipoles shown in Fig. 5.15 is

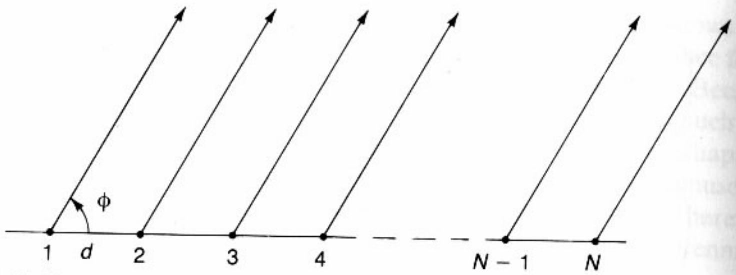

Fig. 5.15 Geometry for radiation from a linear array of N dipoles.

$$\boldsymbol{E} = \boldsymbol{E}_0(1 + e^{j\psi} + e^{2j\psi} + \cdots + e^{(N-1)j\psi}), \tag{5.74}$$

where ψ is given by (5.71) as before. The series in (5.74) is a geometrical progression whose sum is

$$\boldsymbol{E} = \boldsymbol{E}_0\left(\frac{1 - e^{Nj\psi}}{1 - e^{j\psi}}\right) \tag{5.75}$$

so

$$|\boldsymbol{E}| = |\boldsymbol{E}_0|\left|\frac{\sin N\psi/2}{\sin \psi/2}\right| \tag{5.76}$$

which has a maximum value of $N|\boldsymbol{E}_0|$ when $\psi = 0$. The actual direction in space of the maximum radiation depends upon the relative phases of the dipoles. The direction of maximum directivity is given by

$$\psi = k_0 d \cos\phi + \alpha = 0. \tag{5.77}$$

If $\alpha = -k_0 d$ then $\phi = 0$ and the maximum directivity is in the direction of the array and the antenna is described as an endfire array. If the dipoles are fed in phase with each other $\phi = 90°$ and the maximum signal is in a direction at right angles to the array which is then called a broadside array.

The function in (5.76) has zeroes when

$$\psi = \frac{2\pi}{N}, \frac{4\pi}{N}, \frac{6\pi}{N}, \text{ etc.} \tag{5.78}$$

and sidelobe maxima when

$$\psi = \frac{3\pi}{N}, \frac{5\pi}{N}, \frac{7\pi}{N}, \text{ etc.} \tag{5.79}$$

Figure 5.16 shows the polar diagram for the electric field strength of a six-element array. It must be remembered that this diagram only shows the radiation pattern in the plane normal to the dipoles. In planes at right angles to this one the pattern is that of an individual dipole scaled by the

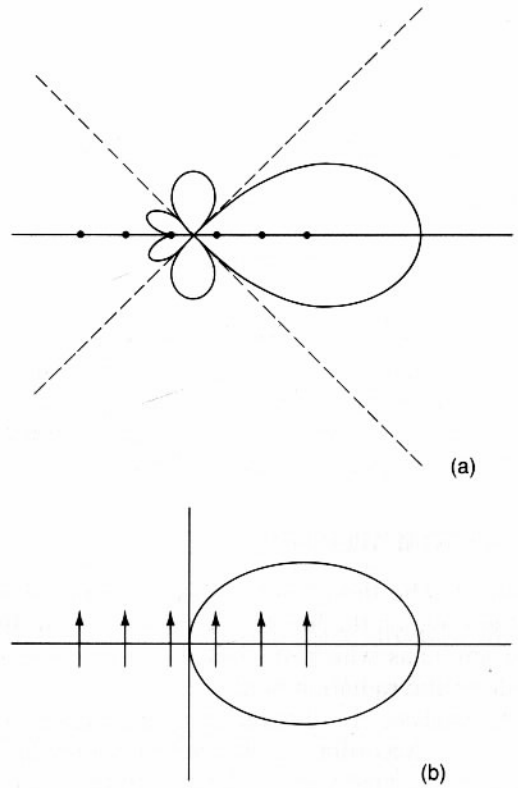

Fig. 5.16 Polar diagrams for radiation from a six element linear array: (a) in the plane bisecting the dipoles at right angles and (b) in the plane containing the dipoles.

variation given by (5.76). This is rather difficult to represent graphically so it is usual to display the radiation patterns in principal planes. Figure 5.16(b) shows the radiation pattern in the direction of the main lobe and in a plane parallel to the dipoles.

A very common example of an array is the Yagi–Uda array (Rudge *et al.*, 1982–3), shown in Fig. 5.17, which is used for television reception. The array has only one active element, normally a half-wave dipole. All the other dipoles are parasitic elements excited by the electromagnetic field. The spacing between the elements is only a small fraction of a wavelength. Since a conductor cannot have an electric field tangential to its surface each parasitic dipole must be excited in antiphase with the exciting field. The power extracted from the incident wave is then re-radiated and the complete field of the antenna is found by superimposing the fields of the dipoles. A closely spaced pair of dipoles excited in antiphase has an endfire field pattern so, by extension, a Yagi–Uda array is an endfire array. If such an

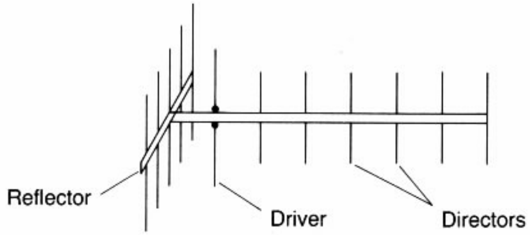

Fig. 5.17 Arrangement of a Yagi–Uda antenna.

array were arranged with a conducting plane normal to its axis and close to the active dipole the result would be an increase in the directivity because of the reflection from the plane. A practical antenna uses either a small array of parasitic dipoles (as shown in Fig. 5.17) or sometimes just one slightly longer dipole to achieve the same effect.

5.9 RADIATION FROM APERTURES

A radiating dipole can be thought of as a termination of a two-wire line which matches the wave on the line to the radiation field. In a similar way we can imagine antennas which are based on matching the fields in a hollow waveguide to the radiation field.

The basis of the analysis of radiation from apertures is Huygens' theory of secondary sources. According to this theory a propagating wavefront can be regarded as being made up of a large number of secondary sources each radiating a spherical wave as shown in Fig. 5.18. The superposition of these waves forms a new wavefront and so on. It turns out that, in order to model this process correctly each secondary source should have a radiation pattern which varies as $(1 + \cos\theta)$, where θ is the angle measured from the normal to the wavefront, and a phase which is 90° ahead of that of the wavefront (Jordan and Balmain, 1968). Note that the angular variation ensures that there is no radiation back towards the source.

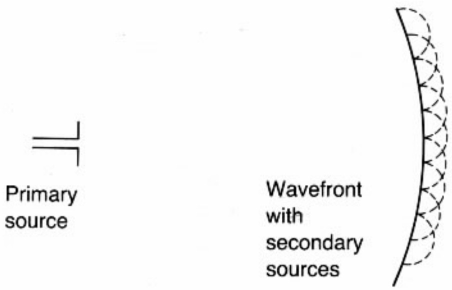

Fig. 5.18 Representation of wave propagation by superposition of secondary wavelets.

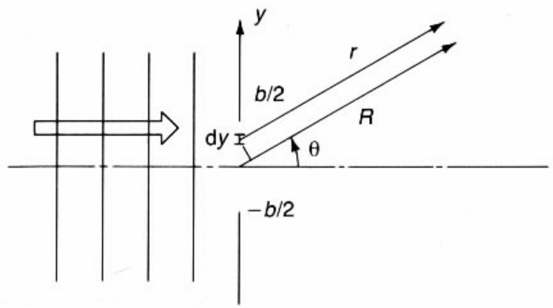

Fig. 5.19 Geometry for radiation from a uniformly illuminated slot.

The simplest example of radiation from an aperture is that which occurs when a gap in a conducting plane is illuminated by a plane wave as shown in Fig. 5.19. The field at a remote point due to the secondary source dy is given by

$$dE = \frac{jk_0E_0a\,dy}{4\pi}(1+\cos\theta)\frac{e^{-jk_0r}}{r} \tag{5.80}$$

where a is the width of the aperture in the x direction and

$$r = R - y\sin\theta. \tag{5.81}$$

The field radiated from the aperture is thus

$$E = \frac{jk_0E_0a(1+\cos\theta)}{4\pi}\int_{-b/2}^{b/2}\frac{e^{-jk_0(R-y\sin\theta)}}{R-y\sin\theta}\,dy. \tag{5.82}$$

If $R \gg b$ the bottom line of the integral is approximately equal to R and

$$|E| = \frac{k_0E_0a(1+\cos\theta)}{4\pi R}\left|\int_{-b/2}^{b/2}\exp(jk_0y\sin\theta)dy\right| \tag{5.83}$$

which can be integrated to give

$$|E| = \frac{k_0E_0ab}{4\pi R}(1+\cos\theta)\left[\frac{\sin(\frac{1}{2}k_0b\sin\theta)}{\frac{1}{2}k_0b\sin\theta}\right]. \tag{5.84}$$

The angular variation of this expression is dominated by the last term. The dependence of the power density on θ is shown in Fig. 5.20, where $u = (k_0b/2)\sin\theta$. The field nulls are given by

$$\tfrac{1}{2}k_0b\sin\theta = \pm N\pi, \tag{5.85}$$

where $N = 1, 2, 3$, etc. Hence

$$\theta = \pm\sin^{-1}\left(\frac{N\lambda}{b}\right). \tag{5.86}$$

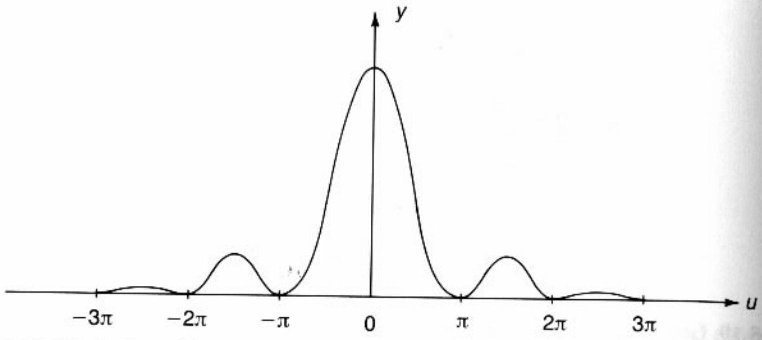

Fig. 5.20 Variation of intensity of radiation from a uniformly illuminated slot with the parameter $u = (k_0 b/2) \sin \theta$.

Clearly the maximum number of nulls is given by

$$N = b/\lambda \tag{5.87}$$

so that a narrow aperture will produce only a single lobe in the radiation pattern. Conversely a wide aperture produces many lobes with a narrow main lobe. It is generally true that the beam width in a particular plane is inversely proportional to the width of the aperture in that plane. Thus the parabolic reflector shown in Fig. 5.21 has a beam which is narrow in the horizontal direction and wide in the vertical direction somewhat as shown.

In general for an aperture illuminated by a wave whose intensity varies as $\boldsymbol{E} = \boldsymbol{E}_0(y)$ we have

$$\boldsymbol{E} = \frac{\mathrm{j}k_0 a(1 + \cos\theta)}{4\pi R}\, \mathrm{e}^{-\mathrm{j}k_0 R} \int_{-b/2}^{b/2} \boldsymbol{E}_0(y)\, \mathrm{e}^{+\mathrm{j}k_0 y \sin\theta}\, \mathrm{d}y. \tag{5.88}$$

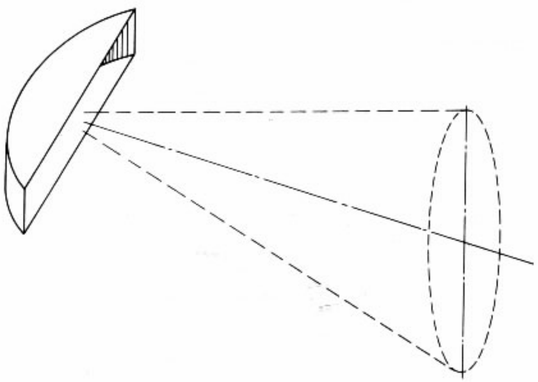

Fig. 5.21 Illustrating the relationship between the dimensions of a parabolic horn antenna and the horizontal and vertical beamwidth.

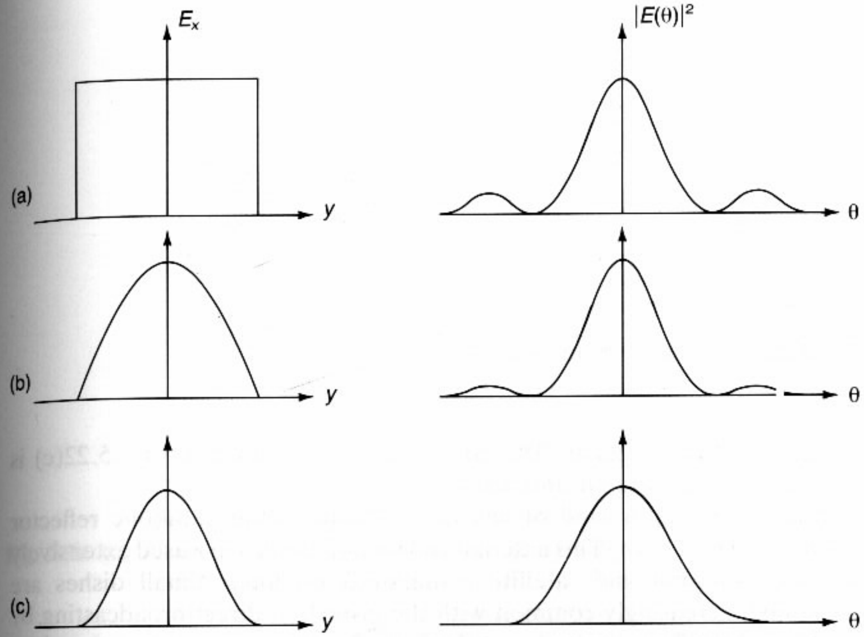

Fig. 5.22 Radiation patterns for a slot with various distributions of illumination: (a) uniform, (b) cosine and (c) cosine squared illumination.

The integral is the Fourier transform of the source distribution. Figure 5.22 shows some source distributions and the corresponding radiation patterns. The first two are those for a uniformly illuminated aperture and for radiation from the open end of a rectangular waveguide. Now the width of a standard waveguide is of the order of magnitude of the free-space wavelength so the width of the beam radiated from an open end is large. To obtain a narrower beam we expand the end of the waveguide into a 'horn' as shown in Fig. 5.23. Provided that the expansion is gradual and the aspect ratio of the horn is the same as that of the waveguide there will be a negligible mismatch between them. Moreover the wave impedance is given by

$$Z_w = Z_0 \frac{\lambda_g}{\lambda_0} \tag{5.89}$$

from (2.45) and (2.46). As the wave travels down the horn λ_g tends to λ_0 and the wave impedance tends to that of free space. Thus the horn serves as a means of matching the fields of the waveguide to those of the radiated wave. A more careful examination shows that the wavefront in the horn is cylindrical so that there are small mismatches at the transitions from a plane wave and to a spherical wave. It is also necessary to remember that the surface at the mouth of the horn on which the phase is constant is a

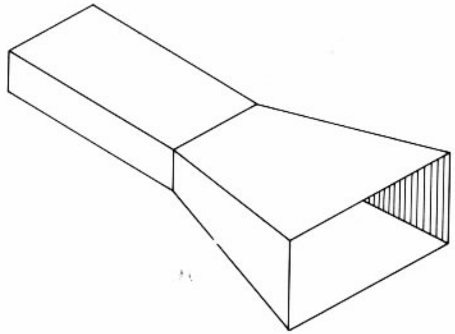

Fig. 5.23 A waveguide horn antenna.

cylinder and not a plane. The $\cos^2$ illumination shown in Fig. 5.22(c) is commonly used in radar antennas.

Another common kind of aperture antenna is the parabolic reflector illuminated by a horn. This antenna and variations on it are used extensively for point-to-point and satellite communication links. Small dishes are becoming increasingly common with the growth of direct broadcasting by satellite. If the horn generates a spherical wave whose apparent source is at the focus of the parabola then the waves reflected from the dish have plane wavefronts. The radiation pattern of the dish may therefore be derived by assuming illumination of its aperture by plane waves. The theory is slightly more involved because the aperture is circular rather than rectangular but the general principles are the same. The directivity of a circular parabolic dish is given by

$$D_{max} = K\left(\frac{2\pi a}{\lambda}\right)^2 \tag{5.90}$$

where K is a constant in the range $0.61 \leqslant K \leqslant 0.865$. Its effective area is typically around half of its physical area.

5.10 SLOT ANTENNAS

In Chapter 3 we saw that one possible TEM transmission line is the slot line formed by the gap between two parallel plates. If a section of this line is closed off by a pair of short circuits the result is a resonant section of line which can be excited by connecting a signal between the centres of the two sides of the slot as shown in Fig. 5.24(a). The resulting standing wave in the slot can be thought of as the superposition of a pair of equal and opposite travelling waves passing through the slot regarded as a short length of rectangular waveguide. It follows that a slot excited in this manner may be

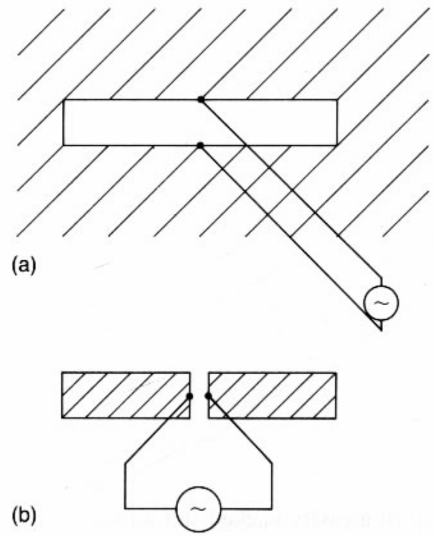

Fig. 5.24 (a) A slot antenna and (b) its complementary dipole.

expected to act as an antenna which radiates on both sides of the conducting sheet in which it is cut.

Booker's principle (Jordan *et al.*, 1968) states that the radiation from a slot antenna is the same as that from a dipole antenna which would just fill the slot (Fig. 5.24(b)) with the electric and magnetic fields interchanged. This dipole is called the complementary dipole. It can be shown that the radiation resistances of the slot antenna and its complementary dipole are related by

$$R_s R_d = Z_0^2/4. \tag{5.91}$$

We do not usually require the slot to radiate on both sides of the plane. A single-sided antenna can be made by backing the slot with a resonant cavity as shown in Fig. 5.25 (resonant cavities are discussed in Chapter 7). Because the slot now radiates only on one side and the resonant cavity presents an open circuit on the other side the radiation resistance is double that of the open slot. The slot can be excited by exciting the cavity with a probe or by connecting the two wires of a transmission line to opposite sides of the slot. Thus the radiation resistance of a cavity-backed half-wave slot radiator is

$$R_s = \frac{2 \times (377)^2}{4 \times 73} = 973\,\Omega. \tag{5.92}$$

This impedance is too high for it to be easy to match the antenna to a coaxial cable. This problem can be solved by connecting the cable across the slot close to one end where the voltage is much lower. This arrangement

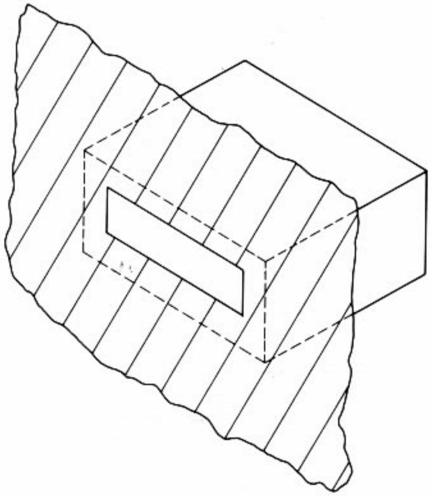

Fig. 5.25 Arrangement of a cavity-backed slot antenna.

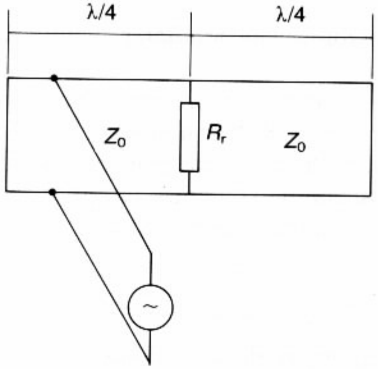

Fig. 5.26 An off-centre fed slot antenna.

has the equivalent circuit shown in Fig. 5.26 where Z_0 is the characteristic impedance of the slot line.

Another way of exciting a slot antenna is to cut it into the wall of a waveguide, as shown in Fig. 5.27(a), so that it intercepts the current circulating in the wall. An array of slots (Fig. 5.27(b)) can act as a compact highly directional antenna. The slots are carried over on to the broad walls of the guide to make them long enough to resonate within the waveguide band. If they are arranged at half guide-wavelength intervals then angling them as shown ensures that they are excited in phase with each other so that the array radiates in a broadside pattern.

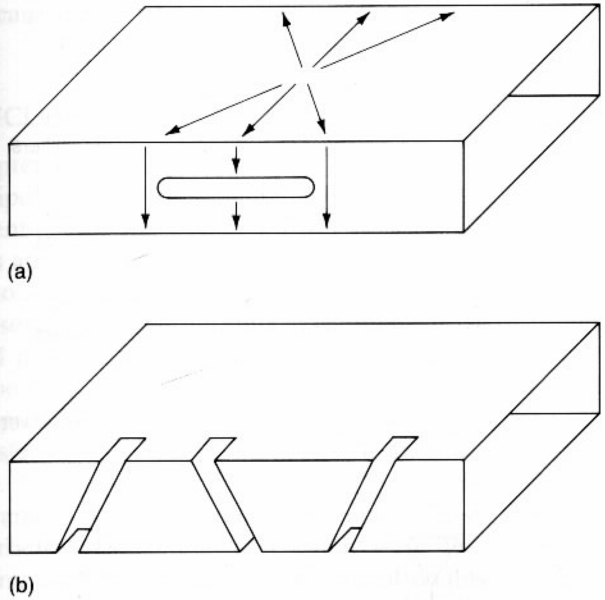

Fig. 5.27 Slot antennas in waveguide: (a) showing how a slot is excited by the currents flowing in the walls of the guide, and (b) a multi-slot array.

5.11 PHASED ARRAY ANTENNAS

From what has been said earlier about the radiation patterns of dipole arrays it will be clear that the radiation pattern can be altered by varying the relative phases of the array elements. This idea can be extended to cover arrays of other types of radiator. By making a two-dimensional array of elements it is possible, in theory, to synthesize any desired radiation pattern within the limits imposed by the overall size of the array. One reason why this idea is very attractive is that it allows the beam of the antenna to be steered electronically instead of mechanically. Mechanical scanning is limited by the inertia of the antenna. Electronic scanning offers the possibility of steering the beam almost instantaneously. This is clearly very attractive for military radar systems because it allows several fast-moving targets to be tracked simultaneously.

The disadvantage of phased arrays is that they must be very complex if they are to achieve good resolution. A typical array might have 100 × 100 elements. It could scan to a little over 45° either side of the normal to the array. Beyond this point the resolution deteriorates because the effective aperture of the antenna is smaller. Thus to achieve full coverage of the sky without moving the antenna five arrays arranged in a truncated four-sided pyramid are required.

Two different implementations of phased arrays are possible as shown in

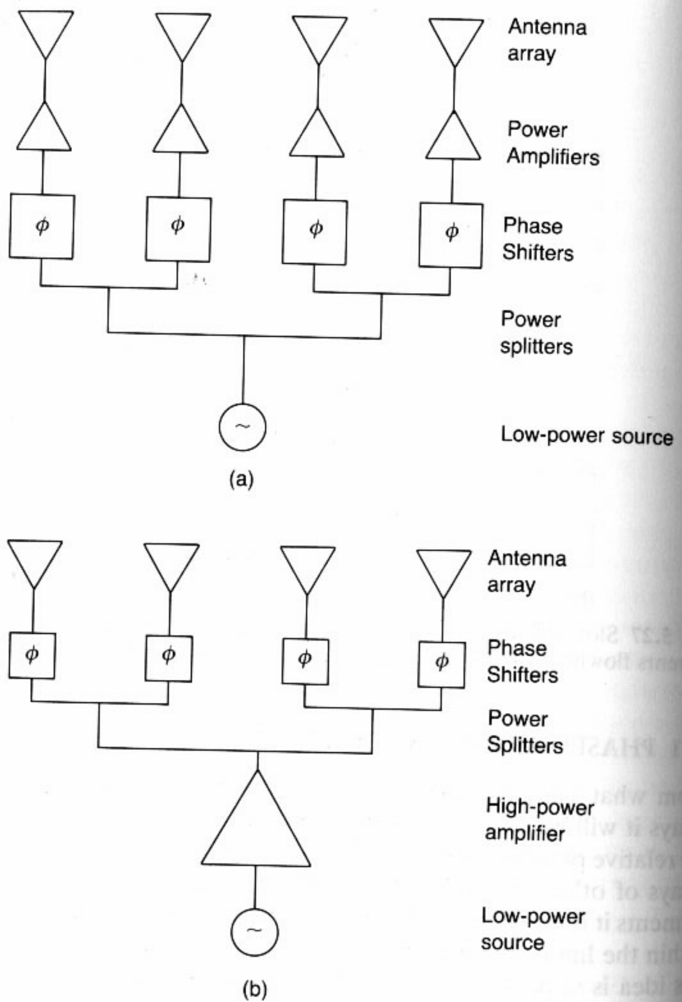

Fig. 5.28 Phased array antenna block diagrams: (a) system with low-power phase-shifters and distributed power amplifiers, and (b) system with a single high-power amplifier and high-power phased-shifter.

Figs. 5.28(a) and (b). The first employs a low-power signal source and carries out the power splitting and phase shifting at low power. There are then as many power amplifiers as antennas and each can be of quite low power. For example a 100 × 100 array of 10 W amplifiers would radiate 100 kW. The second arrangement uses a single high-power amplifier, probably a microwave tube, and carries out the splitting and phase shifting at higher power levels. The question of which of these two approaches is to be preferred is a complex one involving factors such as cost and reliability

which we cannot pursue here. For further information see Rudge *et al.* (1982–3).

5.12 CONCLUSION

In this chapter we have seen how the alternating current in an electric or magnetic dipole can produce a radiating spherical TEM wave. The magnetic vector potential was introduced as a way of dealing with this kind of problem. It was also noted that the finite time taken for a wave to travel from one point to another means that the phase of the wave must be related to that of the source at an earlier time. This can be done by using the concept of retarded potentials. The power radiated from a dipole appears to the source to be dissipated in a resistive load whilst the energy stored in the near field provides a reactive component of impedance. For maximum power transfer the radiation resistance of the antenna must equal the source impedance. The reciprocity theorem was used to link the concept of the gain of a transmitting antenna to the effective area of a receiving antenna. The properties of a small magnetic dipole were deduced from those for a small electric dipole by using the principle of duality.

The radiation from apertures such as waveguide horns and dish antennas was considered based on Huygens' method of secondary sources. It was shown that the radiation pattern of these antennas is the Fourier transform of the illumination of the aperture so that narrow beams require large-aperture antennas for their generation.

Other practical antennas were discussed including the half-wave dipole, quarter-wave monopole and the slot antenna regarded as a complementary wire dipole. The use of arrays of antennas to provide particular radiation characteristics was introduced and the subject of active phased arrays was briefly touched on.

EXERCISES

5.1 Compute the effective areas of antennas having 20 dB gain at frequencies of 500 MHz, 4 GHz and 35 GHz.

5.2 Calculate the radiation resistance of a 100-turn coil wound on a non-magnetic former 10 mm in diameter at a frequency of 1 MHz.

5.3 Plot the polar radiation diagrams and calculate the maximum directivities for antennas comprising two parallel quarter-wave dipoles with the following spacings and phase differences: (a) $\lambda/4$, zero, (b) $\lambda/4$, 180° and (c) λ, 180°.

5.4 Calculate the directivities of endfire arrays of two, four and eight quarter-wave dipoles at frequencies 10% above and below the frequency at which they are a half-wavelength apart.

5.5 Plot polar diagrams for the radiation from a uniformly illuminated slot 100 mm wide at frequencies of 5 GHz, 10 GHz and 60 GHz.

5.6 Find the correct point for connecting the feed to a cavity-backed slot antenna as illustrated in Fig. 5.26 if the radiation resistance is 1000 Ω, the characteristic impedance of the slot is 100 Ω and the source impedance is 50 Ω.