Intoduction to Finance

Mark B. Shackleton

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0 Course Outline

The full course aims to treat the principal topics and issues in Finance that are of concern to the general manager and all those needing to make financial decisions. These include, but are not limited to, Net Present Values and related financial measures, Risk and Return, the Capital Asset Pricing Model, Valuation of financial and non–financial assets (equity, debt, futures, forwards, options etc., firms, projects etc.), Market Efficiency and its implications, Modigliani Miller/Dividend Irrelevance, Capital Structure, Firm Financing/Capital Budgeting and some personal finance (valuing your education!?).

It aims to do this with a minimum of algebra and mathematics although a proper understanding of these issues does require some investment in the few formulae used. Numerical examples and spreadsheets will be used to demonstrate those formulae used.

At the end of this module students will be able to:

- Apply discounting formulae to value simple corporate and personal finance situations
- Produce the relevant firm or salary cashflows to feed into such calculations
- Understand in broad terms, the factors present in risk return tradeoffs
- Use simple risk return formulae to produce an appropriate risk adjusted discount factor
- Discuss market efficiency, examine in outline more sophisticated finance products (options).

0.1 Course Reading

Readings will be from the set text: Finance, Zvi Bodie and Robert Merton, 2000, Prentice Hall, (ISBN 0–13–310897–X).

0.2 Course Outline

The course will be based around the set text, students are expected to have read the relevant chapters before each lecture, course notes will be distributed and placed on the web. The schedule for the week by week lecture outline is as follows (the readings from the relevant chapters in Bodie and Merton are listed):-

1. What is Finance? (B&M 1)
    - Markets for capital and financial discipline

- Financial intermediaries
- Ownership and control
- Regulation and Government

2. The Financial system (B&M 2)
    - The composition of financial markets (analysis)
    - Historical rates of asset return (analysis)
    - The role of finance in the economy (discussion)
3. Time value of money and discounted cashflow analysis (B&M 4)
    - Compounding, from discrete to continuous
    - Net present values (NPV) and the time value of money
    - Annuities, perpetuities and amortising flows
4. Time value of money and discounted cashflow analysis (B&M 4), continued
    - Internal rate of return (IRR)

- Exchange rates and inflation
- Rent or buy a house (analysis)
- Present value/equivalent annual cost of equipment (analysis)
- Why NPV? Why not IRR or payback or similar? (discussion)

5. Life-cycle financial planning (B&M 5)
   - Project cashflows
   - Sensitivity analysis
   - Dependence between projects
   - Your investment/consumption decision
   - Valuing your education
   - Present value of gains from study (analysis)
   - Sensitivity to investment returns (analysis)
6. How to analyse investment projects (B&M 3, 6)

- Profit/Loss Statements
- Cash Flow Statements
- Balance sheets

7. How to analyse investment projects (B&M 3, 6)
    - Relevant cash flows for decision making
8. Principles of asset valuation (B&M 7)
    - The law of one price "no arbitrage"
    - Interest rates and exchange rates
    - Information and security prices
    - Efficient Markets
9. Valuation of bonds & common stocks (B&M 8, 9)
    - Bond yields (analysis)
    - Changing the dividend yield on a stock (analysis)

- Market efficiency (discussion)
- Market efficiency (game)

10. An overview of risk management (B&M 10)
    - Definitions of risk
    - Risk transfer
    - Portfolio theory
    - Mean and standard deviation of returns
11. Hedging, insuring and diversifying (B&M 11, 12)
    - Diversification
    - Returns on a stock and a portfolio (analysis)
    - Gains from diversification and rewards for risk (discussion)
12. Choosing an investment portfolio, The Capital Asset Pricing Model (CAPM) (B&M 13)

- Risk return trade-offs
- The efficient frontier and the market portfolio
- The tangency, security market line and the CAPM
- Risk return across two assets (analysis)
- The security market line (analysis)
- Is beta dead? Was it ever alive? (discussion)
- Which stock or investment is "best"? (discussion)

13. (B&M 16)

- Using company reports and accounts, ratios etc.
- Financial planning & forecasting
- Managing liquidity
- Modigliani Miller
- Allocating the cost of capital

- Firm valuation

14. Forward and futures prices (B&M 14)
    - How forwards and futures differ
    - Currency parity relations
    - Elimination of risk
15. Options and contingent claims (B&M 15)
    - Video: Horizon, "The Midas Formula"*
    - Option definitions
    - Pricing with binomial trees
16. Options and contingent claims (B&M 15) continued
    - The Black Scholes model and its importance
    - Implied volatility

*http://www.bbc.co.uk/science/horizon/1999/midas.shtml

- Analysis Binomial tree (analysis)
- To what extent is dynamic replication possible? (discussion)

17. Discussion & course summary review

1 Finance (B&M1)

1.1 Why study finance? (Bodie & Merton)

- Manage your personal resources
- Deal with the business world
- Pursue interesting and rewarding career opportunities
- Make informed choices as a citizen
- Expand your mind!

1.2 What is finance?

Allocation of scarce resources over time where cost and benefits are:-

- Spread out over time (investment is required before subsequent realisation of returns)
- Not known in advance (return realisation is uncertain)

1.3 Why is finance different to economics?

- Economics shows how today's goods and services are allocated through a price mechanism
  - In the markets for guns and butter there are limited production possibilities

- Aggregate preferences for the two tell us the relative worth of the two as determined in a market where consumers are free to chose
- More frequently, a common numeraire is used to express prices, money in today's terms
- Thus each commodity has a price which reflects its worth to consumers
- Competition should drive prices toward long run marginal cost and there should be no economic rents (abnormally large profits)

• Finance shows how future uncertain return are allocated between investors through expected returns.

- Future:–
- There is also a market for money today and money tomorrow with limited production possibility
- Some prefer money today and some tomorrow, we allow them to trade by lending money

- Aggregate preferences determine the relative value of money today and tomorrow
- Both commodities have a “price” but their rate of exchange is called the interest rate
- Competitive markets for borrowing and lending should mean that the financial markets should offer a fair return
- Uncertain:–
- There is also a market for certain returns and uncertain returns
- Some prefer certainty, others uncertainty
- Preferences will determine the relative “price” of certain returns compared to uncertain returns
- Competitive markets for investment should mean that the financial markets should offer a fair return on risk (need to define risk carefully later).

1.4 Personal financial objectives

- Maximise your future wealth (maximise your asset return and minimise your costs), while still sleeping well at nights (not taking too much risk that may endanger personal bankruptcy)

1.5 Corporate financial and the managers objectives

- Maximise the value of the firm on behalf of the owners

1.6 Separation of ownership & control

- Asset ownership used to demand that control was also exerted by the owner; sole proprietorship or partnerships (it was difficult to get someone to mind the business for you)
- The arrival of the joint stock company allowed separation of ownership from control and new enterprises were formed that specifically had investors who did not manage
- This allowed people without capital to manage, managers to specialise (or specialist managers to be hired) and owners to diversify (invest in many businesses)
- The resulting form of the firm is undoubtedly more efficient but whether it will survive the next 100 years of corporate evolution is anybody's guess

1.7 Financial discipline

- Companies that thrive and prosper will attract more resources
  - Credit, labour, managers, equity and debt capital
- Companies that perform poorly will be starved of resources and will be subject to takeover
  - More shareholders will try to sell the firm's stock pushing the acquisition price to a competitor down

2 The Financial System (B&M2)

2.1 Principals and Agents within the economic system

- Principals are those who ultimately own an interest in a firm. All firm ownership can be tracked down to individuals (via intermediaries) in the end
- Agents are those to whom the principals delegate control. These include firm managers (on behalf of the firm owners) and all government officials (on behalf of everyone)
- Even though it is more efficient to delegate the managerial role, it is not without its incentive problems
- Table 1 shows the most important parties in a modern financial system and the flows of money and services that link them

2.2 Incentive problems

- Moral hazard exists because the effect of purchasing insurance is to reduce incentives for the insured to take preventative measures against loss
- Adverse selection exists where the amount of information that two parties have differs (say a buyer and a seller) causing some sellers or some buyers to be disadvantaged
- Principal agent problems exist because the agent although charged with representing the principals best interests, has incentives of his own and therefore may not pursue the strategy that is best for the principal

from ↓ to→	**Individuals**	**Operating Companies**	**Banks**	**Investment Funds**	**Insurance Companies**	**Gover**
Individuals	Trade	Labour	1.Interest 2.Deposits	Investment	Insurance Premia	Pers Taxe
Operating Companies	Wages	Trade	1.Interest 2.Deposits	Dividends	Dividends	Corp Taxe
Banks	1.Loans 2.Interest	1.Loans 2.Interest	Trade	1.Dividends 2.Interest	1.Loans 2.Interest	Corp Taxe
Investment Funds	Retirement Benefits	Equity Investment	1.Equity Inv. 2.Deposits	Trade	Equity Investment	Invest Taxes
Insurance Companies	Insurance Benefits	Equity Investment	1.Interest 2.Deposits	Dividends	Trade	Bond Invest
Government	Services	Services	Services	Services	Bond Interest	-

Table 1: Flows between parties within an economy

2.3 Adam Smith's Invisible Hand

Acting in one's own self interest will promote overall good for society (welfare) e.g.

- Buying goods from a low cost producer promotes efficient producers who waste less
- Investing in Education provides incentives to improve education
- Investing in stocks with good prospects promotes efficient projects selection
- Seeking to exploit information promotes efficient capital markets

BUT beware of market failure

- Not all goods and services can be traded in markets, e.g.
- Defence, each individual has an incentive to avoid buying this good
- Healthcare and Education are also public as well as private goods, i.e. of benefit to all

- Consumption of drugs, although possibly individually desirable, imposes costs on others
- Not everyone values a pension sufficiently to save for one

Hence Government intervention and provision of certain services, e.g.

- Defence, Healthcare, Primary Education
- Basic pension provision

Government attitude to these is changing and pensions are becoming a private not public concern!

2.4 Financial markets

- Deposits/short term certificates of deposit (CDs)/short term bills/money market accounts are all short term (less than one year) investments whose rate of return is know at the beginning of the investment. So long as the issuer does not default, this means we can treat these rates of return as being risk free for the period of investment.

- Bonds/long term fixed rate investment accounts or loans are all long lived investments which despite having a known long term rate of interest, can fluctuate in value due to shorter term changes in interest rates. e.g. if you are locked into a fixed rate bond at 10% and rates rise to 15%, your investment must fall in value to yield 15% to compete with the current opportunity cost. Because short term interest rates fluctuate, longer term bond prices also change.
- (residual) Equity claims on limited liability firms represent the share or stock value of a stake in a firm. Value is returned to the investor by the payment of dividends and by capital appreciation driven by future dividends, both of which are uncertain, therefore equity returns are far from riskless.

- Derivative claims are financial instruments (package or legal vehicle) that depend on others, maybe in a complex manner for their value. Options and futures are examples discussed later.

2.5 Calculation of returns and index levels

N.B. rates of return are normally quoted as percentages $\% = \frac{}{100}$ and therefore are normally less than one, $10\% = 0.10$, $20\% = 0.20$ etc only $100\% = 1$!

2.5.1 From returns to an index

- Set the initial wealth to $W_0 = 100$ units (arbitrary)

- Given a total return over a year of 10% the closing wealth will be a multiple of $1 + 10\% = 1.1$ of the initial wealth, $W_1 = 110$
- Given a total return over a year of $r_{01}\%$ the closing wealth will be a multiple of $1 + r_{01}$ of the initial wealth

$$W_1 = (1 + r_{01})\, W_0$$

- Repeat for W_2 from W_1 etc. from r_{12}

2.5.2 From an index to returns

- If the end of period one wealth $W_1 = 110$ then the return from period zero is the gain divided by the initial investment $10\% = \frac{110-100}{100} = \frac{110}{100} - 1$

$$\begin{aligned} r_{01} &= \frac{W_1 - W_0}{W_0} = \frac{W_1}{W_0} - 1 \\ 1 + r_{01} &= \frac{W_1}{W_0} \end{aligned}$$

- Repeat for r_{12} etc from W_2,W_1.
- We will see that for stocks with dividends, W_1/W_0 can be thought of as $(P_1 + D_{01})/P_0$ and therefore the % return on wealth invested in stocks is the same if one or a million stocks are held.

$$\begin{array}{ccccccccccc} W_0 & & & & W_1 & & & & W_2 & & & & W_3 \\ & \searrow & & \nearrow & & \searrow & & \nearrow & & \searrow & & \nearrow & \\ & & r_{01} & & & & r_{12} & & & & r_{23} & & \end{array}$$

2.6 Calculation of mean of returns

- Get or calculate returns, $r_{01}, r_{12}...r_{N-1,N}$ etc
- Add them up

$$r_{01} + r_{12} + ... + r_{N-1,N} = \sum_{i=1}^{N} r_{i-1,i} = r_{0,N}$$

- Divide by the number of observations N to get the mean return (in %) which has a label of $\overline{r}$ (r bar)

$$\overline{r} = \frac{r_{0,N}}{N}$$

2.7 Calculation of standard deviation of returns

- Take the mean return away $(r_{01} - \overline{r}), (r_{12} - \overline{r}) \ldots \left(r_{N-1,N} - \overline{r}\right)$ from each return
- Square each term $(r_{01} - \overline{r})^2, (r_{12} - \overline{r})^2 \ldots \left(r_{N-1,N} - \overline{r}\right)^2$ and add them up to get the total variance

$$(r_{01} - \overline{r})^2 + (r_{12} - \overline{r})^2 + \left(r_{N-1,N} - \overline{r}\right)^2 = \sum_{i=1}^{N} \left(r_{i-1,i} - \overline{r}\right)^2$$

- Get the variance by dividing by the number of observations less one (one observation is required and therefore used up to fix the mean $\overline{r}$)

$$
\begin{aligned}
\text{Variance} &= \sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(r_{i-1,i}-\overline{r}\right)^2 \\
\text{Standard Deviation} &= \sigma = \text{Square Root of Variance} \\
&= \left(\frac{1}{N-1}\sum_{i=1}^{N}\left(r_{i-1,i}-\overline{r}\right)^2\right)^{\frac{1}{2}}
\end{aligned}
$$

- The square root of the variance is the standard deviation. The standard deviation has the same units as the return itself (%) and is often labelled σ (sigma, yes it helps to know some Greek!)
- Perform both mean and standard deviation calculations on <u>returns</u> r not on wealth levels W! Mean wealth levels are not useful in finance, returns are.

2.8 Distribution of returns

- Future assets returns are random are not predictable! (Market Efficiency)
- At a first guess, asset returns seem to be <u>normally</u> distributed (Bell Shape)
- This is the most common distribution in nature
- There are statistical reasons why it crops up again and again
- (Many coin tosses will yield a normal distribution for the number of heads)
- (In fact the sample mean from any distribution will be normally distributed)
- It is convenient to work with mathematically
- It is quite hard to reject that asset returns are normally distributed

2.9 Present Values

- With an interest rate of r% per annum, just as we can say that £1 today is worth $\pounds 1 + \pounds r = \pounds(1+r)$ next year, we can calculate the equivalent of £1 next year in today's terms, it is $\frac{\pounds 1}{1+r}$

	today	next year
$\pounds 1$ today	$\pounds 1$	$\pounds(1+r)$
$\pounds 1$ next year	$\frac{\pounds 1}{1+r} = \pounds 1 * (1+r)^{-1}$	$\pounds 1$

- That is to say that $\frac{\pounds 1}{1+r}$ today will grow to $\frac{\pounds 1}{1+r}(1+r) = \pounds 1$ next year and $\frac{\pounds 1}{1+r}$ is called today's present value of £1 next year.
- For two years we can say that £1 today is worth $\pounds(1+r)(1+r) = \pounds 1 + \pounds 2r + \pounds r^2 = \pounds(1+r)^2$ in two years time with a present value of $\frac{\pounds 1}{(1+r)^2}$

- For a cashflow coming T years into the future the result can be generalised to $\pounds\,(1+r)^T$ and the present value that would grow to $\pounds 1$ over T years is $\frac{\pounds 1}{(1+r)^T}$ (on page 110 B&M use i instead of r and n instead of T $\frac{1}{(1+i)^n}$)
- Examples
  - An IOU (I owe you) of \$100 dated for next year is worth (bankable) only $\$90.91 = \frac{100}{1.1}$ if interest rates are 10% pa
  - Although it may cost \$250,000 to purchase your pension annuity when you retire in 30 years time, this only has a present value of $\$24,844 = \frac{\$250{,}000}{1.08^{30}}$ if investments return 8% pa
  - If you purchase a car for \$10,000 and expect to sell it for \$5,000 in three years time the depreciation rate (negative return) is $\left(\frac{5{,}000}{10{,}000}\right)^{\frac{1}{3}} - 1 = -20.63\%$ pa (this must be included in the cost of motoring–a lease deal where no purchase were necessary would probably cost ~20% pa of the current cost of the car!)

3 Time value of money, discounted cashflow analysis (B&M 4)

3.1 Using your financial calculator

- Compound returns require calculation of tedious products $1.1 \times 1.1 \times 1.1 \times 1.1 \ldots$
- Using the y^x button on the calculator simplifies this considerably $1.1^5 = 1.1\ y^x\ 5 = 1.6105$
- We say that 1.6105 is the Future Value (FV) of the Present Value (PV) 1.0 after 5 years (N) with an interest rate of 10% (r or I/Y)
- Discount factors are the inverse or reciprocal of growth rates, the five year discount factor is $\frac{1}{1.1^5} = 1.1^{-5} = 1.1\ y^x\ -5 = 0.6209$, if we had 1 unit due in five years time (FV) it would have a PV of 0.6209

- If the flow were to come in 10 years the PV is only 1.1 y^x $-$10 $=$ 0.3855, the further ahead a cashflow is, the less it is worth!
- 8% over 45 years yields a growth factor of 1.08 y^x 45 $=$ 31.92, however
- 9% over 45 years yields a growth factor of 1.09 y^x 45 $=$ 48.33!
- Over a long horizon, small yield differences can make a huge difference (over 50% in this case!)
- (see B&M p 106/7 for the Rule of 72)

3.2 Compound returns

Over many periods returns can compound up because of interest on interest. If a bank account pays interest at 10% p.a. at the end of the year, it doesn't take too long to realise that £110 will be left for every £100 deposited. How about if the 10% is calculated (on a pro rata basis) every 6 months and interest earns more interest thereafter, you might be quick and come to the answer £110.25 which is in fact £100$*1.05^2$. It would take longer to calculate the effect of quarterly or monthly interest compounding and only a mathematician would be able to guess if you would become infinitely rich or not if interest were calculated every second of the day or even more frequently. Table 2 shows the effect on £100 of every increasingly frequent compounding, in fact the limit is $100e^{0.1} = 110.5171$ which can be shown mathematically from the limit of the series or by taking a large number of n.

$e = 2.71828..$ is a clearly a special number in growth theory and is the basis of Napierian logarithms. Frequently in finance it is easier to assume that interest and other flows are compounded continuously rather than discretely. This is because the amount in the account at the end of a year is just e^r.

Compounding	**Calculation**	**Closing value**
Annual	$1 + 0.10$	110.00
Semi–Annual	$(1 + 0.05)^2$	110.25
Monthly	$(1 + 0.10/12)^{12}$	110.471
Daily	$(1 + 0.10/365)^{365}$	110.5156
n times a year	$(1 + 0.10/n)^n$	
$n \to \infty$ continuous	$\lim_{n\to\infty}(1 + 0.10/n)^n = e^{0.1}$	110.5171

Table 2: The effect of more frequent compounding

Compounding Frequency	**Calculation**	**Closing value**
Annual	$1\,(1.1-1)=10.0\%$	\$110.00
Semi–Annual	$2\left((1.1)^{\frac{1}{2}}-1\right)=9.76\%$	\$110.00
Monthly	$12\left((1.1)^{\frac{1}{12}}-1\right)=9.57\%$	\$110.00
Daily	$365\left((1.1)^{\frac{1}{365}}-1\right)=9.53\%$	\$110.00
n times a year	$n\left((1.1)^{\frac{1}{n}}-1\right)$	
$n\to\infty$ continuous	$\log_e 1.1=\ln 1.1=9.53\%$	\$110.00

Table 3: Quoted rates decline as compounding frequency increases

- Continuous returns lead the investment to grow with e^r each year so after T years the investment is worth $e^r e^r ... e^r$ T times or $(e^r)^T = e^{rT}$ and the present value that leads to $\pounds 1$ at the end of T years is $\frac{\pounds 1}{e^{rT}} = e^{-rT}$.
- We call the factor by which present money is less valuable than future money, the discount factor, the discount factor for continuous compounding is just e^{-rT} while for discrete compounding it is $\frac{1}{(1+r)^T}$ for small rates r or short times T these two are roughly the same.

3.3 Time Value of Money (TVM) calculations

- Financial calculators have the following five buttons n i PV PMT FV

n	the number of years between PV and FV
i	the rate of interest per period
PV	the initial/first or present value
PMT	the payment per period
FV	the final or closing payment

- Set the calculator options to assume that PMT payments come at the end of each period and that i is quoted and applied per whole period (this will vary from machine to machine but will account for the different values your machine may return!)
- These five buttons can be used to perform a range of TVM calculations

3.3.1 PV

- e.g. computing ? yields 68.06 as the PV of a FV of 100 ($\frac{100}{1.08^5}$)

n	i	PV	PMT	FV
5	8	?	0	100

3.3.2 FV

- e.g. computing ? yields 110.20 as the FV of a PV of 75 (75×1.08^5)

n	i	PV	PMT	FV
5	8	75	0	?

3.3.3 i

- e.g. computing ? yields 5.922% as the i that turns a PV of 75 into a FV of 100 ($i = \left(\frac{100}{75}\right)^{\frac{1}{5}}$). We require the outflow FV and the inflow PV to be of different sign because one is an investment and the other is a return.

n	i	PV	PMT	FV
5	?	-75	0	100

3.3.4 PMT

- Use of the PMT button is for multiple cashflows and returns

3.4 Annuities

- These are periodic and even flows from now until some specified future time when they stop
- Examples include pension payments (with a fixed termination date) and mortgage payments

$$\begin{aligned} PV &= \frac{PMT}{1+i} + \frac{PMT}{(1+i)^2} + \frac{PMT}{(1+i)^3} \cdots + \frac{PMT}{(1+i)^n} \\ &= \sum_{t=1}^{n} \frac{PMT}{(1+i)^t} \end{aligned}$$

n	i	PV	PMT	FV
10	10	?	10	0

- yields -61.445 for the PV

3.5 Zero Coupon Debt

- This is a debt instrument with a one time repayment of principal at maturity
- Examples include some corporate debt and zero interest repayment loans

$$PV = \frac{FV}{(1+i)^n}$$

n	i	PV	PMT	FV
10	10	?	0	100

yields -38.554 for the PV

3.6 Coupon Debt

- This is debt which not only repays a principal on maturity but also pays (annual) regular coupons in-between.
- Most corporate debt is of this form and an interest only mortgage would require repayment of principal at loan maturity
- Coupon debt can be synthesised from an Annuity and a Zero Coupon Bond. 10 years at 10% yields -100 for the PV! (Why do you think this is? Are you surprised?)

$$\begin{aligned} PV &= \frac{PMT}{1+i} + \frac{PMT}{(1+i)^2} + \frac{PMT}{(1+i)^3} ... + \frac{PMT}{(1+i)^n} + \frac{FV}{(1+i)^n} \\ &= \sum_{t=1}^{n} \frac{PMT}{(1+i)^t} + \frac{FV}{(1+i)^n} \end{aligned}$$

n	i	PV	PMT	FV
10	10	?	10	100

3.7 Perpetuities

- These are periodic and even flows from now with no final payment!
- Examples include some UK Government Debt and a stock with a constant dividend (preferred stock)
- A current start and a forward start perpetuity can be used to synthesize an annuity

$$PV = \frac{PMT}{1+i} + \frac{PMT}{(1+i)^2} + ... = \frac{PMT}{i}$$

- This can be shown using some algebra (not too difficult!)

n	i	PV	PMT	FV	
999	10	?	10	X	$\frac{PMT}{i} = \frac{10}{0.1} = 100$

yields -100 for the PV again! (Why do you think this is? Are you surprised?). Think about a bank account earning 10% pa, how much could you draw from it each year leaving the principal untouched?

3.8 Forward start perpetuities and annuities

- These are valued at the forward time as a perpetuity and then brought back to the present using a further discount factor

$$
\begin{aligned}
PV &= \frac{PMT}{(1+i)^{n+1}} + \frac{PMT}{(1+i)^{n+2}} \cdots \\
&= \frac{1}{(1+i)^n} \frac{PMT}{i}
\end{aligned}
$$

- The annuity between now and n is worth the difference between the current and forward $n+1$ start perpetuities (B&M p 120)

$$PV = \frac{PMT}{i} - \frac{1}{(1+i)^n} \frac{PMT}{i} = \frac{PMT}{i} \left(1 - \frac{1}{(1+i)^n} \right)$$

3.9 Growing Perpetuities

- These are periodic but growing flows from now with no final payment!
- Examples include stocks with growing dividends

$$PV = \frac{PMT}{1+i} + \frac{PMT\,(1+g)}{(1+i)^2} + \frac{PMT\,(1+g)^2}{(1+i)^3}\ldots$$
$$= \frac{PMT}{i-g}$$

- This can be shown using some algebra (not too difficult!)
- Note that the growth rate g cannot exceed the interest rate i!!
- We need to input the rate i less the growth rate g (i.e. a positive number)

n	i	PV	PMT	FV	
999	$10-3=7$	?	10	X	$\frac{10}{0.07} = 142.857$

3.10 Growing Perpetuities (contd)

- Dividing the prospective (annual) dividend by the current observed price, yields the current dividend yield which according to the growing perpetuity formula must be equal to the required rate of return less the expected capital gain.

$$\text{Div Yield } \frac{PMT}{PV} = i - g$$

- Alternatively, the sum of the expected capital gain and the current dividend yield must be equal to the total required rate of return

$$\text{Div Yield} + \text{Ex Cap Gain } = \frac{PMT}{PV} + g = i \text{ Required rate}$$

3.11 Growing Annuities

- These are periodic but growing flows from now until some specified future time when they stop
- Examples include inflation linked Government Debt
- Growing annuities can be synthesised from a current start and a forward start growing perpetuity

$$PV = \frac{PMT}{1+i} + \frac{PMT\,(1+g)}{(1+i)^2} + \frac{PMT\,(1+g)^2}{(1+i)^3} + ... \frac{PMT\,(1+g)^{n-1}}{(1+i)^n}$$

- There is a formula like the annuity formula for this sum

3.12 Internal rate of return

Flow	n	i	PV	PMT	FV	IRR is the solution for i
Annuity	10	?	-61.445	10	0	$PV = \left(1 - \frac{1}{(1+i)^n}\right)\frac{PMT}{i}$
Zero Coupon Debt	10	?	-38.555	0	100	$PV = \frac{FV}{(1+i)^n} \Longleftrightarrow$ $i = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$
Coupon Debt	10	?	-100.00	10	100	$PV = \frac{1}{(1+i)^n}\frac{PMT}{i} + \frac{FV}{(1+i)^n}$
Perpetuity	999	?	-100	10	X	$PV = \frac{PMT}{i} \Longleftrightarrow i = \frac{PMT}{PV}$
Growing Perpetuity	999	?	-142.857	10	X	$PV = \frac{PMT}{i-g} \Longleftrightarrow$ $i - g = \frac{PMT}{PV}$

- For a given set of cashflows PMT, FV and the amount required to purchase them PV, the rate of return that sets the discounted sum (PV) equal to the purchase price is called the Internal Rate of Return (IRR)

- For all the examples (but one) we have calculated, the IRR is 10% for the growing perpetuity, the IRR is 7% which is the 10% rate of return less the growth rate of 3%.
- For a given level of investment PV and future cashflow returns $PMTs$ and capital repayments FV (which is n years away) the IRR (i) is a measure of the average annual rate of return over the n years
- i.e. setting $i = IRR$ will make the Net Present Value zero!

$$\begin{aligned} \text{Present Value } (PMTs, FV) - PV &= 0 \\ \text{Present Value } (PMTs, FV) &= PV \end{aligned}$$

3.13 Net Present Value

- Net Present Value (NPV) is defined as the present value benefit of future cashflows $PMTs, FV$ net of their investment expense $-PV$
- As just stated if the rate of return is set equal to the IRR the $NPV = 0$
- However the required rate of return i on these cashflows can be above or below the IRR in which case the NPV can be below or above zero

$$i > IRR \iff NPV < 0$$
$$i = IRR \iff NPV = 0$$
$$i < IRR \iff NPV > 0$$

To see this value a 10 year 10% Coupon Bond at 10%, you get 100 right? You should expect that. Now revalue it at $i = 5\%$ and the value has gone up to 138.61, i.e. a 10% bond is worth more than 100 when rates are below 10% (5%).

This is because the 10% bond is more attractive (valuable) than current bonds (5%) once rates have fallen. Conversely if rates rise to 15%, the bonds price falls to 74.91 because it is now less attractive than current investments (15%).

- In competitive markets (such as bond markets) we assume that marginal benefit is driven down to marginal cost and that $NPV = 0$
- This allows us to take the price and infer the appropriate yield as the IRR (value of i that solves for the price)

3.14 NPVs in different currencies

- You cannot directly compare the price of goods in different currencies and so it is with $NPVs$

- A car may be £10,000 but \$13,000 the units are different
- When taking $NPVs$ in different currencies the rates of interest and discount may differ between the two
- e.g. a project requires \$10,000 but could be situated in Japan where it yields ¥575,000 pa or in the US where it yields \$6,000 both for five years. Where to chose? Required rates of return are 4% in ¥ and 6% in \$ and the exchange rate is \$0.01 $= 1$¥

n	i	PV	PMT	FV	$\Longrightarrow NPV$
5	6%	$\mathbf{\$25,274}$	\$6,000	0	$\mathbf{\$15,274}$
5	4%	$\mathbf{¥2,559,798}$	¥575,000	0	$\mathbf{¥2,559,798 \times 0.01\frac{\$}{¥} - \$10,000}$ $\mathbf{= \$15,597}$

- The NPV is higher from investing in Japan (but not if you use $n = 5$, $i = 6\%$, $PMT =$ ¥575,000)

3.15 Markets care about inflation

- Investment returns must be positive to induce investors to save (what strategy could you adopt if they were negative?) but they must also beat the inflation rate or people will chose to hold physical assets instead (which appreciate at the inflation rate). The real rate of return is often used to indicate the rate of return net of inflation and is calculated from the nominal rate and the inflation rate

$$1 + \text{Real rate} = \frac{1 + \text{Nominal Rate}}{1 + \text{Inflation Rate}}$$
$$\text{Real rate} \approx \text{Nominal Rate} - \text{Inflation Rate}$$

  The real rate is approximately the nominal rate less the inflation rate so long as the latter is small, we would expect it to be positive if investors are willing to hold financial assets instead of physical assets.

- The nominal rate of interest is normally above the rate of inflation in a country

- As part of their anti inflation programme, governments now offer inflation linked bonds which pay a small real rate of interest on a balance that increases with inflation. If inflation rises their debt becomes more expensive to service, investors who are concerned about inflation buy them.
- Interest rates are different in different countries principally because inflation is different in different countries
- Both real and nominal analysis are consistent (but mixing real flows with nominal discount rates and vice versa is wrong). Both require an inflation assumption but in different places (see Table 4)

3.16 Investment Criteria

- Some managers have tended to use a payback rule to analyse investments which is the number of year taken for the net cashflow to become positive

		Discount Rate	
		Real	Nominal
Cash	Real	correct ✓ include inflation in discount rate	incorrect ×
Flows	Nominal	incorrect ×	correct ✓ include inflation in cashflows

Table 4: Real and Nominal Analysis

$-PV, PMT, PMT, PMT$

$$\text{Years Payback} = \frac{-PV}{PMT}$$

With -100,10,10... etc we clearly have a 10 year payback. Fast (low years) paybacks are preferred

- $\frac{1}{\text{Pay Back}} = \frac{PMT}{-PV}$ is very similar to the IRR in this case (which would be exactly 10% for this perpetuity), high $IRRs$ are preferred
- However both are scale independent (give no information about the size of the project), NPV however does just that and so is a preferred measure when comparing projects for evaluation (high NPV preferred to low). When dealing with projects that are mutually exclusive, this will ensure that we do not turn down a large marginally profitable opportunity at the expense of a highly profitable but small project

Total Assets	\$,000	\$,000	**Total Liabilities**
Cash/Investments			Bank Overdraft
Car			Personal Loans
House			Mortgage
Other			**Net Worth**

Table 5: Net Worth of an Individual

4 Life-cycle financial planning (B&M 5)

- An individual's net worth is defined by what he owns less what he owes, it is a bit like an accounting balance (see Table 5) sheet. Although a useful start point for defining our study of personal finance, we will see that this analysis only represents the past and current situation and can not tell us much about what the future holds for our finance.
- How much do you need to save to secure a happy and prosperous retirement? Assuming the following: Age 35, retirement age 65, age of death 80, current

income \$30,000 current savings and asset nil. Ignoring taxes and assuming that your real income remains at \$30,000 but that the real asset return is 3%, we can estimate how much to save.

4.1 Replacement Rule

- Assuming you want 75% of your salary on retirement from age 65 to 80 we need a PV age 65 that will produce an annuity of \$22,500

n	i	PV	PMT	FV
15	3	**\$268,604**	\$22,500	0

- Next we say that we must produce a FV of this amount over the next 30 years with 3% real returns

$$\begin{array}{ccccc} n & i & PV & PMT & FV \\ 30 & 3 & 0 & \mathbf{\$5,646} & \$268,604 \end{array}$$

So in order to produce sufficient savings to generate 75% of your salary on retirement ($22,500) you need to save $5,646pa over 30 years if real rates of return are 3%. This is some 19%! You may not like the figures but try it with your own inputs (NB you may have savings already in which case decrease the PV in the second calculation from zero since you already have some of the terminal amount saved). Don't be tempted to simply increase the real rate of return, we will show that more return requires taking more risk!

4.2 Constant Consumption

- How about choosing a replacement rate that leaves your salary net of saving the same? Assume that the salary net of saving is C. The amount saved for each of the first 30 years is \$$30,000-C$. One dollar saved each year is worth \$47.58 in 30 years time

n	i	PV	PMT	FV
30	3	0	\$1	**\$47.58**

  so \$$C$ pa will yield $\mathbf{\$47.58}\times$ (\$$30,000-\mathbf{C}$)

- The amount drawn each year form the retirement account is C for each year of 15 one dollar a year has a PV of

n	i	PV	PMT	FV
15	3	**\$11.94**	\$1	0

so that C dollars a year requires a lump sum of $\$11.94 \times \mathbf{C}$. Now solve for C

$$\begin{aligned} \$\mathbf{47.58} \times (30,000 - \mathbf{C}) &= \$11.94 \times \mathbf{C} \\ C &= \$23,982 \end{aligned}$$

Savings over each of the first 30 years is now $\$30,000 - C = \$6,018$ or 20%

4.3 Income and Human Capital

$$\sum_{t=1}^{45} \frac{C}{(1+r)^t} = \sum_{t=1}^{30} \frac{\text{Income of } 30,000}{(1+r)^t} \Longleftrightarrow C * 24.52 = 30k\$ * 19.60$$

- C also solves this equation $((45, 3, -24.52, 1, 0), (30, 3, -19.60, 1, 0))$ which says that the Present value of lifetime consumption (over 45 years) is equal to the present value of labour income (over 30 years), terms which have become known as Human Capital and Permanent Income (consumption). They are related

$$PV\text{ (Human Capital)} + PV\text{ (Financial Capital)} = PV\text{ (Permanent Income)}$$

Since financial markets value the present value of financial instruments every day, PV (Financial Capital) is given by their market value so the present value of our Permanent Income (Consumption) is equal to our Human Capital plus the market value of whatever Financial Capital we own. The more Human Capital or Financial Capital you own, the higher your Permanent Income or Consumption (assuming you leave no bequest to you children).

- What is known as the budget constraint can be expanded to include a Bequest B and initial wealth W_0, labelling income Y. If T is the number of years of life and R the numbers of years to retirement (See Table 6)

$$W_0 + \sum_{t=1}^{R} \frac{Y}{(1+r)^t} = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{B}{(1+r)^T}$$

PV Income	\$,000	\$,000	**PV Outgoings**
Current (net) Financial Wealth PV Working Income			PV Permanent Consumption PV Bequest

Table 6: PV Balance for an Individual

- You can increase your consumption if you...
  - Have higher income Y
  - Lower the bequest B you anticipate leaving your children
  - Can raise your real rate of return r
  - Lengthen the time you expect to work R
  - Expect to die earlier ($T > R$ is lower)
  - Have more initial wealth W_0. W_0 may be negative (borrowing) as it may be possible to borrow limited amounts against your future human capital,

however an individuals borrowing capacity declines with his or her human capital and is strictly limited after retirement

4.4 Valuing your Education

- Why are you sitting here studying finance? (something you have probably pondered greatly!)
- You can either treat this education as a consumption good (like watching a very long and expensive movie!), if this is the case I hope you enjoy it because it will not necessarily increase your wealth or income.
- You can treat this education as an investment good which will reap valuable returns in years to come. What benefits might you expect from studying?

- Increased employment opportunities, greater job flexibility and satisfaction
- In short you expect this study (which is costly and reducing your W_0 or making it more negative if you are already in debt) to increase your Human Capital and therefore increase your expected consumption, otherwise you are just watching an expensive movie!
- By how much might income have to increase to make a years study a profitable ($NPV > 0$) activity? Well the explicit study costs may be around \$20,000 for the year, on top of that you have given up your job for a year so you have an opportunity cost of not working for the year (say \$30,000). Consumption costs over the year can safely be ignored, you would have had to be housed (pay rent), eat, drink and socialise anyway, this level of consumption may be different now compared to last year when you were working but we will ignore that.
- If your Human Capital depends on the next 30 years of income we clearly need

an investment of some \$50,000 to pay off over 30 years with a real return of 3%

n	i	PV	PMT	FV
30	3	$-\$50,000$	$\mathbf{\$2,551}$	0

That is we only need a base salary rise of $\$2,551$ on \$30,000 (and the subsequent indexation to be based on the new level†). Take heart, this is only 8.5% while most studies show that students can increase their salary by more than this.

- Conclusion? Studying is great value, it increases your Human Capital by more than it costs!
- Corollary? We are clearly not charging enough for our courses! In competitive markets the price of any good rise to its marginal benefit ;-).

†Or your MBA may allow you to consume more cheaply; 45 year annual savings of \$2,039 will pay for your MBA $(45, 3, -50k, 2.039k, 0)$. Finally it may reduce your career risk (reducing 3% to 2%).

4.5 Buy or rent a house

- Objective is provide yourself with the lowest cost (NPV) housing (which is part of consumption). Taxes etc are ignored. It assumed that because you need housing until you die, the horizon is long and the perpetuity formula can be used
- Buying a house to house yourself may cost \$100,000 (£62,500) and the real interest rate in \$ or £ is currently about 3%, the effective annual "rent" for purchasing the property which is a perpetuity of rent is 3%×\$100,000 (£62,500) $= \$3,000$ (£1,875)
- Renting may involve paying \$5,000 (£3,000) pa which will increase with inflation and therefore will need discounting at the real rate $\frac{\$5{,}000(£3{,}000)}{0.03} = \$166,667$ (£100,000)
- Transaction costs aside, over the long term aside it is usually cheaper to buy rather rent

- The element of PV(consumption) that is generated by housing is the PV of the rent or the house price itself
- The good news about living in a nice big house is that it is enjoyable, the bad news is that it is costly because the capital could be employed elsewhere

5 Financial Statements (B&M3)

5.1 The Function of Financial Statements

Who are the stakeholders in a firm who need information?

5.2 The three main Financial Statements

- Profit and Loss
  - What has been earned over a period
  - To record the change in economic value of the firm
- Balance sheet
  - What is owned and what is owed
  - To record the cumulated economic value of the firm
- Cashflow statement
  - What cash changes have occurred over a period
  - To record the cash changes of the firm
- The three are linked

5.3 Market versus Book values

- All three accounting statements are based on accounting rules. Accountants produce book values
- Market values, however, are based on willingness to pay and may be very different to book values. Markets produce market values!

5.4 Returns to shareholders versus return on book equity

- Shareholder returns are determined by dividend distributions and capital gains
- Return on book equity is determined by accounting profit divided by accounting book value
- The latter can be systematically higher than the former if book values lag market values

5.5 Ratio analysis

- Ratios are useful for analysing the historical performance of a firm, either over time or across firms
- They are not however forward looking like equity values
- The Efficient Market Hypothesis would say that the future explanatory power of past or current ratios (public info?) is nil!

5.6 Working capital management

- Until recovered during downturn in sales or termination, WC is sunk capital
- Measures that reduce WC produce immediate firm PV, JIT and KanBan

6 How to analyse investment projects (B&M 6)

6.1 Examples of Investment Ideas

- Launch a new line of business (increase assets)
- Replace existing plant with more efficient plant (replace assets)
- Close a loss making subsidiary (decrease assets)
- Even if they do not necessarily increase book assets, all three involve sinking cash now in order to gain a future expected benefit

6.2 Which cashflows?

- Estimate the cashflows without the investment
- Estimate the cashflows with the investment
- Take the difference
- This will yield only future, differential, cashflows, profit and non–cash items, past and sunk costs will be excluded
  - e.g. you know that investing in your business education will be profitable in the long run, however NPV cares about cash and the timing of cashflows not profit
  - Also you invested considerable time, effort and money in researching your education before you undertook this course, however until you decided to actually come (pay the fee) you could have changed you mind, therefore these sunk costs should not have affected your decision after they had been

incurred. These sunk costs purchased you an option, but not an obligation to proceed

6.3 Profit & Loss statement

Seven year project to sell 4,000 units pa @ \$5,000 each (marginal cost of \$3,750 each), the annual P/L will look like Table 7

6.4 Balance Sheet

The year by year balance sheet (in \$M) is shown in Table 8 is

Sales (4,000 @ $5,000)	$20,000,000
Cost of Sales (4,000 @ $3,750)	($15,000,000)
Gross Margin	$5,000,000
SG and A	($3,100,000)
Depreciation	($400,000)
Profit before Tax	$1,500,000
Tax @ 40%	($600,000)
Profit after Tax (Net Income)	$900,000
"Dividend"	($1,300,000)
Retentions	($400,000)

Table 7: Profit statement for each of the seven years of operation

Year End	0	1	2	3	4	5	6	7
Fixed Assets	2.8	2.4	2.0	1.6	1.2	0.8	0.4	0.0
Working Capital	2.2	2.2	2.2	2.2	2.2	2.2	2.2	0.0
Total Assets	5.0	4.6	4.2	3.8	3.4	3.0	2.6	0.0
Owners Interest	5.0	4.6	4.2	3.8	3.4	3.0	2.6	0.0
Profit / Loss	0.9							
Distributions, divs	(1.3)							
Total Liability	5.0	4.6	4.2	3.8	3.4	3.0	2.6	0.0

Table 8: Evolution of the Balance Sheet over the seven years of operation

6.5 Cash Flow Statement

The cash flow statement reconciles the P/L with the opening and closing Balance Sheets is shown in Table 9

Using an interest rate of 15%, this has a NPV $(PV - I)$ of \$1.236M = \$6.236M − \$5M

n	i	PV	PMT	FV
7	15	**\$6,236,000**	1.3	2.2

The Cashflow (or free cash flow) can be deduced two ways

$$\begin{aligned}\text{Cash Flow} &= \text{Revenue} - \text{Cash Expenses} - \text{Taxes}\ (20 - 15 - 3.1 - 0.6 = 1.3) \\ \text{Cash Flow} &= \text{Revenue} - \text{Total Expenses} - \text{Taxes} + \text{Non Cash Expenses} \\ &= \text{Net Income} + \text{Non Cash Expenses}\ (0.9 + 0.4 = 1.3)\end{aligned}$$

Year	0	0–1	1–2	2–3	3–4	4–5	5–6	6–7
Capital exp	(2.8)	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**
Working cap	(2.2)	0.0	0.0	0.0	0.0	0.0	0.0	**2.2**
Total investment	(5.0)	0.0	0.0	0.0	0.0	0.0.	0.0	**2.2**
Op prof bef tax		1.5	1.5	1.5	1.5	1.5	1.5	1.5
Non cash items		0.4	0.4	0.4	0.4	0.4	0.4	0.4
Total op cashflow		1.9	1.9	1.9	1.9	1.9	1.9	1.9
Taxes		(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)
Distributions, divs		(1.3)	(1.3)	(1.3)	(1.3)	(1.3)	(1.3)	(1.3)
Change in cash		0.0	0.0	0.0	0.0	0.0	0.0	0.0
Project cashflow	(5.0)	1.3	1.3	1.3	1.3	1.3	1.3	3.5

Table 9: Cashflows over the seven years of operation

6.6 Ranking projects with different lives

- If we have to chose between competing projects (expand operations under plan 1 or plan 2) and both plans have the same life, there is no problem. We can simply chose the one with the greatest NPV $(PV - I)$
- However there is a problem if the projects have different lives say 5 & 10 years, effectively one NPV will represent the present value cost/benefit of 5 years operation while the other will represent 10 years of operation and will therefore probably be bigger!
- The concept of equivalent annual cost or annualised capital cost takes care of this for us. We assume that the shorter project is repeated until its final time horizon matches the longer (or they are both are repeated indefinitely if their lives have no common denominator). Assume that both machines save the same amount pa

- A machine with a five year life costs \$2M, its annualised capital cost at 10% is \$0.528M while the machine with a ten year life costs \$4M, its annualised capital cost at 10% is \$0.651M. Two five year machines (one after the other) are preferred because the annual capital charge is lower that way. If the machines had different benefits, they could be included in the computation in order to evaluate a net annual capital charge

n	i	PV	PMT	FV
5	10	\$2M	**\$0.528M**	0
10	10	\$4M	**\$0.651M**	0

6.7 Inflation & Capital Budgeting

- Again to repeat, if future cashflows are stated in real terms then a real rate must be used

- If future cashfows are stated in nominal terms then a nominal rate must be used
- The two should not be mixed!
- To go between the two don't use real $=$ nominal $-$ inflation, (it will not give the same NPV under each method) use

$$1 + \text{Real} = \frac{1 + \text{Nominal}}{1 + \text{Inflation}}$$

7 Principles of asset valuation (B&M 7)

- In economics the <u>law of one price</u> says that competition will drive the price of identical goods to the same level
- In finance this law is enforced through <u>arbitrage</u> an activity pursued by speculators or other investors seeking to purchase something for less than its intrinsic value and sell it for more
- Physical goods (foods, equipment etc) can be difficult to arbitrage (buy in one location and sell in another) because of transaction costs
- Financial assets (stocks, bonds, currencies) are easy to arbitrage, if they sold for different prices in different locales, it would be easy to exploit price differentials

7.1 Measures of value

- Accounting Book Value involves conservatism so understate value
  - Lower of cost or net realisable value plus accrued profits less distributions
- Earnings multiples, price/earnings (P/E) and earnings per share (EPS)
  - $P/E = \frac{P}{EPS}$ with $EPS =$ \$2 and an industry $P/E = 10$ one estimate of the share price would be \$20
- Cashflow models from company and public information, analysts forecasts and all other available information
  - Lay out all future cash flows and take an NPV using a required rate of return (interest rate)
- Ask around others for their opinion! Guess!
- Look at the price at which others are prepared to trade, this is the Market Value
- Market to Book values are normally greater than one and can easily exceed 5!

7.2 Efficient Markets

- The efficient markets hypothesis (EMH) says that prices must reflect the amount of information know by the market participants, if this were not the case, someone with better information would be able to exploit his arbitrage opportunity! There are three levels of market efficiency attributable to Fama (1970 [**?**], 1991 [**?**]).
- **Weak form:** No investor can earn excess returns by developing trading rules based on historical prices or return information. In other words, the information in past prices is not useful or relevant in achieving excess returns. Those with superior public or private information can still potentially profit.
- **Semi-strong form:** No investor can earn excess returns from trading rules based on publicly available information. (e.g. investment advice, annual reports and accounts and of course past prices). Those with superior private information (insiders) can still potentially profit.

- **Strong form:** No investor can earn excess returns using any information (i.e. including private insiders), whether publicly or not.
- **Weak** ⊂ **Semistrong** ⊂ **Strong**. The weak form is contained in the semi–strong form which is it itself contained in the strong form.

7.3 Random walks

- One consequence of the EMH is that when markets fully anticipate all available information they appear to follow a random walk, Samuelson (1965) [**?**] was one of the first academics to show this.
- New (unanticipated) information arrives randomly so that in efficient markets prices follow **random walks**. e.g.
- Asset prices go up if unexpected news is good and down if unexpected news is bad (e.g. unexpected change in interest rates, fall in seasonally adjusted employment, change in anticipated monetary policy and inflation etc.).
- Go up if a head is tossed, go down if a tail is tossed in a repeated game (if the coin is unbiased, no matter how far ahead we look we expect – on average – to be where we are now).
- Spread bet on point differential in a basketball game, I pay you £1 for each point your team score and you pay me £1 for each that my team score (unbiased?).

Time Toss	0	 1st	1	 2nd	2	 3rd	3	 4th	4: End of game p'off($\widetilde{H}$)	×perms	×prob	= exp.val
									\$8 (4)	1	$\left(\frac{1}{2}\right)^4$	\$0.5
							\$7	↗↘				
					\$6	↗↘			\$6 (3)	4	$\left(\frac{1}{2}\right)^4$	\$1.5
			\$5	↗↘			\$5	↗↘				
Start	\$4	↗ $\frac{1}{2}$H ↘ $\frac{1}{2}$T			\$4	↗↘			\$4 (2)	6	$\left(\frac{1}{2}\right)^4$	\$1.5
			\$3	↗↘			\$3	↗↘				
					\$2	↗↘			\$2 (1)	4	$\left(\frac{1}{2}\right)^4$	\$0.5
							\$1	↗↘				
									\$0 (0)	1	$\left(\frac{1}{2}\right)^4$	\$0.0
Total	$E\left[\$2\widetilde{H}\right] = \$4$				\$4		\$4			16	1	\$4.0

Table 10: Four round binomial tree of Heads ($\$2 \equiv +\$1$), Tails ($\$0 \equiv -\$1$)

7.4 50 half–years (25 years), quarters, months, weeks, days and hours of foreign exchange returns

If you don't believe that markets could behave randomly or in this way at all, then try to say how you could distinguish the graphs from a simulated random walk and also try to discern which of the following six graphs corresponds to 50 Deutsche Mark/U.S.Dollar exchange rates over 25 (half) years, 50 quarters, 50 months, 50 weeks, 50 days, 50 hours! Mandelbrot has written about the fractal nature of asset returns [**?**] (self similarity of scaled variance $\frac{\sigma_T^2}{T}$ over any time horizon).

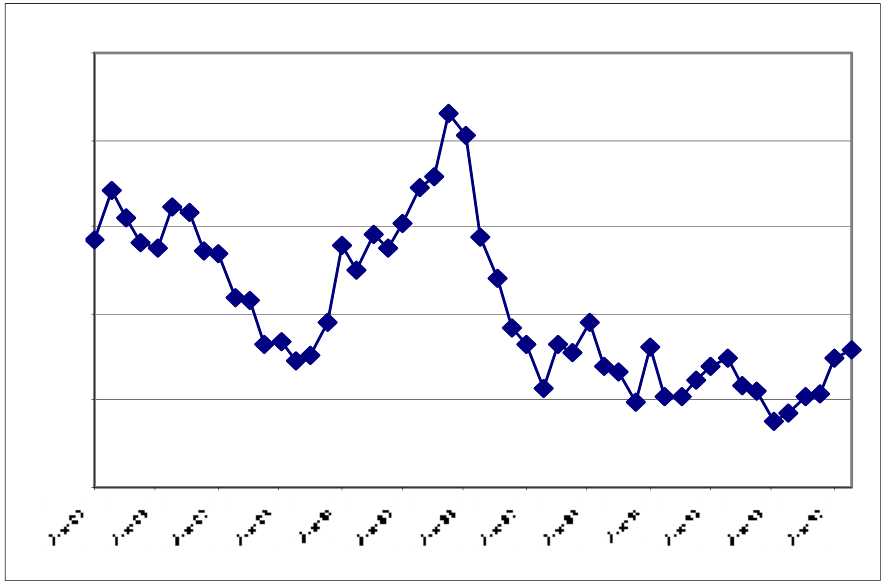


Figure 1: First series

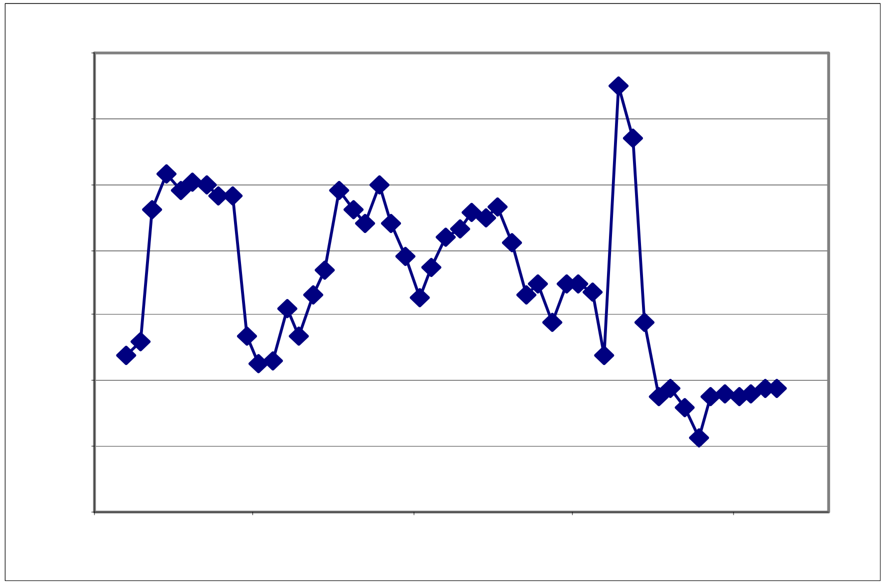


Figure 2: Second series

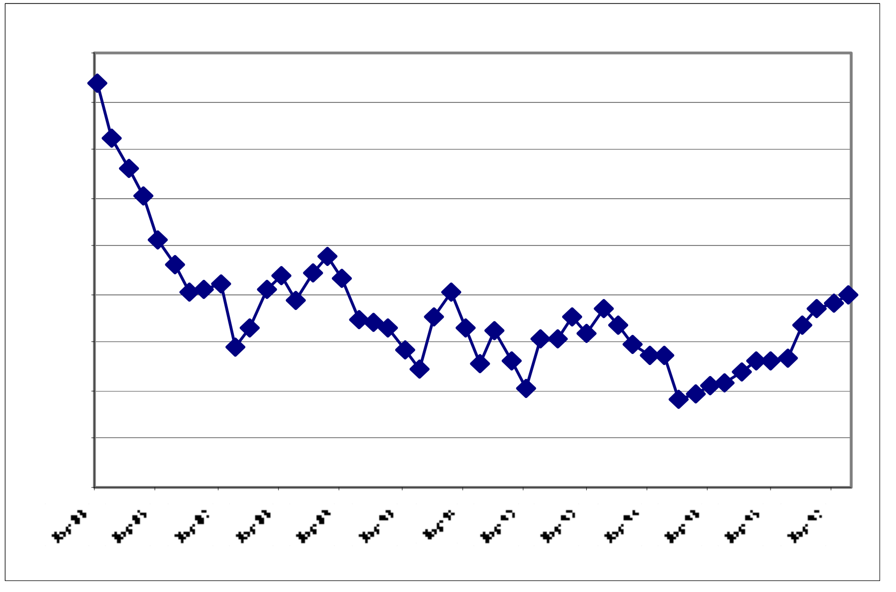


Figure 3: Third series

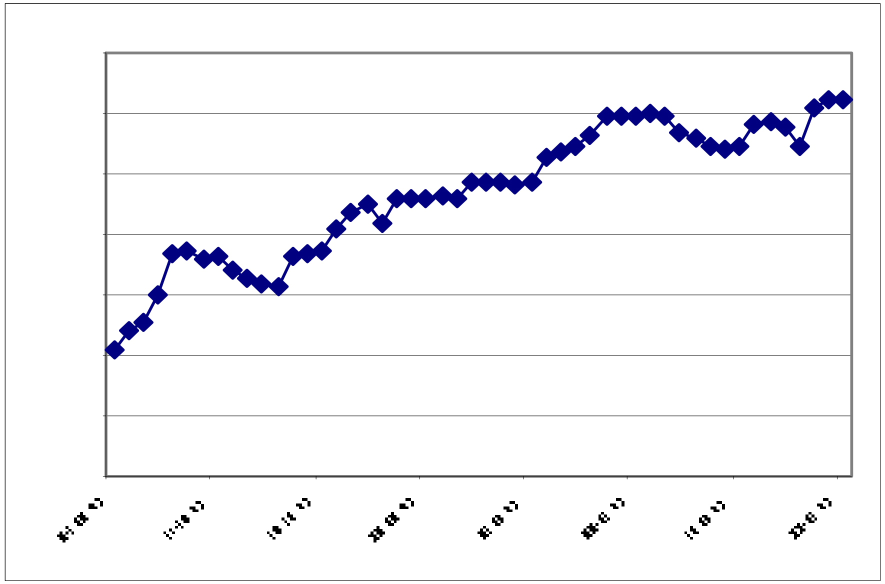


Figure 4: Fourth series

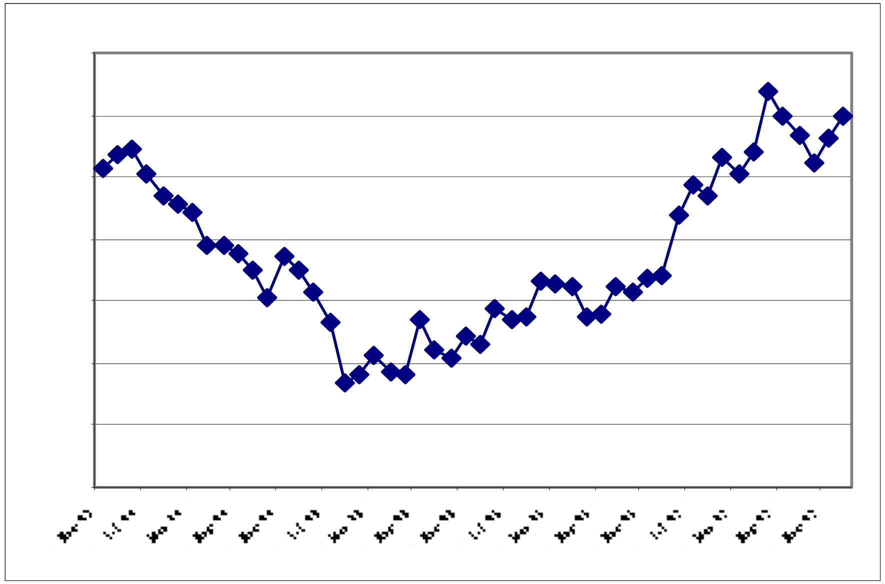


Figure 5: Fifth series

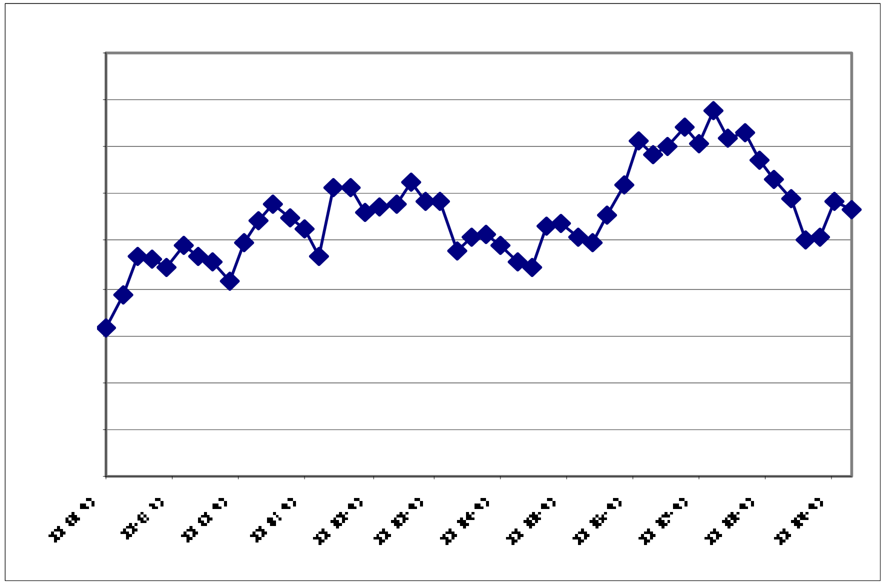


Figure 6: Sixth series

7.4.1 DEM/USD Time Series

7.5 Is it possible to beat the market?

- Sure! Somebody is doing it every day, it is just that it is not the same guy every day and we can't say in advance who will do it (i.e. it is like winning the lottery)! The existence of big winners (and losers) does not contravene efficiency, only if there were systematic winners might we get interested. Darwinian selection takes care of the systematic losers.
- Efficient market say that on average you will get a return that compensates you for the risks (we have yet to define which risk matters) you take and your **expected** return is fair

- After the risks are realised, you may or may not have been lucky and out or underperformed the market, this does not mean that it was not a fair bet when you invested.
- In efficient markets, it should not be possible to **consistently** beat the market.

7.6 Tests of Market Efficiency and the CAPM

Roll's critique (1977) [**?**] says that Market Efficiency and the CAPM cannot be tested separately, i.e. tests of CAPM is always a joint test of ME & CAPM. Fama MacBeth (1973) [**?**] was one of the earlier papers attempting to test the CAPM, in practice more attention is focussed on the composition of the market portfolio than the CAPM per se which can always be considered correct in a theoretical world. The practical conclusion is that you always need to account for risk; larger excess returns can always be produced by the larger risks.

7.7 Market Efficiency debate

- EMH says:–
  - Prices reflect underlying value or the consensus value
  - Market prices may follow a random walk since unanticipated news by definition is random
  - The market risk premium must be positive at all times to persuade investors to hold the risky market instead of the riskless bank account
  - You cannot fool all the people all of the time
- EMH does not say:–
  - Prices are uncaused, they are caused by new news items
  - There is no upward trend in the market. In fact the market drifts up by R_m and the risk free asset increases at a rate $R_f < R_m$.

  - All shares have the same expected return, different firms can offer different rates of return
  - Investors should throw darts to select stocks
- Why are there still disbelievers?
  - There are optical illusions, mirages and apparent patterns in stock prices. Our brains have developed to detect patterns, however there are no "patterns" in random walk series!
  - The truth is less interesting than fiction (contrary to popular belief: the Truth is not out there!)
  - From past data, there is some weak evidence against efficiency (seasonality, insider trading); tests of the EMH are weak
- Market efficiency is the null hypothesis everyone would like to be able to reject

8 Valuation of bonds (B&M 8)

8.1 Coupon Debt

$$PV = \frac{PMT}{1+i} + \frac{PMT}{(1+i)^2} + ...\frac{PMT}{(1+i)^n} + \frac{FV}{(1+i)^n} = \sum_{t=1}^{n} \frac{PMT}{(1+i)^t} + \frac{FV}{(1+i)^n}$$

n	i	PV	PMT	FV
10	?	-100	10	100

- The Yield to Maturity (YTM) or IRR is the rate i that solves the present value to the price of the bond and represents the required rate of return on debt investment. This may be close to the risk free rate but it may also be larger.

- The current yield is defined as the ratio of current income to price $\frac{PMT}{PV} = 10\%$
- Because future interest rates may be higher or lower than today's interest rates, long and short term bonds may have different $YTMs$. A plot of the YTM against maturity is called the Yield Curve
- Finally some bonds may not pay future coupons and principal with 100% probability (what we assume for our Governments!), they bear credit risk and therefore must yield more than default free bonds

9 Valuation of Common stocks (B&M 9)

9.1 Growing Perpetuities[‡]

$$PV = \frac{PMT}{1+i} + \frac{PMT\,(1+g)}{(1+i)^2} + \frac{PMT\,(1+g)^2}{(1+i)^3} ... = \frac{PMT}{i-g}$$

- i should be the appropriate (required) rate of return for stock investments. The (prospective) dividend yield is about 2% currently for the market as a whole and defined as

$$\begin{aligned} DY &= \frac{PMT}{PV} = \frac{\text{Next Div}}{\text{Price}} = i - g \\ i &= DY + g \end{aligned}$$

[‡]see page 236 for proof of this formula

- Thus investors receive equity returns i via two means, a dividend yield and a capital gain g (the capitalisation of future dividend increases)
- Investors can transform a capital gain into cash now (virtual dividend) or a cash dividend into a capital gain (increased holding) by selling or buying more stock. Thus the expected dividend policy is actually irrelevant (Modigliani Miller)
- Thus Microsoft (which had 0 dividend yield until 2003) returned capital gains to investors who were implicitly capitalising future (large) dividends which will come about when the firm changes its dividend policy. Investors who want a dividend now can effectively bring these future dividends forward by selling some stock to produce cash now 11

9.2 Trading off capital gain against yield

This can be done by increasing or decreasing your holding in a stock, for instance if Microsoft are paying no dividend, how is it possible to retrieve a dividend yield? All that needs to be done is to sell enough stock each and every period to generate the required dividend yield, the remaining stock will be subject to the same %age future returns but from a smaller base leading to the extracted dividend decreasing over time. This works at least in theory when applied to uncertain returns.

9.3 Estimating g & i

- The dividend yield of a stock can be observed but g & i cannot

div	Zero			Z/F	Full			High	Low	Neg.
yld %	0			0, 10	10			15	5	-5
g %	10			10, 0	0			-5	5	15
time	4	3	2	1	2	3	4	4	4	4
0	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100
1	0	0	0	110	10	10	10	15	5	-5
2	0	0	121	0	110	10	10	14.3	5.3	-5.8
3	0	133.1	0	0	0	110	10	13.5	5.5	-6.6
4	146.4	0	0	0	0	0	110	94.3	127.3	167.3
IRR	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

Table 11: Different Yield and Capital Gain(Loss) Strategies with equal IRRs

- Sometimes shares rise and produce a capital gain $+g$, sometimes they fall $-g$. However past trends may not be continued, therefore we have to be careful when analysing historical data. On average they tend to produce $+g$ and for the future we normally expect g to be positive unless the dividend yield is in excess of the required rate of return.
- g can have a short term value and a different longer term value when the dividend policy may change
- In practice, we can estimate g by estimating the required rate of return on equity and then subtracting the current dividend yield. We will say more about estimating i later.

9.4 Summary of asset yield, gain, return

- Assets typically return value to their owners via a cash yield δ and a capital gain g. The total expected return is the sum of the two $i = \delta + g$ (derived from Gordon Model). We have still not said where i comes from! (A model of returns is needed).
- Different assets may have different total return $i \gtrless r$ i.e. may be above or below the risk free rate and they may have different yield gain δ, g balances (see Table 12).
- Sometimes we infer i from δ, g sometimes other way round
- For risk free assets $i = r$, we can use a rate of about 5% and we can estimate the implied dividend yield from assets if we know characteristics
- (note that the δ, g balance can be adjusted by changing fractional shareholdings)*

i, δ, g	**Asset**	**TotRet** $i =$	**DivYld** $\delta +$	**ExCpGn** g
Obs g est i	Car (computer etc)	5% $= r$	25,45%	–20,–40%
infer δ	House	$5, 10$%	0,5%?	5%
Untraded!–	Human capital (study)	5%	–10%	15%?
valuation?	Human capital (work)	$10, 5$% age	10%	0,–5%
Obs $\delta\,(CY)$	Risk free debt*	5% $= r$	5%	0%
est. i,	Risk free zero cpn.*	5% $= r$	0%	5%
infer g,	Risky (corp.) debt*	7% $> r$	7.5% (CY)	–0.5%
(switch g,i)	Risky (corp.) zero*	7% $> r$	0%	7.0%
Obs δ,	Microsoft stock*	10% $> r$	∼0%	10%
est i,	Typical stock, index*	10% $= r_m$	2.5%	7.5%
infer g	Mat./decl. ind. stk.*	10% $> r$	5%	5%
Obs δ,	(Index) call options*	25% $> r, r_m$	0%	25%
est i,	(Index) put options*	-10% $< r$!	0%	–10%
infer g	Insurance purchase	$i = -0$%	–%premia	$+$%claims

Table 12: Expected return and yield–gain across assets

10 Risk Management (B&M 10)

We haven't yet said where discount rates come from so this section tells us. It results from work on Portfolio Theory by Markowitz (1959) [**?**] and the Capital Asset Pricing Model (CAPM) of Sharpe [**?**], Lintner [**?**] (1963) et al.

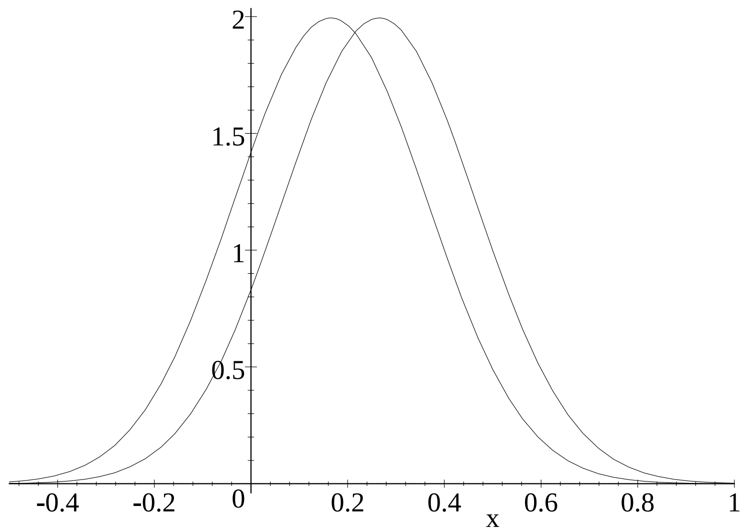
10.1 Market returns & risk

- Market returns are risky, normally distributed with mean/variance statistics

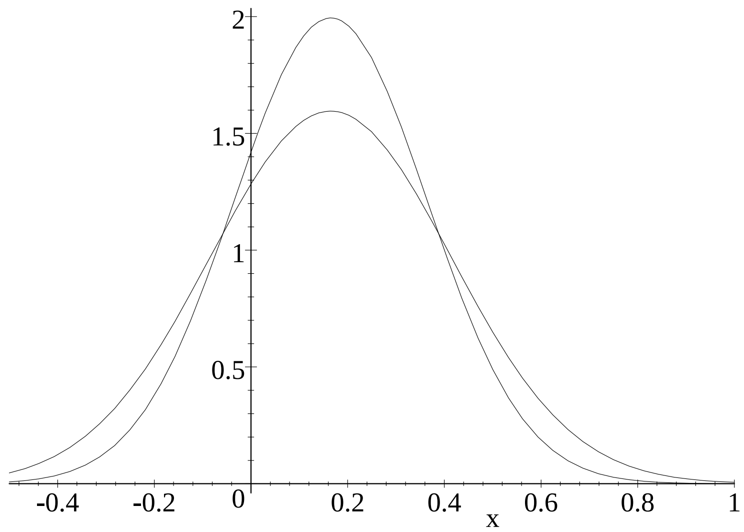
$$\begin{aligned}\%\text{ return } r_1 &= \frac{P_1 + D_1 - P_0}{P_0} = \frac{P_1 + D_1}{P_0} - 1 \\ &= \%\text{ capital gain } + \%\text{ dividend yield}\end{aligned}$$

$$\begin{aligned}\text{Mean}\,(r_i) &= \overline{r_i} = \frac{1}{N}\sum_{i=1}^{N} r_i \\ \text{Var}\,(r_i) &= \frac{1}{N-1}\sum_{i=1}^{N}(r_i - \overline{r_i})^2 \qquad \text{etc.} \\ \text{SD}\,(r_i) &= \text{Var}\,(r_i)^{\frac{1}{2}}\end{aligned}$$

$$\begin{aligned}\overline{x} &= 0.16, \sigma = 0.20 \\ \overline{x} &= 0.26, \sigma = 0.25\end{aligned}$$

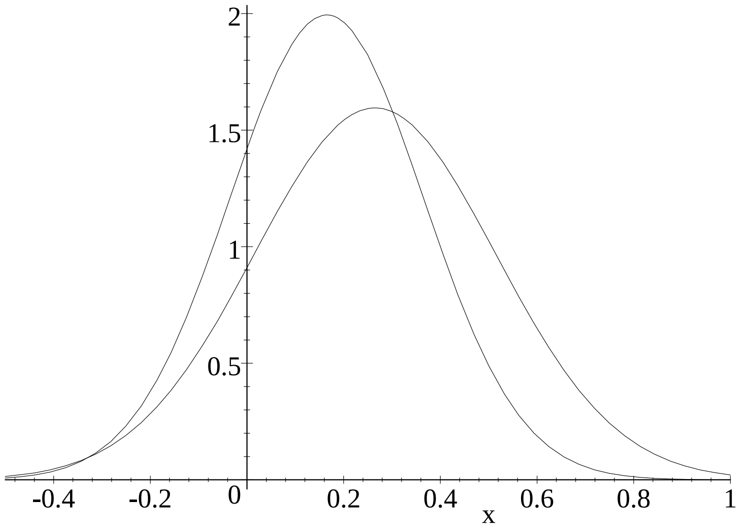


Two normal distributions for returns with different means



Two normal distributions with different variances

Investors prefer higher expected return and lower risk but what about combinations?



Two normal distributions with different means and variances

10.2 Portfolio theory & risk return space

- When forming portfolios we use covariance and Pythagoras to determine portfolio risk

$$\sigma_p^2 = (\sigma_1 + \sigma_2) \times (\sigma_1 + \sigma_2)$$

- For equal weights

$$\sigma_p^2 = \sigma_1^2 + 2\rho_{12}\sigma_1\sigma_2 + \sigma_2^2 = \text{Var}\,(r_1) + 2\text{Cov}\,(r_1, r_2) + \text{Var}\,(r_2)$$

- Special correlation cases (for equal weights)

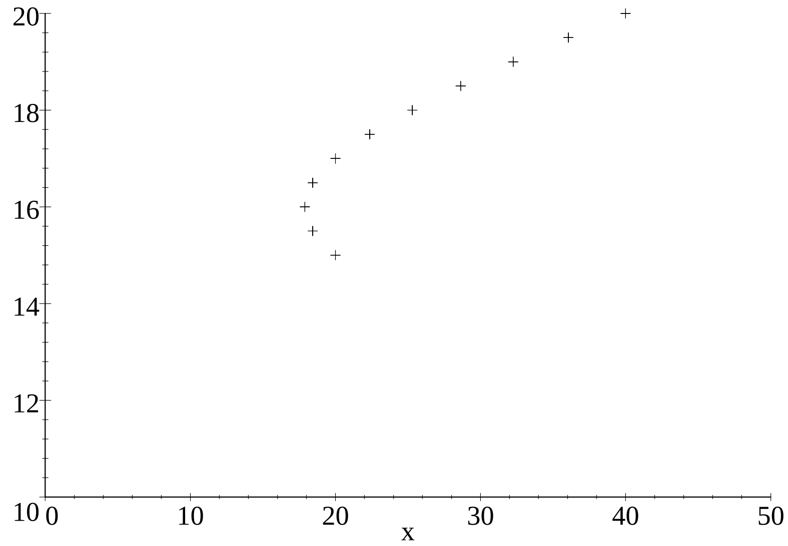
$\rho_{12} = 1$	risks perfectly aligned	$\sigma_p = \sigma_1 + \sigma_2$
$\rho_{12} = 0$	risks at right angles (perpendicular)	$\sigma_p = \left(\sigma_1^2 + \sigma_2^2\right)^{\frac{1}{2}}$
$\rho_{12} = -1$	risks perfectly anti–aligned	$\sigma_p = \lvert\sigma_1 - \sigma_2\rvert$

- Applying weights w_1, w_2, (usually $w_2 = 1 - w_1$) the Weighted Variance is

$$\begin{aligned}\sigma_p^2 &= w_1^2\sigma_1^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 + w_2^2\sigma_2^2 \\ &= w_1^2\text{Var}\left(r_1\right) + 2w_1w_2\text{Cov}\left(r_1, r_2\right) + w_2^2\text{Var}\left(r_2\right)\end{aligned}$$

- Special correlation cases (for weight cases)

$\rho_{12} = 1$	risks perfectly aligned	$\sigma_p = w_1\sigma_1 + w_2\sigma_2$
$\rho_{12} = 0$	risks at right angles (perpendicular)	$\sigma_p = \left(w_1^2\sigma_1^2 + w_2^2\sigma_2^2\right)^{\frac{1}{2}}$
$\rho_{12} = -1$	risks perfectly anti–aligned	$\sigma_p = \lvert w_1\sigma_1 - w_2\sigma_2\rvert$



- An efficient frontier

The total risk $\sigma_p\ (x)$, expected return (y) fontier

- This frontier can be calculated for many combination from many assets and the path of minimum risk for a given return is known as the **efficient frontier**.

10.3 Covariance & regression

- Covariance is defined by

$$\begin{aligned}\text{Cov}\left(r_1, r_2\right) &= \frac{1}{N-1}\sum_{i=1}^{N}\left(r_{1i}-\overline{r_1}\right)\left(r_{2i}-\overline{r_2}\right)=\rho_{12}\sigma_2\sigma_1 \\ \text{Cov}\left(r_1, r_1\right) &= \frac{1}{N-1}\sum_{i=1}^{N}\left(r_{1i}-\overline{r_1}\right)^2=\text{Var}\left(r_1\right)=\sigma_1^2\end{aligned}$$

- and the correlation coefficient ρ (rho)

$$\rho_{12}=\frac{\text{Cov}\left(r_1, r_2\right)}{\sigma_1\sigma_2}$$

- the final variable from regression that is needed is the slope β (beta)

$$\text{slope of b on a } \beta_{12}=\frac{\text{Cov}\left(r_1, r_2\right)}{\text{Var}\left(r_1\right)}=\frac{\rho_{12}\sigma_2}{\sigma_1}$$

10.4 Example of regression

- For example consider 500 points generated by a regression model with unit slope, $N(0,1)$ are variables with a mean of 0 and s.d. of 1. These variates can be generated in Excel using $NORMSINV(RAND())$.

$$\begin{aligned} r_{2t} &= r_{1t} + \epsilon_t \\ r_{1t} &\sim N(0,1) \\ \epsilon_t &\sim N(0,1) \\ \text{Cov}(r_{1t}, \epsilon_t) &= 0 \end{aligned}$$

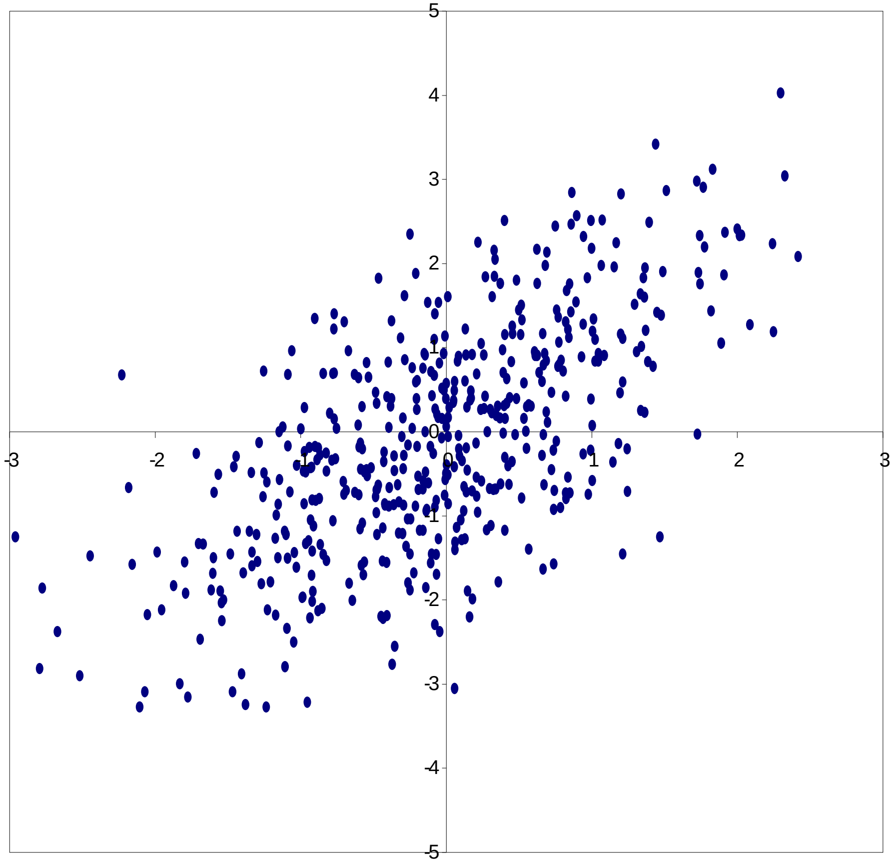


Figure 7: Scatter plot from regression model

$$\begin{aligned}
\text{Var}(r_1) &= 0.97\ (1.00) \\
\text{Var}(r_2) &= 1.83\ (2.00) \\
\text{Cov}(r_1, r_2) &= 0.89\ (1.00) \\
\rho_{12} &= 0.67\ (0.71) \\
\text{slope } \beta_{12} &= 0.92\ (1.00)
\end{aligned}$$

- For stocks, we need to regress the % stock return on the % return of "the market", typically we use the (entire) Equity Market (Index) as the regressor but theoretically we need the total return across all assets (bonds, gold, property, works of art etc.). The slope coefficient of this regression is said to be the beta of the company β, (what happened to alpha α?)

$\times$	σ_1	σ_2	$\cdots$	σ_N
σ_1	σ_1^2	$\rho_{12}\sigma_1\sigma_2$	$\cdots$	$\rho_{1N}\sigma_1\sigma_N$
σ_2	$\rho_{21}\sigma_2\sigma_1$	σ_2^2	$\cdots$	$\rho_{2N}\sigma_2\sigma_N$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
σ_N	$\rho_{N1}\sigma_N\sigma_1$	$\rho_{N2}\sigma_N\sigma_2$	$\cdots$	σ_N^2

Table 13: Variance Covariance Matrix

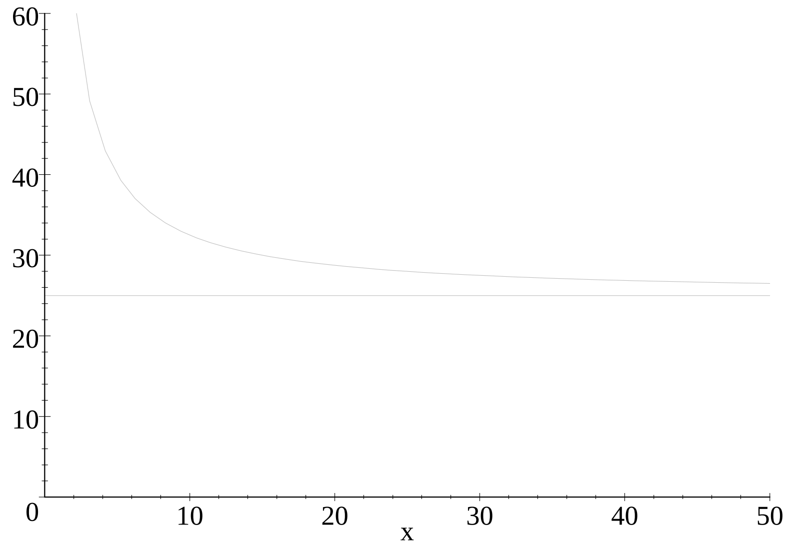
11 Hedging, insuring & Diversifying (B&M 11)

- How is risk affected by diversifying across many stocks in a portfolio?

11.1 Portfolio of many assets

- For the extension to N assets, the portfolio variance $\sigma_p^2 = (\sigma_1 + \sigma_2 \cdots + \sigma_N) \times (\sigma_1 + \sigma_2 \cdots + \sigma_N)$ is given by the sum of all terms in the variance covariance matrix in Table 13

$$
\begin{aligned}
(\sigma_1 + \sigma_2 + \ldots \sigma_N) \times (\sigma_1 + \sigma_2 + \ldots \sigma_N) &= \left(\sum_{n=1}^{N} \sigma_n\right)^2 \\
\frac{1}{N^2}\sum_{n=1}^{N} \sigma_n^2 + \frac{1}{N^2}\sum_{n=1}^{N}\sum_{m=1,\neq n}^{N} \rho_{nm}\sigma_n\sigma_m &= \\
\frac{1}{N^2}N\overline{\sigma^2} + \frac{1}{N^2}\left(N^2 - N\right)\overline{\rho\sigma^2} \overset{N\to\infty}{=} \frac{\overline{\sigma^2}}{N} + \left(1 - \frac{1}{N}\right)\overline{\rho\sigma^2} &\to \overline{\rho\sigma^2}
\end{aligned}
$$



Diversifiable (idiosyncratic) and systematic (market) risk as a function of number of investments (diversification)

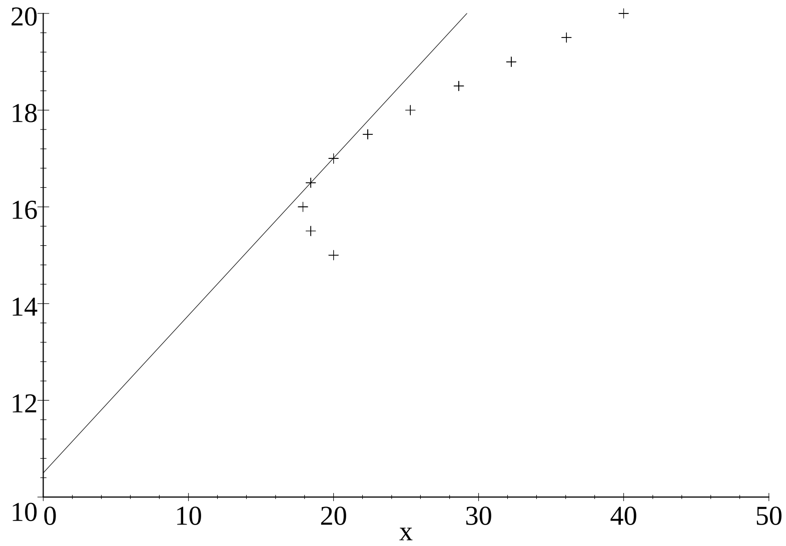
12 Choosing an investment portfolio (B&M 12)

12.1 Risk return trade-offs

- If its seems that higher risk generates higher returns, what is the risk return trade-off?
- How much risk should we take?
- Will this vary over the course of our life and our changing investment needs?

12.2 Asset allocation

- All identical, risk averse investors end up holding a (linear) combination of the risk free asset and the market portfolio (two fund separation) and therefore reside on a security market line (SML). Slopes of this tangency line from the risk free and of the capital market line (CML) efficient frontier must be equal at the point of contact moreover this point of intersection must represent the aggregate market portfolio if all assets are to be held (by someone).



Risk return frontier and tangent in total risk σ_p, expected return space

13 Capital Asset Pricing Model (CAPM B&M 13)

- Considering a portfolio with a fraction w in a risky asset i and $(1-w)$ in the market asset the expected return and risk are given respectively by

$$\begin{aligned} R_p &= wR_i + (1-w)\,R_m \\ \sigma\,(R_p) &= \left(w^2\sigma_i^2 + 2w\,(1-w)\,\rho_{im}\sigma_i\sigma_m + (1-w)^2\,\sigma_m^2\right)^{\frac{1}{2}} \end{aligned}$$

- The rates of change of these variables w.r.t. the fraction w are

$$\begin{aligned} \frac{\partial R_p}{\partial w} &= R_i - R_m \\ \frac{\partial \sigma\,(R_p)}{\partial w} &= \frac{2w\sigma_i^2 + (2-4w)\,\rho_{im}\sigma_i\sigma_m + 2\,(1-w)\,\sigma_m^2}{2\left(w^2\sigma_i^2 + 2w\,(1-w)\,\rho_{im}\sigma_i\sigma_m + (1-w)^2\,\sigma_m^2\right)^{\frac{1}{2}}} \end{aligned}$$

- and their values at $w = 0$ are

$$\left.\frac{\partial R_p}{\partial w}\right|_{w=0} = R_i - R_m$$

$$\left.\frac{\partial \sigma\left(R_p\right)}{\partial w}\right|_{w=0} = \frac{\rho_{im}\sigma_i\sigma_m - \sigma_m^2}{\sigma_m}$$

- If all the assets are to be held in equilibrium there must be no excess or shortfall in demand for each asset i so that the slope of the risk return trade off at $w = 0$

must be equal to the slope of the CML from the risk free to the market portfolio

$$\left.\frac{\partial R_p/\partial w}{\partial\sigma\left(R_p\right)/\partial w}\right|_{w=0} = \frac{R_i - R_m}{\left(\rho_{im}\sigma_i\sigma_m - \sigma_m^2\right)/\sigma_m} = \frac{R_m - R_f}{\sigma_m}$$

$$R_i - R_f = \left(R_m - R_f\right)\frac{\left(\rho_{im}\sigma_i\sigma_m - \sigma_m^2\right)}{\sigma_m^2} + R_m - R_f$$

$$R_i - R_f = \left(R_m - R_f\right)\frac{\rho_{im}\sigma_i\sigma_m}{\sigma_m^2}$$

- We have been writing R_i for the expected return $E\left[R_i\right]$ on asset i so we now (finally) have

$$\begin{aligned}
E[R_i] - R_f &= \frac{\mathsf{Cov}(R_i, R_m)}{\mathsf{Var}(R_m)} \Big(E[R_m] - R_f\Big) \\
E[R_i] - R_f &= \frac{\rho_{im}\sigma_i}{\sigma_m} \Big(E[R_m] - R_f\Big) \\
&= \beta_{im} \Big(E[R_m] - R_f\Big) \\
\text{excess exp. return} &= \text{beta } \times \text{exp. risk premium}
\end{aligned}$$

- and expected excess returns are driven by the individual asset's covariance of returns with the market and a market risk premium
- Although unpriced the idiosyncratic risk of a stock may be large; it is driven by the component of returns orthogonal to the market and linked to the residual

sum of squares, the correlation coefficient and R^2 from a market regression

$$\sigma_s$$

$$\nearrow\uparrow \qquad \sigma_{idio} = \sigma_s \left(1 - \rho_{ms}^2\right)^{\frac{1}{2}}$$

$$\rightarrow$$

$$\sigma_{system} = \beta_{ms}\sigma_m$$

$$\begin{aligned} \sigma_{idio}^2 &= \sigma_s^2 - \sigma_{system}^2 = \sigma_s^2 - \beta_{ms}^2\sigma_m^2 \\ &= \sigma_s^2 - \rho_{ms}^2\sigma_s^2 = \sigma_s^2\left(1 - \rho_{ms}^2\right) \end{aligned}$$

$$\begin{array}{c} \text{error sum} \\ \text{of squares} \end{array} = \begin{array}{c} \text{total sum} \\ \text{of squares} \end{array} - \begin{array}{c} \text{regression sum} \\ \text{of squares} \end{array}$$

13.1 Examples of beta

Find the beta β for a firm of your choice. What about the following assets, what beta might they have?

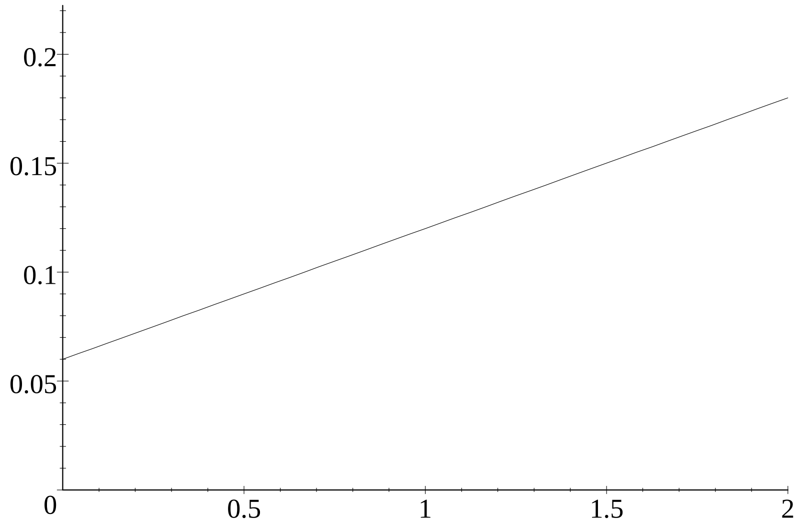
- The average industrial firm
- A firm of higher than average leverage
- Oil or Gold
- Your future wages
- Options?

13.2 The risk premium

- Historically = 8%, 6%, 4%?, but we care about the expected (future) premium?

- Some debate over level although direction is considered to be falling, state your assumption!

$$\begin{aligned} R_f &= 6\%, RP = 6\%, E[R_m] = 12\% \\ E[R_i] &= 0.06 + 0.06\beta_i \end{aligned}$$



The Security Market Line showing **future expected returns** as a function of systematic risk β.

Asset	**beta**	**MV price weights, A**	**portfolio B**
A	1.0	1	1
B	2.0	2	–2
C	1.0	3	3
D	0.5	4	4
	weighted sum	10	2
	sum weights	10	6
	weighted beta	1.0	0.33

Table 14: Weighting betas with Market Values

13.3 Linearity in R_i and β_i

- Since returns are linearly related to betas, the (asset or liability) a weighted average cost of capital (WACC) can be re–expressed in terms of betas from the CAPM and therefore each of the asset or liability components should also lie on the SML with the weighted average residing in the (weighted) middle.

$$R_i = R_f + \beta_i RP \quad R_a = R_f + \beta_a RP \quad \text{etc}$$

$$\frac{B}{A}R_b + \frac{C}{A}R_c = R_a = \frac{D}{A}R_d + \frac{E}{A}R_e \Leftrightarrow \frac{B}{A}\beta_b + \frac{C}{A}\beta_c = \beta_a = \frac{D}{A}\beta_d + \frac{E}{A}\beta_e$$

- Betas can be weighted within a portfolio across assets just as you would with market returns (Table 14)

13.4 If β became important what happened to α?

- Alpha α corresponds to the intercept of the slope line of a regression of excess stock returns against the excess market return. From the CAPM alpha should be zero

$$\begin{aligned} R_i - R_f &= \alpha_i + \beta_i \left(R_m - R_f \right) + \epsilon \\ E\left[R_i - R_f \right] &= \beta_i \left(R_m - R_f \right) \\ \alpha_i &= 0 \end{aligned}$$

  Using historical data, this can be tested by looking at sample α's for systematic deviation from 0 since in an efficient market the CAPM tells us they should be zero.

- On the security market line, you should neither beat nor fall behind beta times the market on average, but you can chose the level of expected return by constructing a portfolio with the appropriate beta. Therefore a better test of risk adjusted outperformance for an investment is to construct the risk adjusted return and to compare this to the risk premium

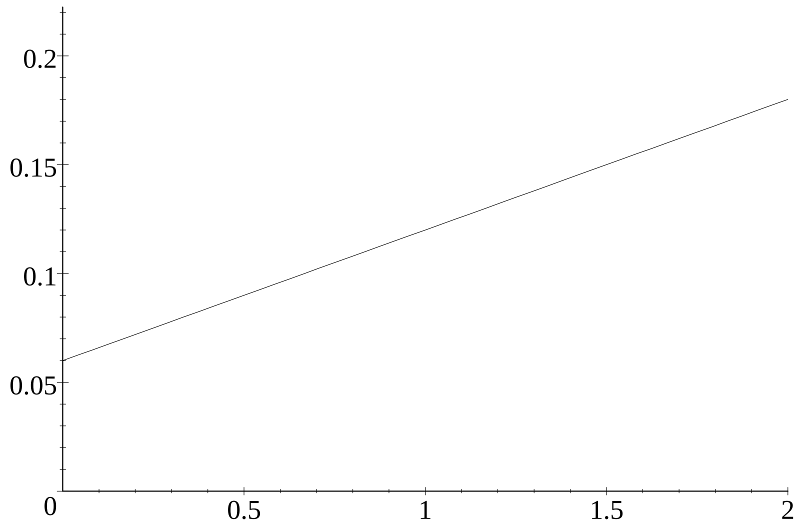
$$
\begin{aligned}
E[R_i] &= R_f + \beta_i \left(E[R_m] - R_f \right) \\
\frac{\text{realised excess returns}}{\text{risk measured by } \beta_i} &= \frac{R_i - R_f}{\beta_i} \lesseqgtr \left(E[R_m] - R_f \right) \text{ ??}
\end{aligned}
$$

- This measure is related to alpha

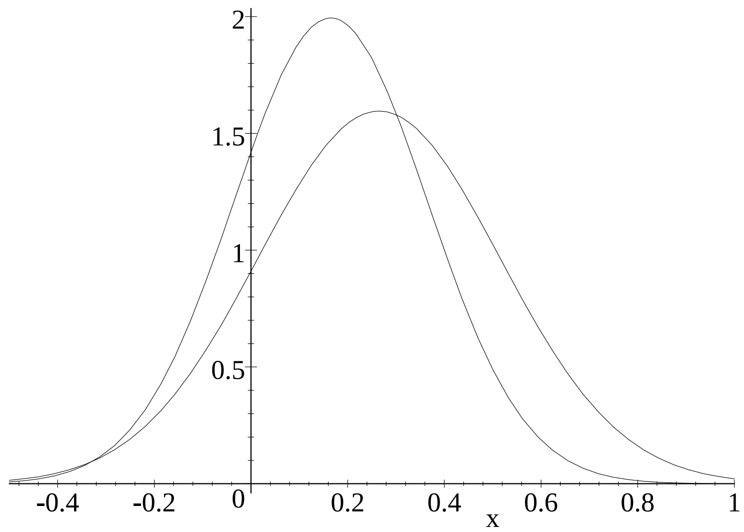
$$\alpha_i = \overline{\left[R_i - R_f\right]} - \beta_i \overline{\left[R_m - R_f\right]} \lesseqgtr 0 \text{ ??}$$

- If significant, then risk adjusted returns may have been in excess of the **historical** SML level, however whether they will continue to be above the **future** SML is anybody's guess

- The easiest way to beat the market is simply to take more risk than the market portfolio and this will lead to higher expected returns! (but also a higher chance of large losses).



Historical return against beta, positive (negative) alpha points lie above (below) the historical SML



High expected return, high risk against low expected return, low risk

14 Corporate Finance/Capital Structure (B&M 16)

14.1 Firm valuation

Now that we have an understanding or how risk and return go together, we can use the CAPM to look at theoretical valuation of the components of firm value. The aim is to use finance rules to produce a Market Value balance sheet. This will differ from a Book Value Balance sheet which is constructed using Accounting rules. Both methods have their place but they may produce similar or dissimilar results.

Total Assets	**Total Liabilities**
Cash and Marketable Securities	Current Interest and Tax
Raw Materials	Creditors
Finished Stock	Senior Loans and Debt
Debtors	Junior Debt and Preferred Equity
Intangible Assets	Shareholders Equity
Fixed Assets	Retained Profit

Table 15: Typical Book Value Balance Sheet (Gross Format)

14.2 Book value balance sheet

Book values are the lower of cost or net realisable value and can be presented in net or gross formats (see Table 15 for example of gross format). The sheet must balance due to accounting identities.

The amount of cash, materials and credit given in normal business less the amount of credit taken is called the working capital requirement (WCR) and is capital that the firm requires to operate. This is separate to the capital required to purchase operating assets, even traders who own no asset may require working capital although some firms get sufficient trade credit and give so little credit themselves (supermarkets) that they have a negative WCR. Profits must reflect a return to this capital that is tied up. Anything the firm can do to diminish the WCR will release cash and create value.

Firms sometimes hold cash as marketable securities for strategic reasons. The shareholders will be seen to be neutral to this strategy but it should not be considered value creating.

Assets and liabilities are increased or decreased over time as their economic change in value is passed through the income statement. Table 16 shows a net book value balance sheet with all current liability holders moved from right to left and now included in Working Capital. This will be generalised to a market value balance sheet.

Net Assets	**Net Liabilities**
WCR = cash and marketable securities	Debt = senior loans and debt
+ raw materials + finished stock + debtors	+ junior debt and preferred equity
– current interest and tax – creditors	Equity = Shareholders equity
Fixed and intangible assets	+ retained profit

Table 16: Net Book Value Balance Sheet

14.3 "Off Balance Sheet" items

Often assets are contracted for via a lease where regular payments (rather than outright purchase) are made. Subcontracting & outsourcing (e.g. rent v. buy) are other examples of a firms activities that do not enter the book value balance sheet but enter firm value implicitly though the cash flows.

Intangibles & growth options, Brands, Research and Development, strategic options and joint ventures are all off the accounting balance sheet as well but on the market value balance sheet so long as their cash flows are anticipated.

Other off balance sheet, Hedge Instruments, Contingent liabilities, non recourse lending, option like investments and the government's tax slice.

14.4 Market values; Cash

The Market value is given by the discounted (present) value of future expected cashflows, i.e. (net) present value of future cashflows. Consider a cash interest account that pays an interest rate r, on an amount I invested, the cashflow at the end of the year is $(1+r)\,I$. The present value is still I because the NPV formula includes cashflows on top and discount rates beneath

$$PV\,(\text{Cash}) = I = \frac{(1+r)\,I}{(1+r)} \tag{1}$$

Thus for cash the book value equals the market value![§] This is not necessarily true of other marketable securities, it depends how the accounting treats the changes in their market value.

[§]This is true not matter what liquidiation time is chosen, the longer the time the higher the cashflow, in just the right proportion to offset the discounting

$$\frac{(1+r)^T I}{(1+r)^T} = I$$

14.5 Debtors & Creditors

Debtors are due within the year, say at the year end (exactly when depends on the terms of trade) so the present (market) value will actually be less than the book value

$$PV\ \text{(Debtors or Creditors)} = \frac{D \text{ or } C}{(1+r')^{t<1}} < D \text{ or } C$$

Raw materials and stock could be valued on its expected sale, invoice and final payment date less the cash flows required to get it to the point of sale.

14.6 Other assets

Intangibles assets are difficult to value on their own (valuation is included with residual equity claims) but fixed assets can be valued if their (potential) sale value or cashflow benefits are known.

14.7 Debts and Loans

Companies can issue loans or bonds as they are known. Once issued bonds with fixed terms will have a value that can fluctuate away from their issue and book value as the interest rate and credit they offer becomes more or less attractive compared to other bonds in issue. The present value is simply the sum across all future interest payments c per bond, for a three year bond with principal

$$\begin{aligned} PV\,(\text{Loan}) &= \frac{E\,[c_1]}{1+r_d} + \frac{E\,[c_2]}{(1+r_d)^2} + \frac{E\,[100 * 1_{ND} + c_3]}{(1+r_d)^3} \\ &= \sum_{t=1}^{3} \frac{E\,[c_t]}{(1+r_d)^t} + \frac{E\,[100 * 1_{ND}]}{(1+r_d)^3} \qquad (2) \end{aligned}$$

and is sensitive to the current rate of interest. If there are m bonds in issue the total market value of the debt is $D = mL$ and the total interest rate expense is $C = mc$. The book value however remains at 100 per bond excluding the interest due within the year (c_1, c_2 per bond etc. which goes under current liabilities), unless the bond is partly or fully repaid in which case the book value of the bond is adjusted accordingly. There are some special cases for the valuation of bonds and some income streams.

14.7.1 Zero coupon bond

For one large "coupon" of 100 at redemption T alone, the NPV formula is

$$PV\,(\text{Zero Coupon Loan}) = \frac{E\left[100 * 1_{ND}\right]}{(1+r_d)^T}$$

14.7.2 Perpetuity

Some UK Government bonds pay interest for ever and have no redemption date, and thus are perpetual in nature. They are called Consols or War Loan because they are government debts consolidated after the Napoleonic Wars. Interest rates then were very low so they only carry coupons of 2–3% but now that their price has fallen to 25% of face value, their yield is around 6%. Indeed if the limit of Equation **??** is taken as the maturity becomes very large, the price of the perpetuity with constant coupon c can be shown to be

$$\begin{aligned} PV\,(\text{Perpetual Loan}) &= \frac{c}{r_d} \\ r_d &= \frac{c}{PV\,(\text{Perpetuity})} \end{aligned}$$

which is the same as Equation 1 for cash if the coupon from a cash holding is considered to be $c = r_d I$ and r_d is the yield on the perpetuity.

14.7.3 Annuity

A stream of cash flows that starts now and finishes at a specific time T can be valued using two perpetuities, one that starts now and runs for ever and a second one that negates the first for all time greater than T. For a riskless perpetual government bond the required rate of return is the risk free rate r

Time 0	1	2	$\rightarrow$	T	$T+1$	$T+2$	$\rightarrow$	∞
Perpetuity 1 $PV\,(1) = \frac{c}{r}$	c	c	$\rightarrow$	c	c	c	$\rightarrow$	∞
Perpetuity 2 $PV\,(2) = -\frac{c}{r}\frac{1}{(1+r)^T}$	0	0	$\rightarrow$	0	$-c$	$-c$	$\rightarrow$	∞
Sum $PV\,(1+2) = \frac{c}{r}\left(1 - \frac{1}{(1+r)^T}\right)$	c	c	$\rightarrow$	c	0	0	$\rightarrow$	∞

14.8 Equity

The book value accounts record the initial amount of equity contributed by shareholders, any further amount contributed by new issue and the amounts earned by but not distributed to shareholders over the course of the years (retained profit/loss). This contrasts starkly with the equity's market value and can often be different by a factor of 10! Using the dividend growth model of Gordon [**?**] with a constant growth rate for dividends of g and infinite horizon

$$P = \frac{d_1}{1 + r_e} + \frac{d_1 \left(1 + g\right)}{\left(1 + r_e\right)^2} ... = \frac{d_1}{r_e - g}$$

d_1 is the next year end dividend per share, if there are n shares in issue the total dividend expense is nd_1 and the market value of Equity is $E = nP$. Book equity B typically remains lower, even though augmented by retained profit because it still does not look at future growth, only what has been contributed or accumulated.

14.9 Book values and clean surplus

For an all equity financed firm, even if book value grows at the same rate as the market value of equity, it is easy to show that the price of a firm's equity will differ from its book value by a constant. Suppose that in addition to the growing dividends d_1 next year, the firms earning that year will be given by e_1, now the amount earned as earnings but not distributed as dividends $e_1 - d_1$ must be reflected in an increase in book value, which must grow at the same rate as earnings and dividends i.e. by an amount $gB = e_1 - d_1$. From the prior relationship, the value of the firms equity P can be decomposed into a book value B and the present value of perpetual residual income

$$P = \frac{d_1}{r_e - g} = \frac{e_1 - gB}{r_e - g} = \frac{e_1 - r_e B}{r_e - g} + \frac{r_e B - gB}{r_e - g} = B + \frac{e_1 - r_e B}{r_e - g}.$$

This is to say that the market value of equity P is a premium to the book value B, the premium is the perpetual capitalised residual income $e_1 - r_e B$ over the next year capitalised at the equity rate r_e next of growth g.

Alternatively the goodwill $P - B$ (on acquisition of a firm worth P but only recognisable in the accounts as B) is the capitalised residual income $e_1 - r_e B$.

If $e_1 - r_e B$ which has an interpretation of value added is positive, then the price of the firm will exceed the book value. If however the firms earnings are insufficient to cover the capital charge $r_e B$ then the firm's price will be less than its book value (negative goodwill).

14.10 Market value balance sheet

Now the market values of Equity and Debt can be added to infer the market value of the assets (net of creditors, i.e. fixed assets and working capital requirement). In practice since the market price of the debt is not always available, it is usually

Market Value of Assets	**Market Value of Liabilities**
	Debt: $D = mL$
Assets: A	
	Equity: $E = nP$

Table 17: Market Value Balance Sheet

necessary to estimate the market value debt as being equal to the book value debt. This cannot be not too bad, especially if the firm is far from default, but this is not an approach we would wish to take for equity.

14.11 Value additivity

Book values add up because of accounting rules, market values add up because they are based on cashflows and they add up. Thus the fact that the cash that the assets produce over their life adds up to the cash available for distribution to liability holders means that the NPV's and market values must also add up. Cash Flows CF_t, Discount rate r, horizon T, Investment I

$$(N)PV = (-I) + \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}$$

$$\begin{aligned} CF_t &= CF_t^A - CF_t^L \\ NPV(A-L) &= \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t} = \sum_{t=1}^{T} \frac{CF_t^A - CF_t^L}{(1+r)^t} = NPV(A) - NPV(L) \end{aligned}$$

and thus for the firm as a whole $CF_t = 0$ and $NPV(A) = NPV(L)$ and the market value of assets is equal to the market value of liabilities (see Table 17)

$$A = D + E$$

14.12 Reconciling asset/liabilities returns; one asset

Consider operation of a firm with assets of value A over one period followed by liquidation. For a given current asset value

$$A = 100$$

and an expected operating cash flow (or profit ignoring investment, cash flow timings etc. in a one period world),

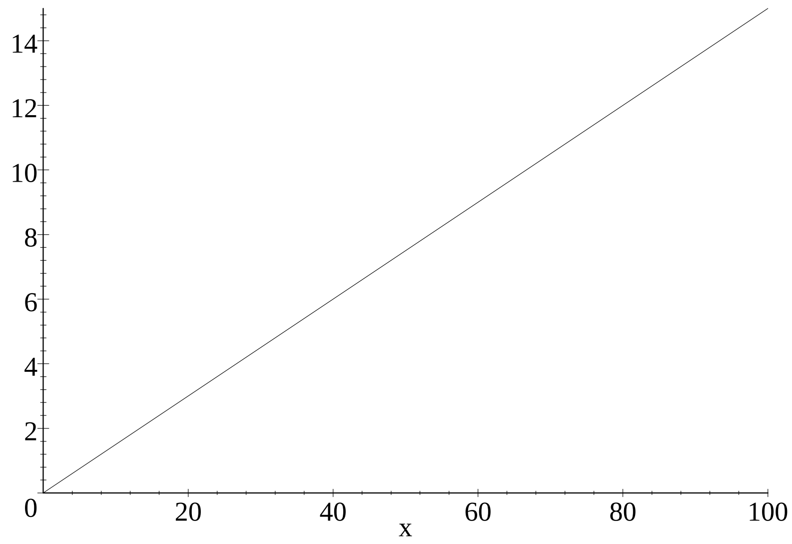
$$\pi_a = 15$$

the expected return on assets is

$$R_a = \frac{\pi_a}{A} = 0.15$$

N.B. this is consistent with the NPV rule

$$\begin{aligned} \text{NPV (Assets)} &= \text{Discounted liquidation value} \\ A &= NPV\,(A) = \frac{A + \pi_a}{1 + R_a} \\ R_a &= \frac{\pi_a}{A} \end{aligned}$$



Flow on y against stock on x (yield as slope)

14.13 Two assets

π_a could be the sum of flows (profits) from two different subassets B and C with individual flows π_b, π_c and current values B, C

$$\begin{aligned} \pi_a &= \pi_b + \pi_c \\ A &= B + C \end{aligned}$$

Assuming

$$B = 20$$

$$\pi_b = 5$$

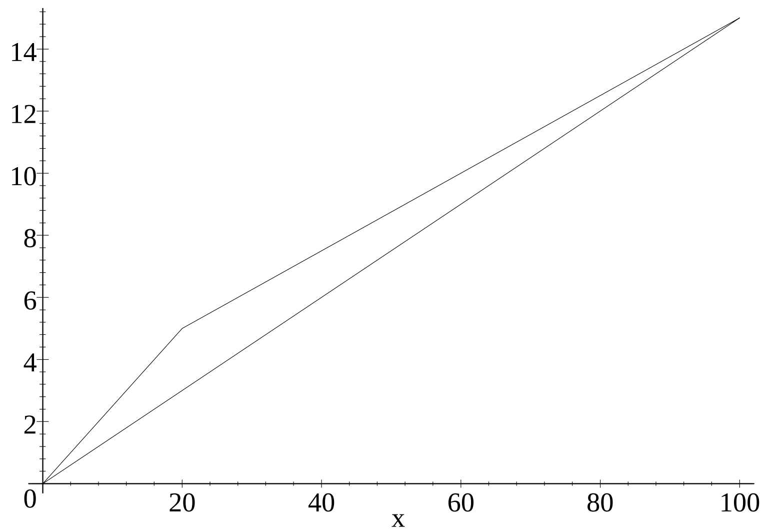
$$C = 80$$

$$\pi_c = 10$$

then the expected rates of return on the individual assets themselves can be defined & calculated

$$R_b = \frac{\pi_b}{B} = \frac{5}{20} = 0.25$$
$$R_c = \frac{\pi_c}{C} = \frac{10}{80} = 0.125$$

so that graphically.



Flow against stock for the sum of two assets

Mathematically this relationship gives the total asset return as a weighted average of the two subasset returns

$$\begin{aligned} \pi_a &= \pi_b + \pi_c \\ R_a A &= R_b B + R_c C \\ R_a &= \frac{B}{A} R_b + \frac{C}{A} R_c \end{aligned} \tag{3}$$

where $\frac{B}{A}$ and $\frac{C}{A}$ are the weights attached to the two individual expected rates of return R_b, R_c. This relationship holds for portfolios of stocks and any other (long lived) investments as well as physical asset, i.e. the expected return operator is linear in its components.

14.14 Return to liability holders; debt

If the asset flow is not entirely attributable to one class of owners, how is the operational flow divided up between these liability holders? If the debt holders are promised interest payments of

$$\pi_d = 5$$

on their (current) face value of Debt

$$D = 50$$

their expected return is

$$R_d = \frac{\pi_d}{D} = \frac{5}{50} = 0.10$$

14.15 Equity

This leaves an expected return attributable to (equity) shareholders

$$\pi_e = \pi_a - \pi_d = 15 - 5 = 10$$

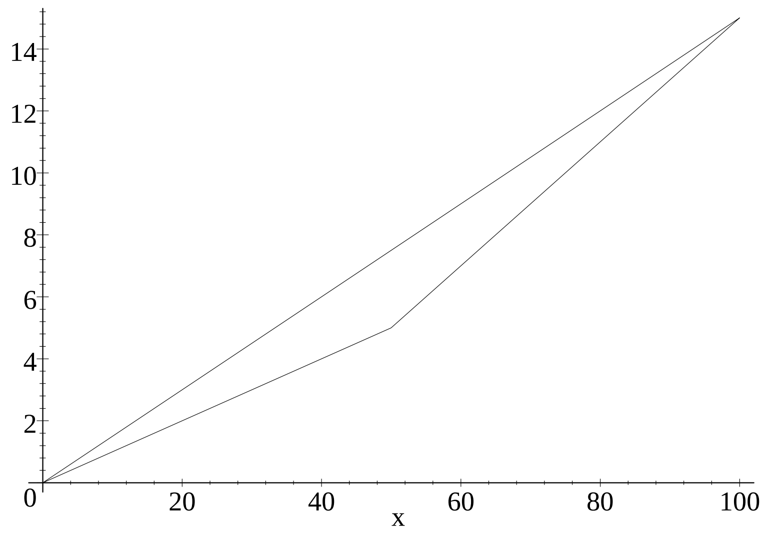
on a (current) value of

$$E = A - D = 50$$

which corresponds to an expected return of

$$R_e = \frac{\pi_e}{E} = \frac{10}{50} = 0.20$$

The return to equity holders is less certain than the return to debtholders (almost certain) so $R_e > R_d$.



Flow against stock for a two liability case

The capital structure is 50% debt and 50% equity ($\frac{D}{A} = \frac{E}{A} = \frac{1}{2}$) so the weighted expected return (asset return) is half that of the return to debt holders and half that of the return to equity holders

$$\begin{aligned} \pi_a &= \pi_d + \pi_e \\ 0.15 &= \frac{1}{2}0.10 + \frac{1}{2}0.20 \end{aligned}$$

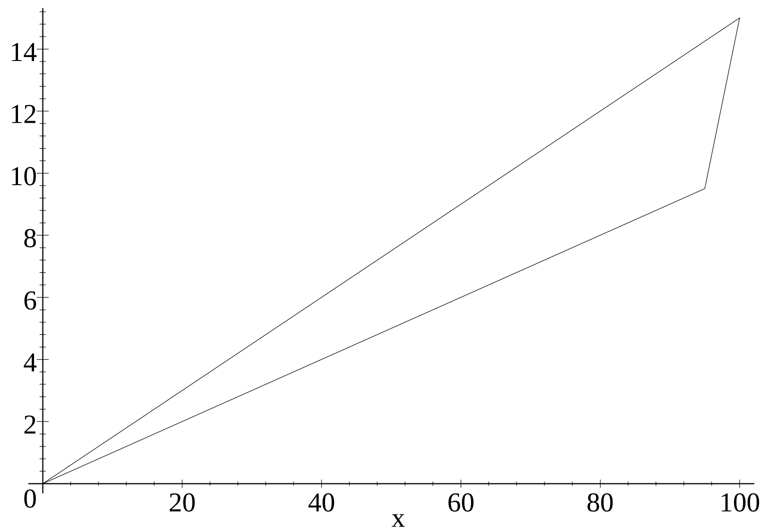
Thus the <u>weighted average cost of capital</u> (across debt and equity) is given by WACC

$$\begin{aligned} R_a A &= R_d D + R_e E \qquad (4) \\ R_a &= \frac{D}{A} R_d + \frac{E}{A} R_e \end{aligned}$$

where it is the <u>market value (not book value)</u> of debt and equity that must be used to weight the expected returns (since expected returns are returns on market investments). Again the expected returns operator is seen to be linear in its components and the return on equity is a linear (but leveraged) function of return on assets less return to debt where new weights are used $\frac{A}{E}, -\frac{D}{E}$ (N.B. these still sum to one)

$$\begin{aligned} R_e &= \frac{A}{E}R_a - \frac{D}{E}R_d \\ 0.20 &= 2 \times 0.15 - 1 \times 0.10 \end{aligned}$$

(N.B. The weighted average asset return - Equation 3 was also the same WACC!) Thus it can be seen that since leverage should not affect asset return R_a and if leverage does not affect the cost of debt (return to debtholders) R_d, R_e can be made arbitrarily large through the choice of extreme leverage!



Flow against stock for the case of extreme leverage

Of course at some extreme point of leverage, the debtholders own virtually of the firm and therefore their claim is as risky as that of the 100% equity financed firm so that

$$E \rightarrow 0 \Longleftrightarrow R_d \rightarrow R_a$$

Thus is absence of taxes, market imperfections (information asymmetries etc.), other frictions (transaction costs), the firm value must be independent of the chosen level of leverage. If it were not, investors could put together a portfolio of some of the firms debt and equity and expect to earn excess returns, but it is the Asset Value and return that is the fixed point from which liability values are derived.

14.16 Other liability holders, e.g. Preferred Equity

What happens if a third tranche of liability holders are present? Denoting R_p for the expected return to preferred equity holders, who own a claim currently worth P and expected cashflow $\pi_p = R_p P$ the weighted average cost of capital formula generalises to

$$\begin{aligned} \pi_a &= \pi_d + \pi_p + \pi_e \\ R_a A &= R_d D + R_p P + R_e E \\ R_a &= \frac{D}{A} R_d + \frac{P}{A} R_p + \frac{E}{A} R_e \end{aligned}$$

where again market weights (not book) must be used. If the preferred claim ranks above equity holders but below debt holders we would expect

$$R_d < R_p < R_e$$

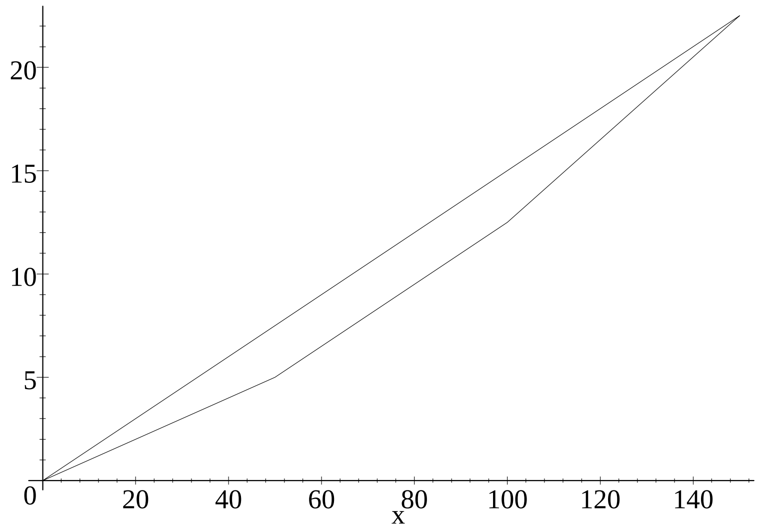
else risk is not rewarded. Selling 50 units of preferred capital to buy 50 units more of existing (like) assets since

$$P = 50$$

and if

$$R_p = .15$$

yields



Flow against stock for the three liability case

Of course any number of asset or liability splits can be implemented this way.

14.17 Earnings per share and other measures

How is the fact that the liabilities can be partitioned anyway without affecting the value of the assets reconciled with the fact that higher earnings per share is favoured? Consider two possible firm structures from the previous case (identical assets worth 100).

Refinancing 45 units of equity by issuing 45 units of debt seems to have greatly increased the per share figure from 0.2 per share to 1.1, a factor of more than five! Is this good? Well the asset value has not changed because the asset cashflows remain unaltered, secondly the equity value has not changed because the required yield has also gone up! All that has changed for the share is its risk return profile and now there is more chance that the firm value will fall below the debt value (equivalently that the profit will fall below the interest expense).

What is really required to add value to a firms equity is to increase the per share dividend while keeping the gearing constant, funnily enough because this usually

50/50 Firm	**Flow**	**Value**	**Yield**
Asset Flow and Value	15	100	15%
Debt Interest and Value	5	50	10%
Equity Dividend and Value	10	50	20%
50 shares in issue	0.2 per share	1.0 per share	

95/5 Firm	**Flow**	**Value**	**Yield**
Asset Flow and Value	15	100	15%
Debt Interest and Value	9.5	95	10%
Equity Dividend and Value	5.5	5	110%
5 shares in issue	1.1 per share	1.0 per share	

Table 18: Effect of leverage on Earnings per Share

involves hard work on the asset side (trimming costs, increasing revenues etc.) slick financial managers are less keen on it as a quick fix, it is easy to increase the gearing and hope no one will notice.

14.18 Modigliani Miller

This statement that the firm value is independent of the capital structure is the seminal work of Modigliani and Miller [**?**]. It is also a statement of linear valuation, the Modigliani Miller proposition and is true for NPV valuation because the NPV rule is a linear operation on it components.

$$NPV\left(B+C\right)=NPV\left(B\right)+NPV\left(C\right)$$

14.19 Weighted Average Cost of Capital (WACC)

In absence of taxes and bankruptcy costs, the WACC remains constant! As the leverage rises, both the cost of equity and the cost of debt will rise! However because the cost of debt is always lower than the return on assets, the weighted average can remain the same.

In the real world we need taxes, bankruptcy costs and other non–linear claims before an optimal capital structure exists.

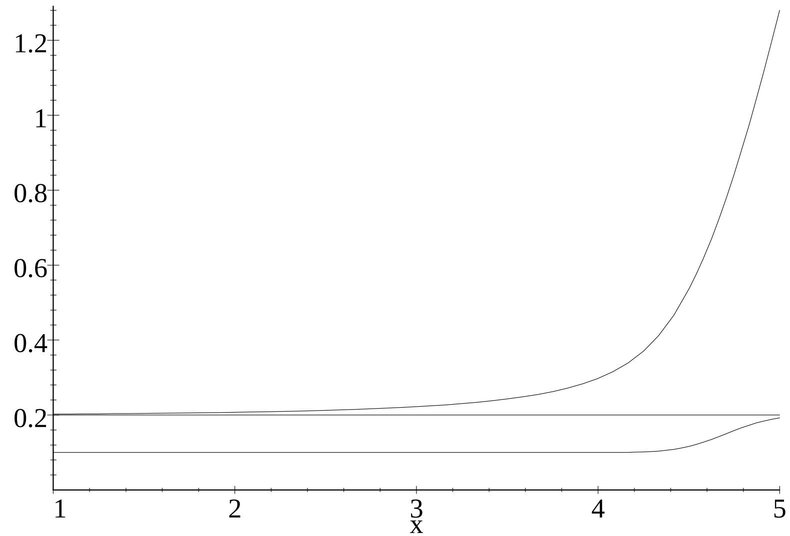


Figure 8: The WACC can remain constant at 20% even though both component costs of Capital (R_E and R_D) rise with increasing leverage ($x = \ln \frac{D}{A}$). NB $R_f = 10\%$.

15 Forwards and futures (B&M 14)

- Spot market: deal today for immediate (maybe tomorrow) settlement (no credit risk beyond settlement)
- Forward market (bi–lateral): deal today for delayed (maybe months ahead) settlement (credit risk until settlement). Zero dividend, stochastic capital gain at expiry.
- Futures (clearing house): deal today for delayed (maybe months ahead) settlement but make good or receive losses each and every day until settlement (no credit risk since all debts squared at the end of each and every day!). Stochastic dividend, zero capital gain at expiry.

The sum of all cashflows is the same for futures and forward contract with the same terms!

15.1 Bilateral v. clearing house markets

- Most spot and forward markets (FX etc.) are self organising and policing so that things are simple but counterparty credit risk is present. Futures markets are run through a clearing house to make sure that losses are made good and therefore counterparty credit is not a risk.

15.2 Futures Exchanges (clearing houses)

- US: Chicago Board of Trade, Chicago Mercantile Exchange, NYMEX, PBOT etc.
- UK: LIFFE, LME, IPE etc.
- Other: MATIF, DTB, TOPIX, SIMEX etc.

15.3 Contract "underlying"

- Stock index futures: S&P500, FTSE100, Nikkei225, CAC, DAX etc.
- Interest rate futures: TBond, TBill, Eurodollar, Gilt, Eurosterling etc.
- Currencies: All major currencies against the U.S.Dollar and cross currencies on local exchanges
- Energy: Oil(s), Petrol, Gas, Coal, other fuels
- Metals: Gold, Silver, Copper, Platinum
- Agricultural: Wheat, Corn, Oats, Soybeans, FCOJ, (Eggs!),Cattle, Hogs, Pork-Bellies!

15.4 Return to speculators and hedgers

- If these (and the other) markets are efficient, everything should be fairly priced (no expected abnormal risk adjusted profit or loss) and the NPV (expected future discounted profit) of any transaction should be zero!
- Speculators should not make excessive risk adjusted returns after costs or there would be incentives for more speculators to enter. Hedgers will also face zero NPV transactions and will neither expect to win or lose from hedging. Hedging should neither create nor destroy value, it will just transfer risk from one party to another.
- If there are returns to risk (CAPM), then the transfer of risk will also involve the transfer of expected return but in a risk adjusted sense parties should be indifferent between hedging and not. Hedgers are thus simply shifting their position of the SML! For example, for a future on a commodity, the CAPM gives

$$E[R_i] = R_f + \beta_i E\left[R_m - R_f\right]$$

but the expected return can be expressed through an expected future spot price $E\left[S_{iT}\right]$ and a current spot price S_{i0}. Setting the forward horizon at $T = 1$ year

$$E\left[R_i\right] = \frac{E\left[S_{i1}\right] - S_{i0}}{S_{i0}}$$

so that real world expectations of future prices need discounting at a risk adjusted rate¶

$$\begin{aligned} \frac{E\left[S_{i1}\right] - S_{i0}}{S_{i0}} &= R_f + \beta_i E\left[R_m - R_f\right] \\ E\left[S_{i1}\right] &= S_{i0}\left(1 + R_f + \beta_i E\left[R_m - R_f\right]\right) \\ S_{i0} &= \frac{E\left[S_{i1}\right]}{\left(1 + R_f + \beta_i E\left[R_m - R_f\right]\right)} \end{aligned}$$

¶This expectation is the so called real world measure $E^P\left[\cdot\right]$

i.e. current market prices are real world expected future prices discounted at a risky rate of return $R_f + \beta_i E\left[R_m - R_f\right]$. Alternatively a so called risk neutral expectation can be formed in which case the risk free rate is the appropriate discount factor‖

$$S_{i0}\left(1 + R_f\right) = E\left[S_{i1}\right] - S_{i0}\beta_i E\left[R_m - R_f\right] \text{ i.e.}$$

$$S_{i0} = \frac{E\left[S_{i1} - S_{i0}\beta_i\left[R_m - R_f\right]\right]}{\left(1 + R_f\right)}$$

- This indicates that under uncertainty, we can formulate spot prices as discounted expected future values using the risk free rate if we adjust the expected value for the systematic risk (subtract $S_{i0}\beta_i E\left[R_m - R_f\right]$). This approach where uncertainty is included in the NPV through the expected cashflows rather than

‖The risk neutral expectation is labelled $E^Q\left[\cdot\right]$ and includes a risk adjustment term.

the discount rate is called the certainty equivalent and is important for futures prices because futures are deferred spot purchases

$$\begin{aligned} {}_0F_{i1} &= \left(1 + R_f\right) S_{i0} \\ &= E\left[S_{i1}\right] - S_{i0}\beta_i E\left[R_m - R_f\right] \end{aligned}$$

- Thus futures prices give us the certainty equivalent of expected spot prices and if the commodity in question has low or zero market β then this will be equal to the expected price but if β is non zero, there will be a risk premium $S_{i0}\beta_i E\left[R_m - R_f\right]$.

15.5 Futures on the index

- If the commodity in question is the market index itself then $\beta_i = 1$ and assuming dividends on physical holdings over the period of $D_{i0,1}$

$$\begin{aligned}
\beta_i S_{i0} E\left[R_m - R_f\right] &= E\left[S_{i1} + D_{i0,1} - S_{i0}\right] - R_f S_{i0} \\
{}_0F_{i1} &= E\left[S_{i1}\right] - E\left[S_{i1} + D_{i0,1} - S_{i0}\right] + R_f S_{i0} \\
&= \left(1 + R_f - \frac{D_{i0,1}}{S_{i0}}\right) S_{i0}
\end{aligned}$$

  so that the futures price of the index is a compounded version of the spot, but using a reduced rate due to futures not benefiting from physical dividends.T

15.6 Currency risk

- There are parity conditions between spot & forward FX rates and interest & inflation rates. By way of summary see Table 19
- The consequence for finance is that foreign discount rates must be used for foreign flows but that due to the parity laws holding (in expectation) this hedged NPV will be the same as the unhedged NPV!
- Consequently if a firm makes a bond issues in a foreign currency it need not necessarily hedge it as the investors (particularly if they have international exposure already) may be indifferent between the unhedged and hedged firm. Even if investors are not indifferent, they can hedge the resulting exposures of all the firms in aggregate rather than individually.
- All three NPVs are expected to be equal. Even if the currency swap remains off the book value balance sheet, because it affects the cashflows of the firm it will

Forward/spot differential $\frac{{}_0F_1}{{}_0S_0}$	FPT expectations and speculation $\longleftrightarrow$	**Expected change in spot** $\frac{E[{}_1S_1]}{{}_0S_0}$
IRPT covered interest rate parity $\updownarrow$	$\searrow\nwarrow$ RRR real rate of return $\nearrow\swarrow$ UIP uncovered interest rate parity	$\updownarrow$ PPPT market law of one (factor) price
$\frac{1+\$r}{1+£r}$ **Interest rate differential**	$\longleftrightarrow$ IFR Fisher equal expected real returns	$\frac{1+E[\$i_1]}{1+E[£i_1]}$ **Expected inflation differential**

Table 19: Interest rate, currency and inflation parities

reside on the market value balance sheet and will transform the exposure of the market value of liabilities from one currency to another.

$$\text{NPV}\begin{pmatrix}\text{foreign discounted,}\\ \text{spot exchanged}\end{pmatrix} = \text{spot rate} \times \frac{\text{£ flows}}{1 + \text{£ rate}}$$

$$\text{NPV}\begin{pmatrix}\text{foreign forward hedged,}\\ \text{domestic discounted}\end{pmatrix} = \frac{\text{£ flows} \times \text{forward rate}}{1 + \text{\$ rate}}$$

$$\text{NPV}\begin{pmatrix}\text{foreign unhedged,}\\ \text{domestic discounted}\end{pmatrix} = \frac{\text{£ flows} \times \begin{matrix}\text{expected future}\\ \text{spot rate}\end{matrix}}{1 + \text{\$ rate}}$$

15.7 Currency forecasts?

- Which currency forecasts (if any) should you use?
- These parity relations have strong consequences for international capital budgeting, namely that projections based on proprietary forecasts are highly questionable and subject to arbitrage.
- The only "forecasts" that are safe to use are those embedded in market prices since they are the only ones which are arbitrage free. To project translation at any other rate is dangerous.
- Foreign flows can either be foreign discounted and spot exchanged, forward hedged and domestic discounted or unhedged and domestic discounted.

16 Options and contingent claims (B&M 15)

16.1 Definition of options vs. forwards

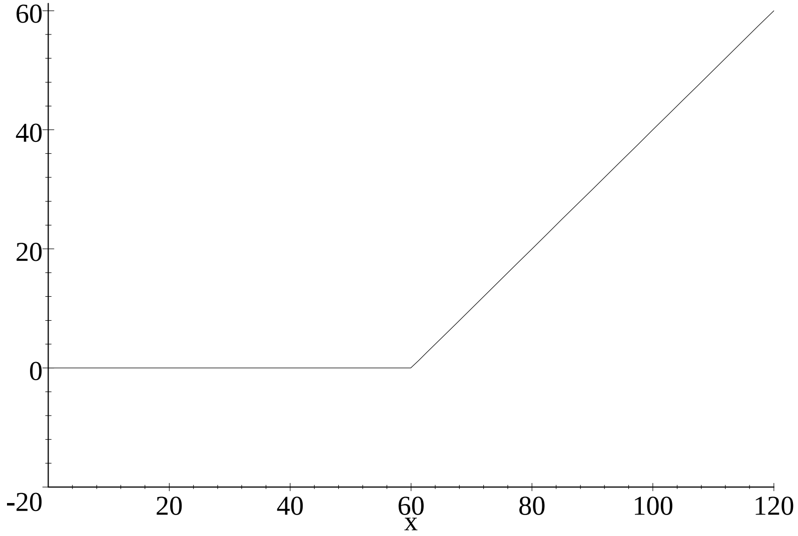
- Call Option: the right but not the obligation to buy (call) a stock (commodity etc.) from someone else at a future date
- Put Option: the right but not the obligation to sell (put) a stock (commodity etc.) to someone else at a future date
- Forward purchase: the right and the obligation to buy at a future date
- Forward sale: the right and the obligation to sell at a future date

Calls (puts) share in 100% of the gains/losses when the final stock price is above (below) the exercise price and none of the gains/losses when the final price is below (above) the exercise price. Being long an option means owning an option, and being short an option means having sold an option.

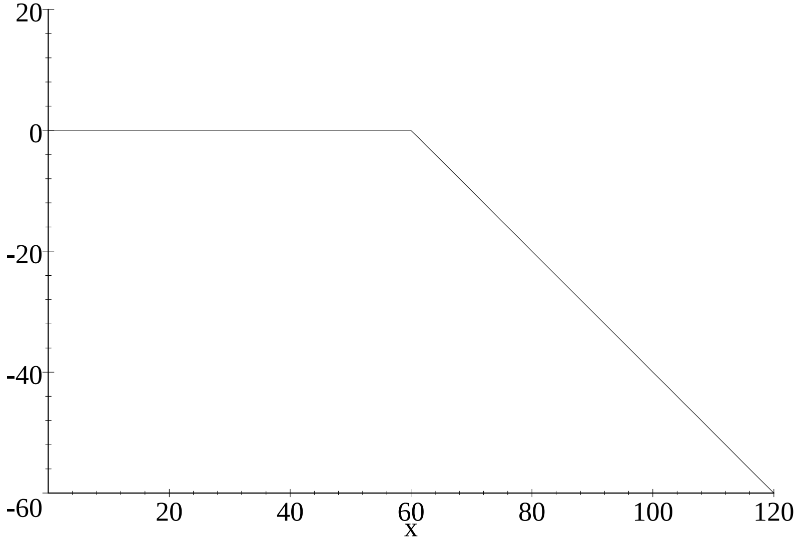
Some Options are exercisable at all times to maturity and these are known as American Options irrespective of their location of issuance or trading or underlying. Other, simpler to price options are known as European, and are exercisable at maturity only, again it is important to note that whether an option has European features may not depend on it's country of origin etc.

16.2 Payoffs at expiry

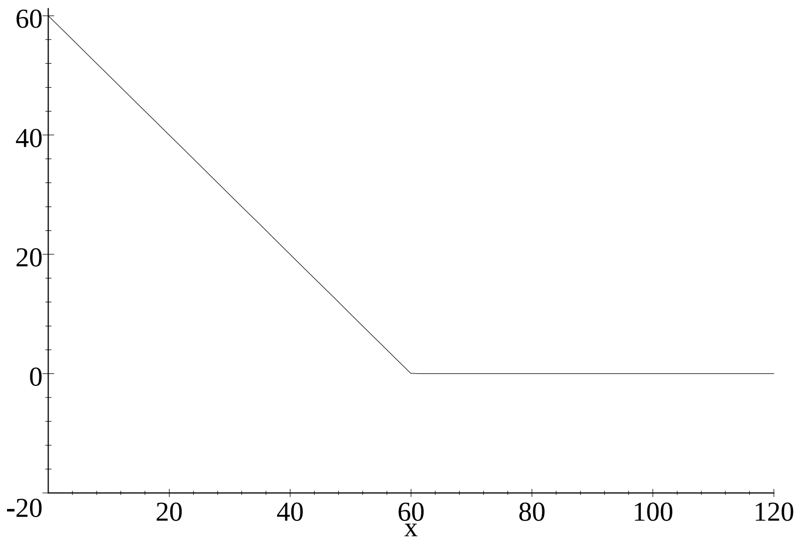
At expiry the option payoffs are graphed



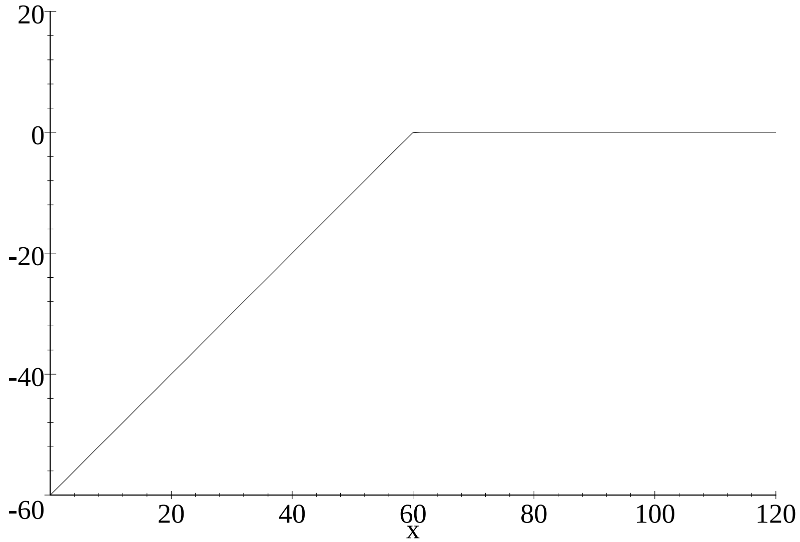
Long a call at 60



Short a call at 60



Long a put at 60



Short a put at 60

- These values at expiry are often referred to as intrinsic values since they refer to the immediate underlying value of the option (N.B. this may not be immediately accessible through early exercise, i.e. may not be European).
- Options are said to be in the money if they have positive intrinsic value and out of the money if their intrinsic value is zero (cannot be negative because an option

once purchased cannot become a liability! why not?).

16.3 Black Scholes Hedging

- The seminal papers are Black and Scholes (1973) [**?**] and Merton (1973) [**?**]. The concept that won the 1997 Nobel Prize for Economics (for Myron Scholes along with Robert Merton; Fischer Black died in 1995) was that options could be priced assuming the option writer (seller) used a dynamic (changing over time) hedging strategy to defray his potential losses.
- If he had written a call that was currently out of the money (zero sensitivity to the stock price) then he had little to worry about or hedge but as the option came into the money and the sensitivity of that position reached 100% of the underlying,

he had better adjust his hedge toward and up to 100% in the underlying (in some smooth and dynamic fashion) since deep in the money option price changes go 1:1 with underlying.

- Thus owning a fraction (delta) of the underlying (between 0 and 1) and adjusting this over time allow neither net profit or loss on the transaction (option sale plus hedge). One way to analyse a trading strategy where hedge ratios can change is to split time to maturity into chunks and form a tree.

16.4 Multiplicative binomial tree

0	1st	1	2nd	2	3rd	3	4th	pff.prms.prb=exvl
								$16.1.\frac{1}{2^4} = 1$
						$12.1.\frac{1}{2^3}\frac{1}{1.1}$	$\nearrow\searrow$	
				$9.1.\frac{1}{2^2}\frac{1}{1.1^2}$	$\nearrow\searrow$			$8.4.\frac{1}{2^4} = 2$
	$\frac{1}{2}$H	$\frac{27}{4}1\frac{1}{2}\frac{1}{1.1^3}$	$\nearrow\searrow$			$6.3.\frac{1}{2^3}\frac{1}{1.1}$	$\nearrow\searrow$	
$\frac{81}{16}\frac{1}{1.1^4}$	$\nearrow\searrow$			$\frac{9}{2}.2.\frac{1}{2^2}\frac{1}{1.1^2}$	$\nearrow\searrow$			$4.6.\frac{1}{2^4} = \frac{3}{2}$
	$\frac{1}{2}$T	$\frac{27}{8}1\frac{1}{2}\frac{1}{1.1^3}$	$\nearrow\searrow$			$3.3.\frac{1}{2^3}\frac{1}{1.1}$	$\nearrow\searrow$	
				$\frac{5}{4}.1.\frac{1}{2^2}\frac{1}{1.1^2}$	$\nearrow\searrow$			$2.4.\frac{1}{2^4} = \frac{1}{2}$
						$\frac{3}{2}.1.\frac{1}{2^3}\frac{1}{1.1}$	$\nearrow\searrow$	
								$1.1.\frac{1}{2^4} = \frac{1}{16}$
$\$\frac{81}{16}\frac{1}{1.1^4}$ $3.458	10% Rm	$\$\frac{81}{8}\frac{1}{2}\frac{1}{1.1^3}$ $3.804		$\$\frac{81}{4}\frac{1}{2^2}\frac{1}{1.1^2}$ $4.184		$\$\frac{81}{2}\frac{1}{2^3}\frac{1}{1.1}$ $4.602		$\$5\frac{1}{16}$ $5.0625
$3.458	PV	$3.458		$3.458		$3.458		$3.458
$3.458	5%Rf	$3.631		$3.812		$4.003		$4.203

Table 20: Four round binomial tree of Heads ($\times 2 \equiv +100\%$), Tails ($\div 2 \equiv -50\%$)

16.5 Black Scholes formula

$$C = SN(d_1) - Xe^{-rT}N(d_2)$$

where the normal areas are defined on two variables

$$d_1 = \frac{\ln S - \ln X + rT + \frac{1}{2}\sigma^2 T}{\sigma T^{\frac{1}{2}}}$$

$$d_2 = \frac{\ln S - \ln X + rT - \frac{1}{2}\sigma^2 T}{\sigma T^{\frac{1}{2}}}$$

$$d_2 = d_1 - \sigma T^{\frac{1}{2}}$$

concisely using $\pm$

$$d_1, d_2 = \frac{\ln S - \ln X + rT \pm \frac{1}{2}\sigma^2 T}{\sigma T^{\frac{1}{2}}}$$

16.6 Option Inputs

- Stock Price S : the traded value of the stock in question
- Exercise Price: the critical price above (below) which calls (puts) will be exercised
- Time to Maturity (expiry): the fraction of years until the option must be exercised
- Interest Rate: the rate of interest (expressed annually) for the period to maturity of the option
- Volatility: the standard deviation (expressed as an annual quantity) of the % stock price movements

Stock Price	$S = 70$	
Exercise Price	$X = 45$	
Interest Rate	$r = 0.06$	
Volatility	$\sigma = 0.2^{\frac{1}{2}}$	$= 0.44721$
Time to Maturity	$T = 3/12$	$= 0.25$

derived inputs (ln means natural log - base e) Cumulative Normal Distribution

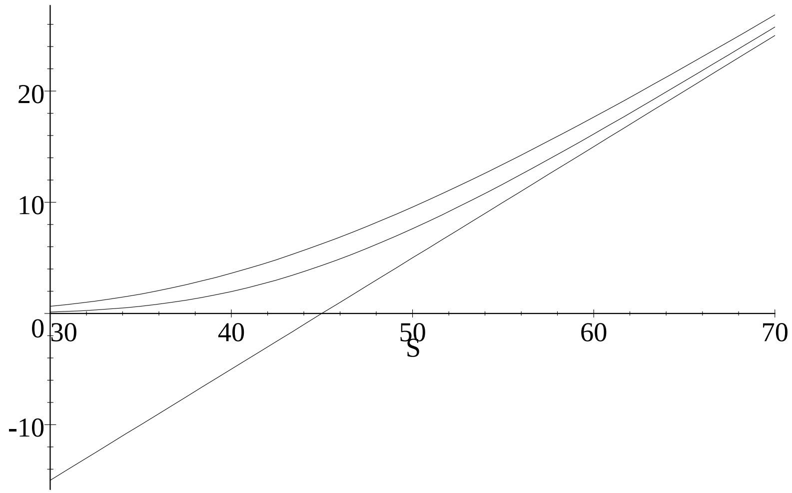
$$N(x) = \mathsf{NormalDist}(x; 0, 1) = \frac{1}{(2\pi)^{\frac{1}{2}}} \int_{-\infty}^{x} e^{-\frac{1}{2}s^2} ds$$

$$N(x) = \frac{1}{(2\pi)^{\frac{1}{2}}} \int_{-\infty}^{x} e^{-\frac{1}{2}s^2} ds$$

$$N(x) = \frac{1}{2}\left(1 + \mathsf{erf}\left(\frac{x}{2^{\frac{1}{2}}}\right)\right)$$

16.7 Call Price

Using values above, the call price can be plotted as a function of the stock price S.



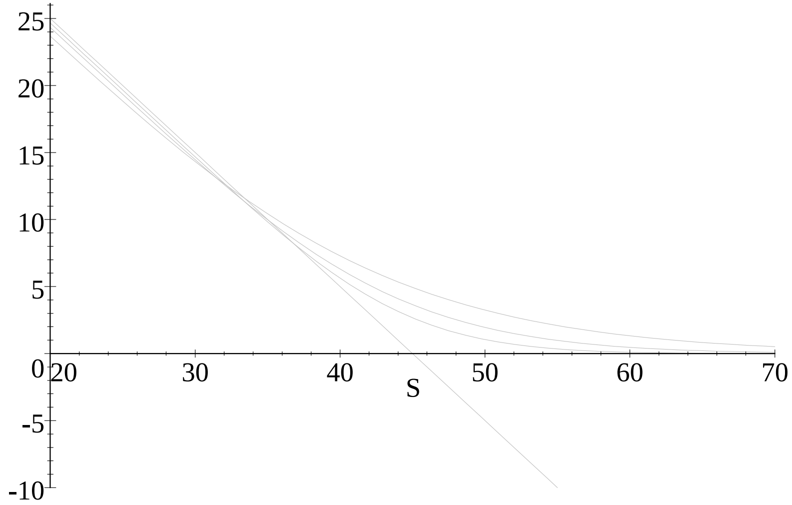
Call at 45

S	$C(S, X = 45, T = 0.25)$	$P(S, X = 45, T = 0.25)$	$S - Xe^{-rT}$
30	0.1325	14.462	$30 - 44.330 = -14.330$
40	1.9762	6.3038	$40 - 44.330 = -4.330$
50	7.6243	1.9517	$50 - 44.330 = 5.670$
60	16.136	0.46458	$60 - 44.33.0 = 15.670$
70	25.764	0.09327	$70 - 44.330 = 25.670$

16.8 Put Call Parity and Put Price

Call less Put (of the same exercise price) is equivalent to owning the stock less a borrowed amount since you either exercise the call (and own the stock) or someone will exercise their (put) option on you and sell it to you (so you will definitely own the stock either way if prices go up or down!).

$$
\begin{aligned}
C - P &= S - Xe^{-rT} \\
C &= SN(d_1) - Xe^{-rT}N(d_2) \\
P &= SN(d_1) - Xe^{-rT}N(d_2) - S + Xe^{-rT} \\
&= S(N(d_1) - 1) - Xe^{-rT}(1 - N(d_2)) \\
&= Xe^{-rT}N(-d_2) - SN(-d_1)
\end{aligned}
$$



Puts of various maturity at 45

16.9 Implied volatility

- If options are no more than leveraged investments in the underlying and they can be replicated rather than purchased (this is precisely how they are priced) why is everyone not replicating instead of actually buying options? (If options are easily replicated are they not a redundant security?).
- Well the answer lies in what (extra) they allow us to trade and the extra item that options allow us to observe and trade is future expected volatility. All other elements in the BS formula are known at the time of purchase but whether the option writer and hedger will profit or lose depends if the realised volatility he experiences over the life of the hedging programme falls short of or exceeds his estimate implied by his sale price.
- Thus option traders are really taking a view on future volatility and if you are long an option (all else equal) you will benefit (lose) if volatility increases (decreases).

- Having bought or sold an option the particular price struck can be used to infer a volatility (implied volatility) which will likely differ from historical volatility because the future market conditions could be more or less risky than the past. Volatility is itself non constant, i.e. volatile! The volatility level negotiated between buyer and seller should represent the best guess about future volatility else arbitrage opportunities may exist by out guessing the level of future volatility.
- Because of this many look at implieds as a forecast of future likely volatility. It is the extra information and opportunities to trade volatility that options afford that causes them to be so widely traded, but strangely if it is the non constancy of volatility that causes this, an option pricing model that specifically allows for changing volatility should be used (not BS). Such a model is again more complicated to implement than BS and so in practice BS (with fudge factors) is

still applied!

Historical Volatility	Future Volatility
$\sigma^2_{-T,0} = \frac{1}{T}\sum_{i=-T}^{0}(r_i - \overline{r})^2$	$\sigma^2_{0,T} = \frac{1}{T}\sum_{i=0}^{T}(r_i - \overline{r})^2$
can be measured at $T = 0$	can only be estimated at $T = 0$

- Solving for the option volatility in BS is a tricky task analytically but on a spreadsheet it is easy, having set up the BS formula allow σ to depend on itself plus a change dependent on the magnitude the calculated option price differs from the traded price (so that it goes up if the calculated price is below the traded and vice versa), recalculate the spreadsheet again and again until the pricing error becomes negligible.
- Alternatively use something like Excel's GoalSeek function. Very often option traders buy and sell simply by quoting the implied volatility and then deriving rather than stating the actual price!