APPENDIX A

Transmission lines

This appendix summarizes some of the equations which relate to the propagation of waves on transmission lines.

Any two-wire line which supports TEM waves can be modelled by the equivalent circuit shown in Fig. A1. Analysis of this circuit (Carter, 1986) shows that the voltage and current obey the equations

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t} \tag{A.1}$$

and

$$\frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t}. \tag{A.2}$$

Eliminating the variables produces the wave equations

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2} \tag{A.3}$$

and

$$\frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2} \tag{A.4}$$

so that waves propagate as $\exp \mathrm{j}(\omega t - kx)$ where

$$k = \pm\omega\surd(LC). \tag{A.5}$$

The ratio of the voltage to the current is the characteristic impedance given by

$$Z_0 = \left(\frac{L}{C}\right)^{\frac{1}{2}}. \tag{A.6}$$

If the line is terminated so that an incident wave V_i produces a reflected wave ϱV_i, where ϱ is the reflection coefficient, then

$$V = V_i \exp \mathrm{j}(\omega t - kx) + \varrho V_i \exp \mathrm{j}(\omega t + kx). \tag{A.7}$$

The amplitude of the voltage at a point on the line x from the termination is given by

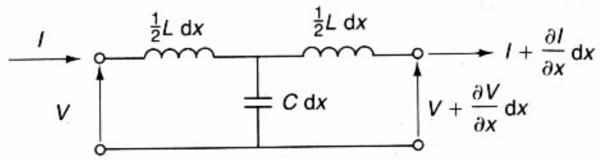

Fig. A1 Equivalent circuit of a two-wire line.

$$|V| = |V_i|\,|1 + \varrho e^{2jkx}| \tag{A.8}$$

which has maximum and minimum values

$$V_{max} = |V_i|(1 + |\varrho|) \tag{A.9}$$

and

$$V_{min} = |V_i|(1 - |\varrho|). \tag{A.10}$$

The ratio of these is the voltage standing wave ratio S so that

$$S = \frac{1 + |\varrho|}{1 - |\varrho|} \tag{A.11}$$

and

$$|\varrho| = \frac{S - 1}{S + 1}. \tag{A.12}$$

The apparent impedance at a point on a transmission line is given by the ratio of the voltage to the current at that point. For a line terminated by an impedance Z_L as shown in Fig. A2 the impedance at the plane A–A is given by

$$\frac{Z'_L}{Z_0} = \frac{Z_L + \mathrm{j}Z_0 \tan kl}{\mathrm{j}Z_L \tan kl + Z_0}. \tag{A.13}$$

A quarter-wave section of line transforms impedance so that

$$Z_0^2 = Z_L Z'_L. \tag{A.14}$$

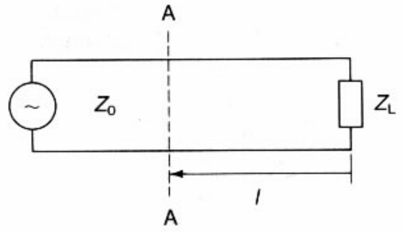

Fig. A2 A line terminated by an impedance.

THE SMITH CHART

A very valuable aid to calculations based on equation (A.14) is the Smith chart. The basis of this chart is a plot of complex reflection coefficient in polar coordinates as shown in Fig. A3. Now as the point of observation is moved along the line away from the termination the only effect is to change the phase so that the tip of the vector in Fig. A3 traces a circle. But the reflection coefficient is related to the impedance by

$$\varrho = \frac{Z - Z_0}{Z + Z_0} \tag{A.15}$$

or

$$\varrho = \frac{z - 1}{z + 1}, \tag{A.16}$$

where z is the impedance normalized to Z_0. When contours of constant impedance are plotted on the polar diagram shown in Fig. A3 the result, the Smith chart, is as shown in Fig. A4. This chart is a graphical representation of equation (A.13). Distance along the transmission line is represented by rotation around the chart with one complete revolution being equivalent to half a wavelength. The standing-wave pattern on a line repeats itself every half wavelength as can be seen from (A.8). Movement towards the generator of the point at which the impedance is measured is represented by clockwise movement around the chart and movement towards the load by anticlockwise rotation.

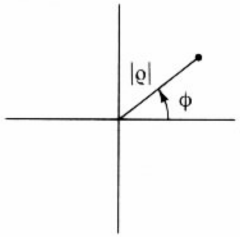

Fig. A3 Plot of complex reflection coefficient in polar coordinates.

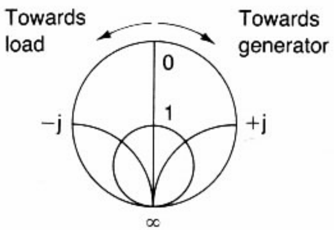

Fig. A4 Graphical representation of equation (A. 13).

Impedances which are wholly real are represented by points on the vertical diameter of the chart. Points on the right-hand half of the chart represent impedances with positive imaginary parts. Impedances with negative imaginary parts are plotted on the left-hand side. Impedances whose value is pure imaginary are plotted around the perimeter of the chart.

There are two points in each revolution of the chart at which the impedance is real. These points correspond to the maxima and minima of the standing wave where

$$z = S \tag{A.17}$$

and

$$z = \frac{1}{S}. \tag{A.18}$$

This property of the chart enables the impedance of a load at a reference plane to be determined from slotted line measurements. Since the value of S and the position of a standing-wave minimum are known the impedance at that plane can be plotted on the chart using (A.18). The transformation of this impedance to the reference plane as shown in Fig. 11.10 is then performed by moving the appropriate angle around the chart at a constant radius.

The admittance at a point is obtained by reflecting the point representing the impedance in the centre of the chart as shown in Fig. A5. The Smith chart can be used to transform admittances using the procedure outlined above for impedances.

A full discussion of the derivation and use of the Smith chart is given by Dunlop and Smith (1984).

LOSSY LINES

The theory given above is only applicable to lossless lines. A lossy line can be represented by the equivalent circuit shown in Fig. A6. The propagation constant is then

$$k = \pm\omega\sqrt{\left[\left(L + \frac{R}{j\omega}\right)\left(C + \frac{G}{j\omega}\right)\right]} \tag{A.19}$$

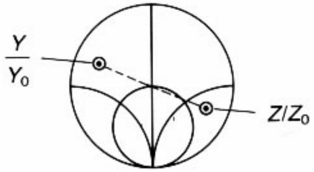

Fig. A5 Obtaining the admittance at a point.

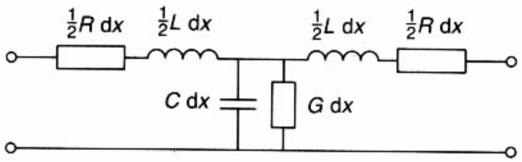

Fig. A6 Equivalent circuit of a lossy line.

by a straightforward extension of (A.5). If the losses are small and dominated by the resistance of the conductors then

$$k = \pm\omega\surd(LC)\sqrt{\left(1 + \frac{R}{\mathrm{j}\omega L}\right)}$$

$$k \simeq \pm\omega\surd(LC)\left(1 + \frac{1}{2}\frac{R}{\mathrm{j}\omega L}\right)$$

$$k \simeq \pm\left[\omega\surd(LC) - \mathrm{j}\frac{R}{2Z_0}\right].$$

A full discussion of propagation on lossy transmission lines will be found in Ramo *et al.* (1965).

APPENDIX B

Vector formulae

This appendix summarizes the principal vector formulae used in electromagnetic theory in Cartesian, cylindrical polar and spherical polar coordinates. It also provides a brief review of Bessel functions. The '^' symbol is used to denote unit vectors in the coordinate directions. All the coordinate systems discussed here are orthogonal systems, that is the three coordinate directions at a point are always at right angles to each other.

CARTESIAN COORDINATES

The system of rectangular Cartesian coordinates for describing the position of a point in space is shown in Fig. B1. Note carefully that this is a right-handed set of axes so that rotation from the x direction to the y direction would cause a right-hand thread screw to advance in the z direction. In Cartesian coordinates the vector formulae are:

Gradient
$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z} \qquad \text{(B.1)}$$

Divergence
$$\nabla\cdot\boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad \text{(B.2)}$$

Curl
$$\nabla\wedge\boldsymbol{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}. \qquad \text{(B.3)}$$

The wave equation is

$$\nabla^2 A = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y_2} + \frac{\partial^2 V}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} \qquad \text{(B.4)}$$

with the general solution

$$V = V_0 \exp \mathrm{j}(\omega t - k_x x - k_y y - k_z z) \qquad \text{(B.5)}$$

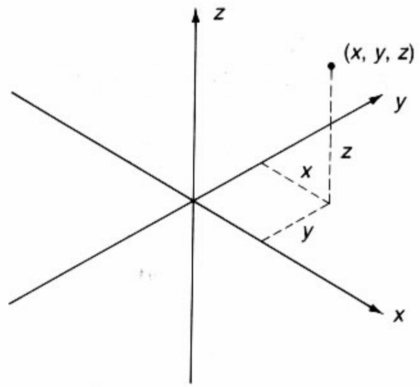

Fig. B1 Rectangular Cartesian coordinate system.

so that

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}. \tag{B.6}$$

CYLINDRICAL POLAR COORDINATES

The system of cylindrical polar coordinates is shown in Fig. B2. These coordinates are related to rectangular Cartesian coordinates by

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z. \end{aligned} \tag{B.7}$$

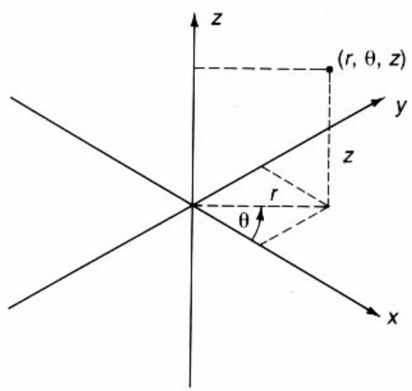

Fig. B2 Cylindrical polar coordinate system.

In cylindrical polar coordinates the vector formulae are:

Gradient $$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \frac{\hat{\boldsymbol{\theta}}}{r}\frac{\partial V}{\partial \theta} + \hat{z}\frac{\partial V}{\partial z} \tag{B.8}$$

Divergence $$\nabla \cdot \boldsymbol{A} = \frac{1}{r}\frac{\partial r}{\partial}(rA_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \tag{B.9}$$

Curl $$\nabla \wedge \boldsymbol{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\boldsymbol{\theta}} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & rA_\theta & A_z \end{vmatrix}. \tag{B.10}$$

The wave equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} \tag{B.11}$$

with the general solution

$$V = V_0\begin{bmatrix} J_n(k_r r) \\ Y_n(k_r r) \end{bmatrix} \exp \mathrm{j}(\omega t - n\theta - k_z z), \tag{B.12}$$

where $J_n(kr)$ and $Y_n(kr)$ are known as Bessel functions of order n of the first and second kind respectively. They are analogous to the sines and cosines which appear in solutions to the wave equation in rectangular coordinates and their values can be looked up in tables. Figure B3 shows

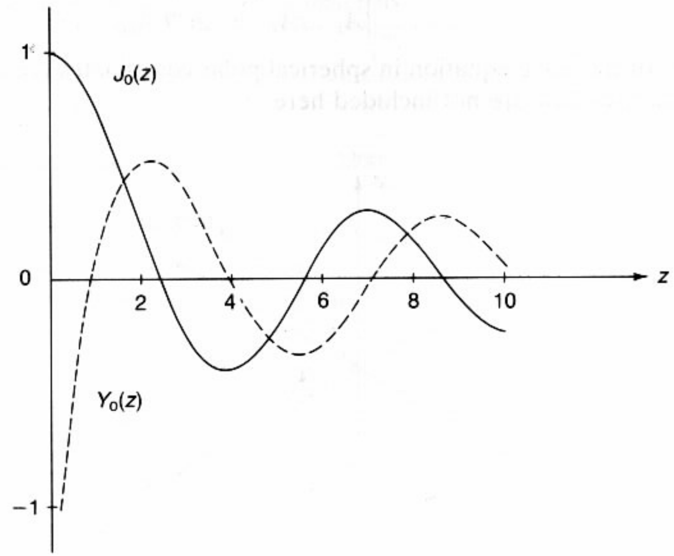

Fig. B3 Graphs of B_0 and Y_0.

graphs of the functions B_0 and Y_0 which occur in problems with cylindrical symmetry. A useful source of information on this subject is McLachlan (1955).

SPHERICAL POLAR COORDINATES

The system of spherical polar coordinates is shown in Fig. B4. They are related to Cartesian coordinates by the equations

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta. \tag{B.13}$$

In this system of coordinates the vector formulae are:

Gradient
$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial V}{\partial \theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial V}{\partial \phi} \tag{B.14}$$

Divergence
$$\nabla\cdot A = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi} \tag{B.15}$$

Curl
$$\nabla \wedge A = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix}. \tag{B.16}$$

Solutions of the wave equation in spherical polar coordinates are not used in this book so they are not included here.

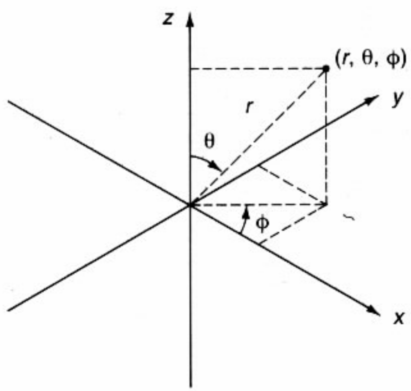

Fig. B4 Spherical polar coordinate system.

Constants and properties of materials

APPENDIX C

Table C1 *Physical constants*

Primary electric constant (ε_0)	8.854×10^{-12} F m^{-1}
Primary magnetic constant (μ_0)	$4\pi \times 10^{-7}$ H m^{-1}
Velocity of light in vacuum (c)	0.2998×10^{9} m s^{-1}
Wave impedance of free space (Z_0)	376.7 Ω
Charge on the electron (e)	-1.602×10^{-19} C
Rest mass of the electron (m_0)	9.108×10^{-11} kg
Charge/mass ratio of the electron (η)	1.759×10^{11} C kg^{-1}

Table C2 Properties of dielectric materials

	ε_r	tan δ (×10^4)
Alumina 99.5%	10	1
Alumina 96%	9	6
Barium titanate	1200	
Beryllia	6.6	1
Epoxy resin	3.5	200
Ferrites	13–16	2
Fused quartz	3.8	1
GaAs (high resistivity)	13	6
Nylon	3.1	200
Paraffin wax	2.25	2
Perspex	2.6	70
Polystyrene	2.54	1.6–2.5
Polystyrene foam	1.05	0.3
Polythene	2.25	3
PTFE (Teflon)	2.08	3.7

Note: the values in this table are typical of those at microwave frequencies. Actual samples of material may have properties which differ from those given here. In some cases they vary appreciably with frequency.

Table C3 Properties of conductors

	Conductivity ($S\,m^{-1}$)
Aluminium	3.5×10^7
Animal body tissue (average)	0.2
Brass	1.1×10^7
Copper	5.7×10^7
Distilled water	2×10^{-4}
Ferrite (typical)	10^{-2}
Fresh water	10^{-3}
Gold	4.1×10^7
Iron	0.97×10^7
Nickel	1.28×10^7
Sea water	4
Silver	6.1×10^7
Steel	0.57×10^7

Table C4 Properties of ferromagnetic materials

	μ_r	B_{sat} (T)
Feroxcube 3	1500	0.2
Mild steel	2000	1.4
Mumetal	80 000	0.8
Nickel	600	
Silicon iron	7000	1.3

APPENDIX D

Answers to selected problems

1 ELECTROMAGNETIC WAVES

1.1 Polystyrene, 229 Ω; alumina, 126 Ω; barium strontium titanate, 3.8 Ω
1.2 21.8 $W\,m^{-2}$; 39.7 $W\,m^{-2}$; 1.32 $kW\,m^{-2}$
1.3 Silver: 9.1 mm, 28.8 μm, 0.91 μm
Graphite: 0.225 m, 0.71 mm, 22.5 μm
Sea water: 35.6 m, 0.11 m, 3.6 mm
1.4 Glass, 3.82 $dB\,m^{-1}$; fused quartz, 0.177 $dB\,m^{-1}$
1.5 8.98 MHz, 898 MHz, 0.148 MHz, 14.8 MHz
1.6 377 Ω, 166 Ω
1.7 1.4 GHz, 2.8 GHz, 5.6 GHz
1.8 $k_+ = 26.4\,j$, $k_- = 10.8\,j$

2 WAVEGUIDES GUIDED BY PERFECTLY CONDUCTING BOUNDARIES

2.1 (1) 5.24 mm; (2) 5.44 mm; (3) 0.66 mm
2.2 Cut-off wavelength, 144.3 mm;
guide wavelengths, 215.7 mm, 138.5 mm, 106.4 mm, 87.7 mm;
phase velocity, $0.414 \times 10^9\,m\,s^{-1}$
group velocity, $0.219 \times 10^9\,m\,s^{-1}$
2.4 72.1 mm, 30.8 mm, 30.8 mm
2.5 At 13 GHz: characteristic impedance 140 Ω, 279 Ω, 419 Ω, 551 Ω
2.6 Cut-off wavelength 95.1 mm (air-filled), 142.7 mm (wax-filled);
characteristic impedance (at 5 GHz) 453 Ω (air-filled), 302 Ω (wax-filled)

3 WAVES WITH DIELECTRIC BOUNDARIES

3.1 32.3°
3.2 1.76

3.3 68°, 62°, 58°
3.4 5.6 mm, 4.6 mm
3.5 14.1 mm, 0.246 (12.2 dB return loss)
3.6 9.3 mm, 0.198 (14.1 dB return loss)
3.7 widths: 7.7 mm, 3.4 mm, 0.45 mm

4 WAVES WITH IMPERFECTLY CONDUCTING BOUNDARIES

4.1 Skin depth
brass 10.7, 3.4, 1.1 μm
gold 5.6, 1.8, 0.56 μm
Surface resistance:
brass 8.5, 26.7, 82.6 $m\Omega\, m^{-1}$
gold 0.4, 13.6, 43.6 $m\Omega\, m^{-1}$
4.2 Transmission loss without film 0.0014 dB; with film 99 dB
4.3 Typical values:
at 10 Hz, S_M = 0 dB, S_E = 285 dB
at 100 MHz, S_M = 343 dB, S_E = 393 dB
4.4 0.11 $dB\, m^{-1}$

5 ANTENNAS

5.1 2.86 m^2, 0.045 m^2, 584 mm^2
5.2 $0.24 \times 10^{-9}\, \Omega$
5.3 6.56, 3.28, 6.56
5.4 6.40, 25.6, 102.4
5.5 Nulls at:
5 GHz; 36.9°
10 GHz; 17.5°, 36.9°, 64.2°
60 GHz, 2.87°, 5.73°, 8.60° etc.
5.6 0.218 of a wavelength from the centre
0.038 of a wavelength from the end
(Note the slot needs to be slightly lengthened)

6 COUPLING BETWEEN WAVE-GUIDING SYSTEMS

6.1 1.68, 1.06
6.2 S reduced by a further 6 dB
6.3 Waveguide height 1.61 mm;
step height 4.41 mm, step length 17.95 mm;
short circuit 17.95 mm behind the junction
6.4 $C = 177\, pF/m^{-1}$, $L = 0.63\, mH\, m^{-1}$;
$C' = 9\, pF/m^{-1}$, $M = 0.03\, mH\, m^{-1}$
Impedances: 61 Ω, 55 Ω
$k = 61.1\, m^{-1}$

7 ELECTROMAGNETIC RESONATORS AND FILTERS

7.1 $L = 21.1$ nH; $C = 0.104$ pF, $R = 270$ kΩ;
37.7 kΩ, 82.0°;
25.5 kΩ, −84.6°
7.2 4.85, 9.71, 14.56 MHz
7.3 21.4, 159 MHz
7.4 11.34, 14.39, 15.29 GHz;
6.06, 8.18, 7.70 GHz
7.5 101 dB
7.6 $r = 0.56$ mm; $h = 0.28$ mm

8 FERRITE DEVICES

8.1 Typical values:
2.36 − 0.009j, 2.07 − 0.0057j at 1 GHz
13.7 − 13.2j, 1.61 − 0.015j at 8 GHz
−4.76 − 1.82j, 1.55 − 0.015j at 10 GHz
8.2 Typical values:
124.7 − 0.24j, 116.8 − 0.16j at 1 GHz
2626 − 1059j, 825 − 3.8j at 8 GHz
333 − 1802j, 1011 − 4.9j at 10 GHz
8.3 Answers at 1 GHz intervals:
y = 6.99, 5.94, 5.20, 4.65, 4.21 mm

10 VACUUM DEVICES

10.1 (1) $41.6 \times 10^6\, m\, s^{-1}$; 25.5 $kA\, m^{-2}$, 555 MHz;
(2) $58.5 \times 10^6\, m\, s^{-1}$, 25.5 $kA\, m^{-2}$, 468 MHz;
(3) $133.8 \times 10^6\, m\, s^{-1}$, 239 $kA\, m^{-2}$, 948 MHz
10.2 (1) 75 mm
13.9, 4.16, 0.69 mm
92.5, 27.8, 46.3 kΩ
(2) 125 mm
19.5, 5.85, 0.98 mm
6.24, 1.87, 0.31 kΩ
(3) 141 mm
44.6, 13.4, 2.23 mm
3.16, 0.95, 0.16 kΩ
10.3 (1) 100 W, 4.5 W
(2) 5 kW, 267 W
(3) 720 kW, 19 kW
10.4 13.6 dB, 114 MHz
10.5 7.6°, 27.9 dB

11 MICROWAVE MEASUREMENTS

11.1 8.059, 9.196, 11.80, 15.61 GHz
11.2 10.39, 11.29, 13.50, 16.94 GHz
11.3 ±0.25 dB
11.4 0.935 + 0.002j
0.89 − 0.13j
1.23 − 0.32j
11.5 64.3
11.6 10.0

12 SYSTEMS USING ELECTROMAGNETIC WAVES

12.1 400 W
12.2 0.6 nW
12.3 8 km (9.2 km if atmospheric refraction is included)
12.4 0.16 nW
12.5 150 m

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