Intermittent Demand Forecasting

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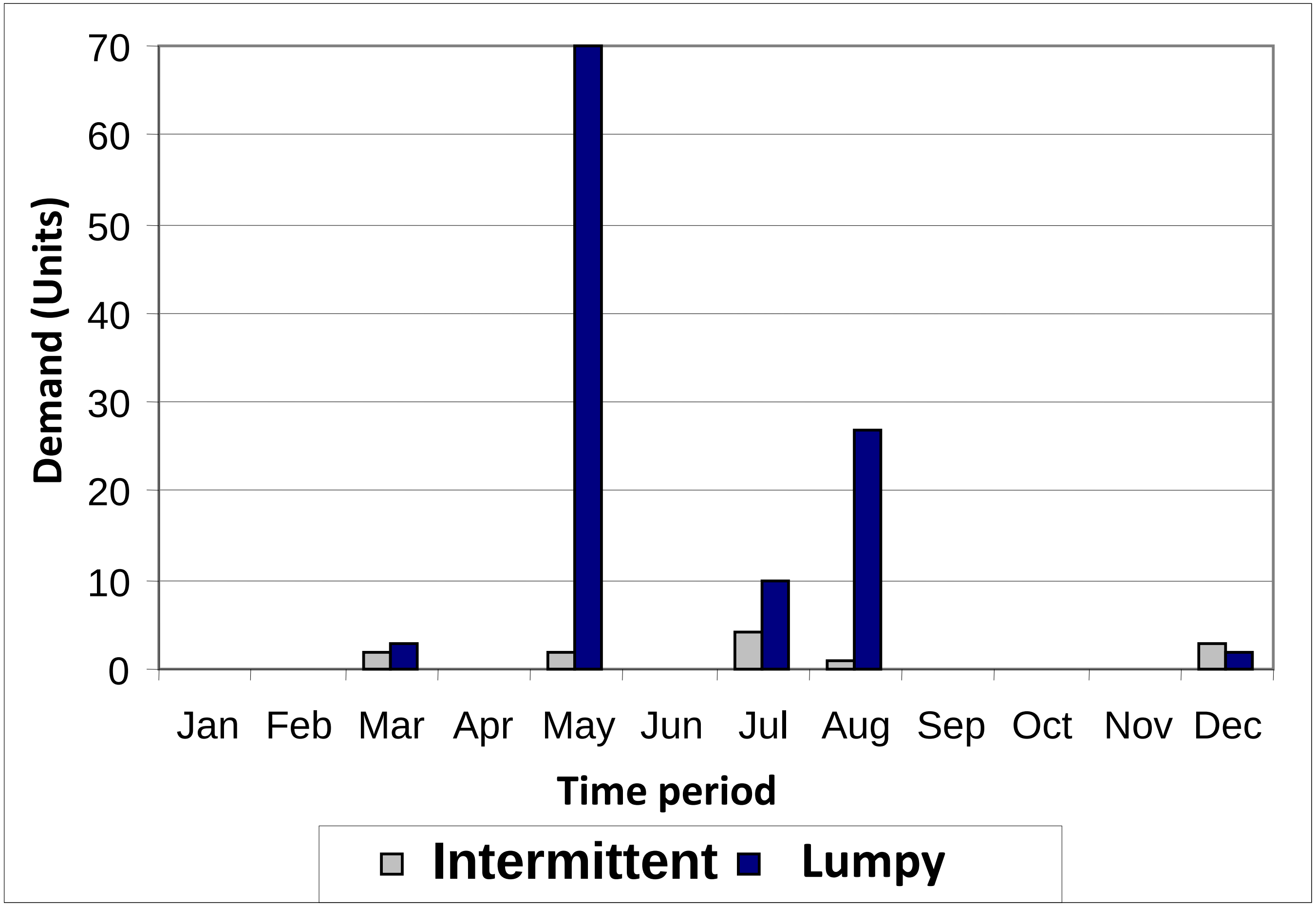
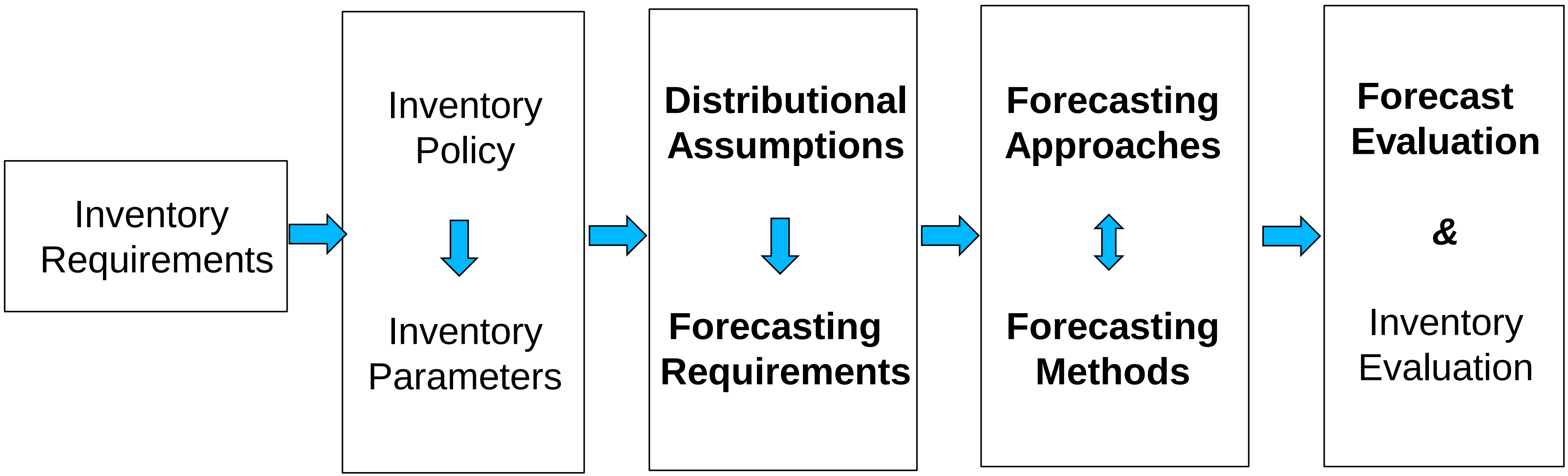
What is Intermittent Demand?



Framework

Need to understand forecasting in broader framework:



Focus of this talk will be on aspects in bold.

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Business Context

- Growth of warehousing to service online demand
- Broadening of stock base at large retailers

- Service becoming ever more critical
- Environmental costs of obsolescence

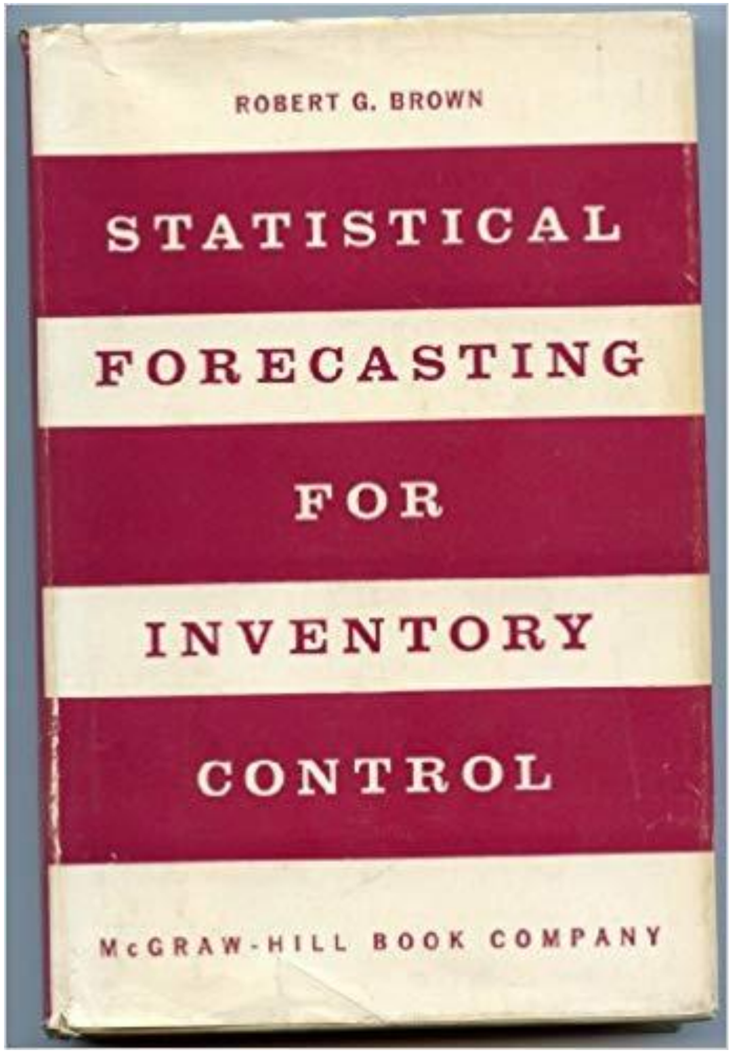
- Greater granularity of data
- ERP systems not geared up for intermittence

Brown (1959)

Brief allusion to slow-moving items on one page.

No discussion of special requirements for forecasting or inventory control.

General recommendation: Exponential Smoothing for mean demand and standard deviation of forecast errors.



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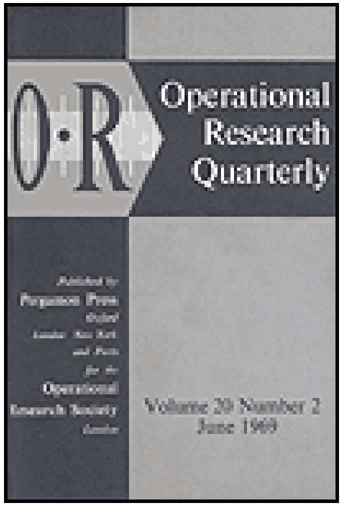
Boothroyd & Tomlinson (1963)

Stock Control of Engineering Spares

Examined all parameters

Total Cost most sensitive to assumed demand rate

Underestimation by 25% - serious impact



"The prediction of demand is ultimately the responsibility of the people who operate the stock control system. We have not offered a mechanical set of rules which will replace the exercise of their discretion".

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Croston (1972)

- **John Croston** – Statistician at P&O, London.
- Audited stock control system, which used Exponential Smoothing for all slow demand items.

$$\hat{d}_t = \alpha d_{t-1} + (1-\alpha)\hat{d}_{t-1}$$

- For some slow demand items, stock levels were excessive.
- Errors were associated with intermittent and lumpy demand items.

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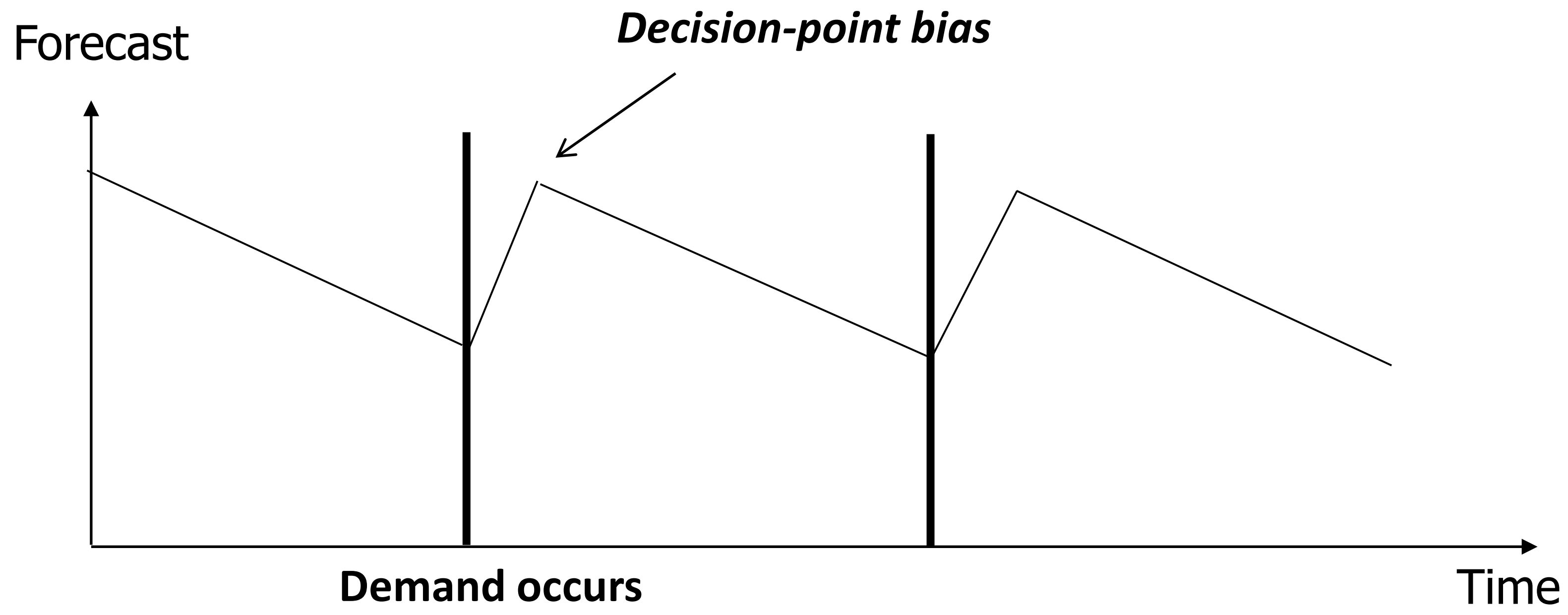
Croston's Basic Finding

If used immediately after a demand occurrence, then Exponential Smoothing is upwardly biased for intermittent demand.



Croston's Method

If demand occurs

1. Re-estimate mean demand size ($\hat{\mu}_S$) using Exponential Smoothing
2. Re-Estimate mean demand interval ($\hat{\mu}_I$) using Exponential Smoothing (same smoothing constant)
3. Re-estimate mean demand : $\hat{\mu} = \hat{\mu}_S \; / \; \hat{\mu}_I$

Else if demand does not occur

Do not re-estimate

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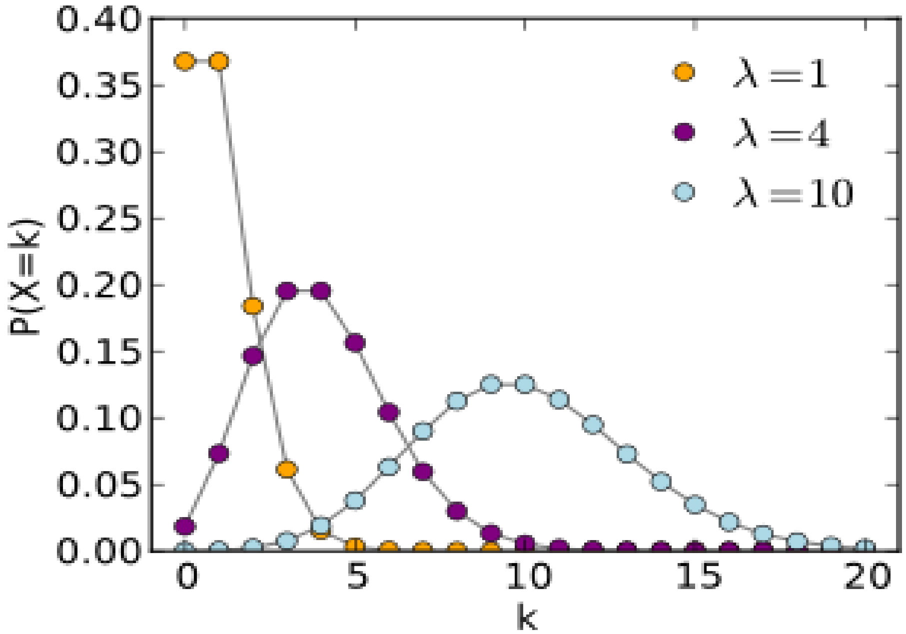
Input to Poisson Distribution

Poisson Distribution:

Input Forecasted Mean to get whole Lead-Time (*L*) Distribution



$\lambda = L \hat{\mu}$

k=Potential Demand Value

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Croston's Method has its own bias

Arises because estimate of Mean Demand Interval is inverted.

$$E(CRO) = E\left(\frac{\hat{\mu}_S}{\hat{\mu}_I}\right) = E(\hat{\mu}_S)E\left(\frac{1}{\hat{\mu}_I}\right) \neq \frac{E(\hat{\mu}_S)}{E(\hat{\mu}_I)} = \frac{\mu_S}{\mu_I} = \mu$$

		Percentage of periods with demand occurrences				
		10%	30%	50%	70%	90%
Smooth Constant	0.1	5%	4%	3%	2%	1%
	0.2	10%	8%	6%	3%	1%

Syntetos & Boylan (2001)

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Syntetos & Boylan (2005)

Syntetos-Boylan Approximation (SBA)

$$SBA = \left(1 - \frac{\alpha_I}{2}\right) CRO = \left(1 - \frac{\alpha_I}{2}\right) \frac{\hat{\mu}_S}{\hat{\mu}_I}$$

- α_I = Smoothing constant for Demand Interval
  (can use different smoothing factors for Size and Interval)

- Approximately unbiased after applying deflation factor.

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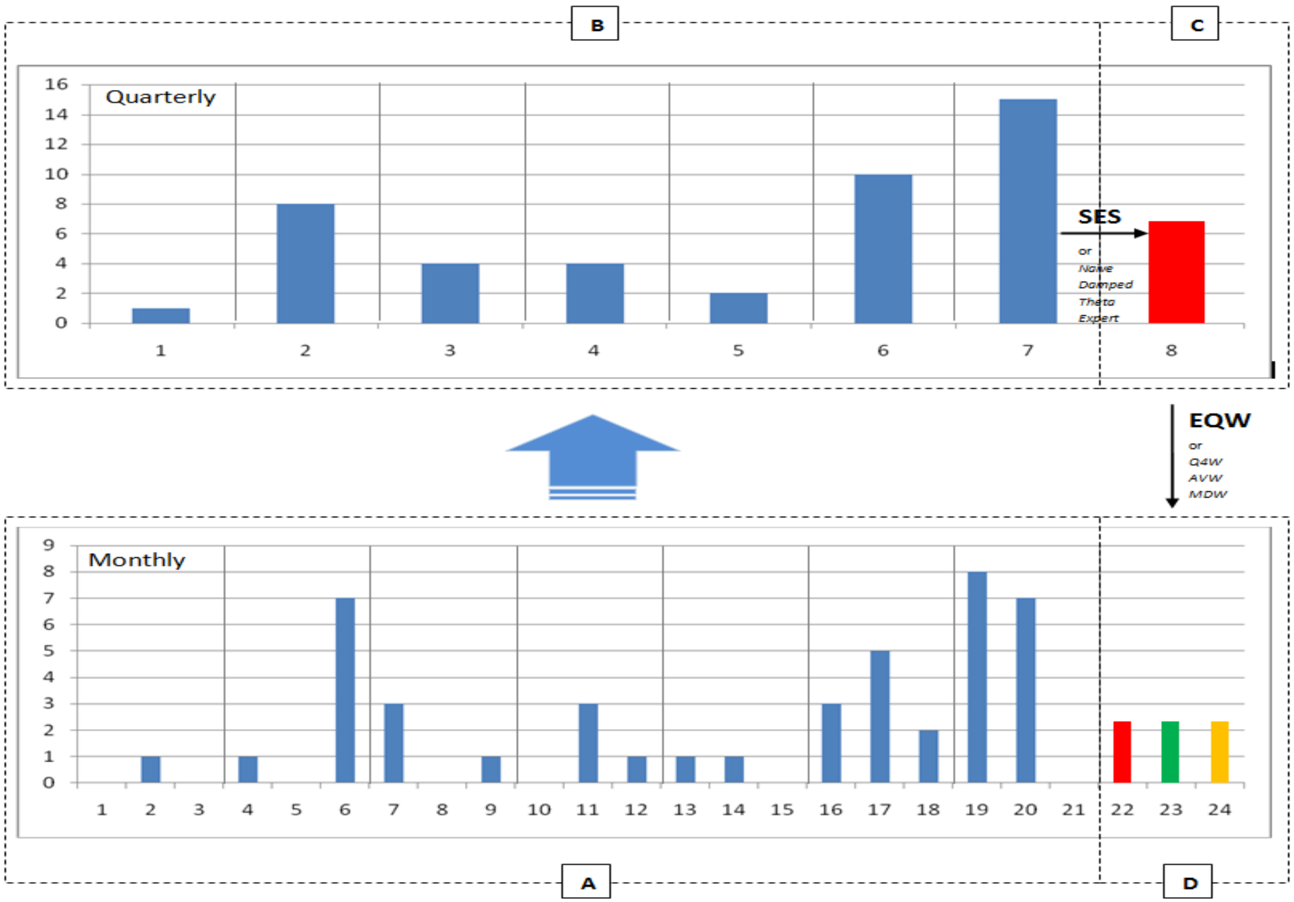
Nikolopoulos et al (2011)

Aggregate-Disaggregate Intermittent Demand Approach (ADIDA)
(A self-improvement approach)



Benefits from ADIDA

Natural approach for Lead-Time Forecasts:

- No need to disaggregate!
- Captures auto-correlation within blocks of time

Empirical analysis (5000 SKUs) shows ADIDA to be a ‘self-improvement’ mechanism for Naïve and SBA

Further empirical study (Babai et al, 2012) found ADIDA to self-improve SES, CRO and SBA in terms of a cost-service analysis.

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Combination of Methods

Combination of Methods (Same Frequency)

- Averaging forecasts for different methods has shown good results for fast-moving data (Makridakis & Hibon, 2000)
- Try averaging CRO, SBA or SMA, CRO, SBA or SES, CRO, SBA
- Empirical study showed no improvement over single method (eg SBA)

Combination of Frequencies (Same Method)

- Small accuracy improvements in Naïve, SMA and CRO by using combinations instead of ADIDA
- Slightly worse accuracy for CRO and SBA

Petropoulos & Kourentzes (2015)

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Parametric Approaches

Parametric distributions often recommended for inventory models

- Normal, Gamma
- Bernoulli (and Compound Bernoulli)
- Poisson (and Compound Poisson)

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Empirical Evidence

Demand per period

Dataset Information		Percentage of SKUs with Strong Fit		
Dataset	Number of SKUs	Poisson	Stuttering Poisson	Negative Binomial
US Defense	4588	20.1%	92.4%	87.2%
RAF	5000	43.8%	99.9%	93.2%
Electronics	3055	14.2%	91.7%	95.4%

Lead Time Demand

Dataset Information		Percentage of SKUs with Strong Fit		
Dataset	Number of SKUs	Poisson	Stuttering Poisson	Negative Binomial
RAF	5000	28.9%	71.9%	59.3%
Electronics	3055	11.1%	88.9%	89.8%

Syntetos et al (2013)

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Compound Poisson Demand Parameters

Stuttering Poisson and NBD are both Compound Poisson:

- Poisson Incidence, Geometric Transaction Size (sP)
- Poisson Incidence, Logarithmic Transaction Size (NBD)

Affords potential to avoid variance estimates and to estimate mean demand incidence and mean transaction size.

Warning: classic application of Croston does not work, as it does not give mean interval between transactions or mean size of transactions.

Prak et al (2018)

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Non-Parametric Approaches

P1	P2	P3	P4	P5	P6	P7	P8	P9	P10

Suppose LT = 3 and Block-Size (m) = 3

Non-Overlapping Blocks

Block 1={P2, P3, P4}, Block 2={P5, P6, P7}, Block 3={P8, P9, P10}

Overlapping Blocks

Block 1 = {P1, P2, P3}, Block 2 = {P2, P3, P4} , ... , Block 8 = {P8, P9, P10}

Overlapping Blocks produce more accurate CDF estimates, except for very short histories (Boylan & Babai, 2016).

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Willemain et al (2004)

Bootstrapping Approach

- Sample h-period ahead demand from historical data (h = 1,…,L ; L=lead time)
- Calculate total lead-time demand
- Repeat a large number of times
- Calculate percentiles of the lead-time demand distribution

Comments

- Method assumes that the underlying distribution – although unknown – is not changing over time.
- May be little non-zero data to sample from

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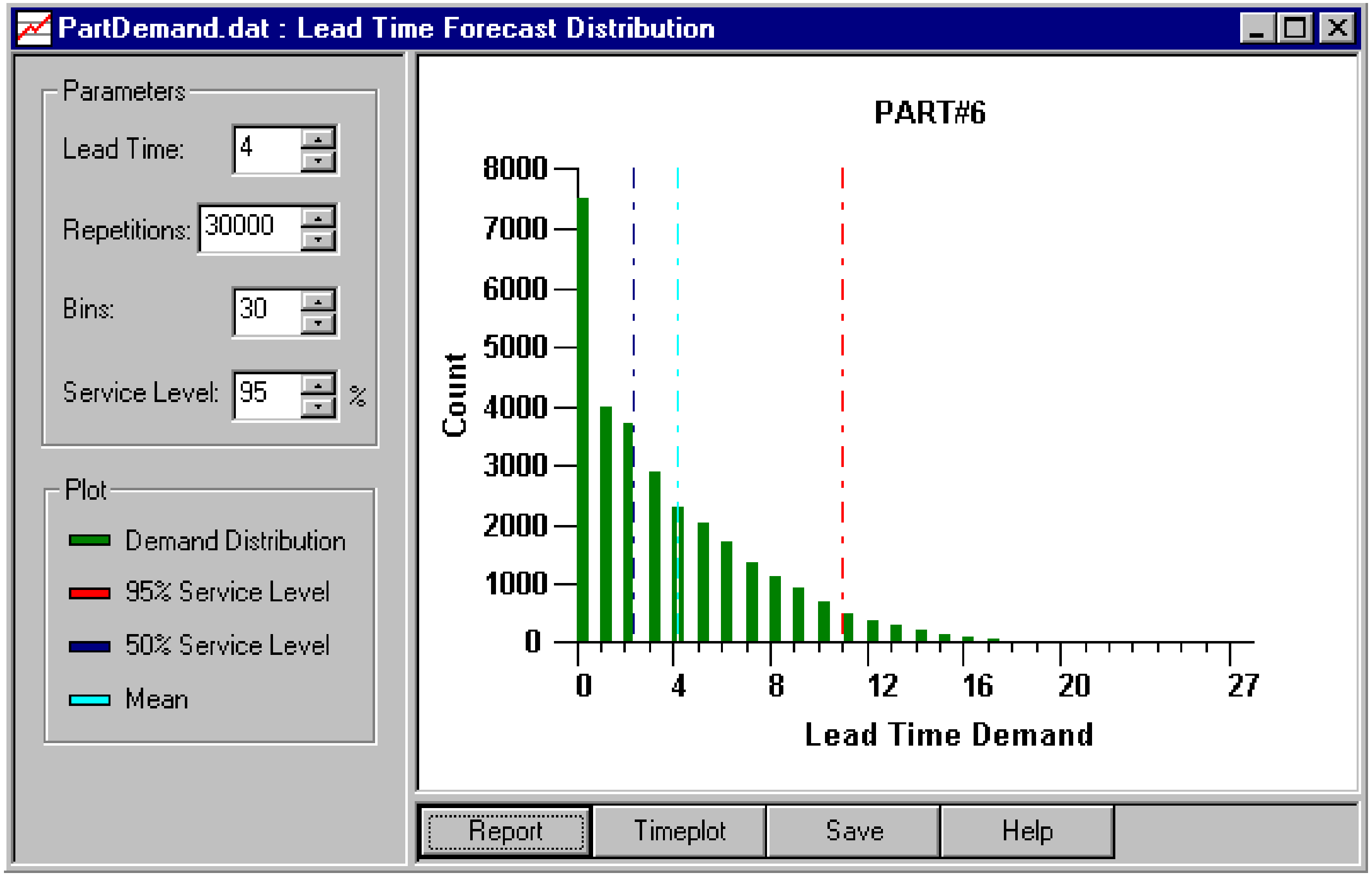
Bootstrapping Software



Incorporates Markov Chain switching (between 'demand ' and 'non-demand' states), and "jittering".

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Modelling Auto-Correlation

- SES, Croston, SBA do not take auto-correlation into account in producing forecasts.
- Empirical study by Willemain (1994) found evidence of auto-correlations in intermittent demand data.

How to model?

- Cannot use ARMA modelling directly because all data non-negative integer.

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Integer ARMA (INARMA) Models

Poisson INAR(1)

$$y_t = \alpha \circ y_{t-1} + z_t$$

z_t Poisson variable (mean λ) – can be thought of as representing new demand

$\alpha \circ Y = \sum_{i=1}^{Y} X_i$ [where $P(X_i = 1) = \alpha$ is a sequence of iid Bernoulli random variables] can be thought of as representing the retention of (some) demand.

Can estimate parameters using Maximum Likelihood.

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Lead-Time Forecast using INAR(1)

$$E\left(\sum_{i=1}^{L} y_{t+i} \mid y_t\right) = \frac{\alpha(1-\alpha^L)}{1-\alpha} y_t + \frac{\lambda}{1-\alpha}\left(L - \sum_{i=1}^{L} \alpha^i\right)$$

Evaluated on empirical data (Mohammadipour, 2009)

Found small but noticeable improvements in forecast error over SES, SBA.

Less promising results for other INARMA models, specifically INMA(1) and INARMA(1,1)

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State-Space Modelling Appraoch

General framework (called iETS), with Bernoulli process of demand occurrence.

Special case, iETS(M,N,N):

$$y_t = o_t z_t$$
$$z_t = l_{z,t-t}(1+\varepsilon_t)$$
$$l_{z,t} = l_{z,t-1}(1+\alpha_z \varepsilon_t)$$

First variable: demand occurrence (1=occurs; 0= not occurs)

Second variable: demand size in a period – if use continuous distribution, then rounded up to next integer. (eg assume model error term to be iid log-normal).

Svetunkov & Boylan (2017)

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Comments on State-Space Approach

- Can estimate parameters using maximum likelihood.
- Can choose between models using Information Criteria (eg AIC)

- Extendable to include trend and seasonality terms, just as in the original ETS framework.

- Current empirical investigation on retail demand forecasting showing benefits from incorporation of trend.

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Forecast Evaluation: Mean Error

- Mean Error takes the average of all the signed errors:

$$ME = \frac{1}{n}\sum_{i=1}^{n}(y_t - \hat{y}_t)$$

- Strictly, this is scale dependent (ie can be expected to be of larger magnitude for series of larger magnitude).
- However, ME is not heavily scale-dependent, because of the cancellation effect of positive and negative errors.
- Useful diagnostic: if this is consistently negative, then forecasts are biased upwards and over-stocking will result.

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Mean Absolute Error

- Mean Absolute Error:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|y_t - \hat{y}_t\right|$$

is optimised on the median value, not the mean.

- If demand is fast, the data is often symmetric, and the mean and the mean and the median are similar.
- If demand is slow, then data is asymmetric and the mean and the median may differ substantially.
- Also, MAE is scale-dependent.

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Mean Absolute Scaled Error

$$MASE = \frac{1}{n}\sum_{i=1}^{n}\frac{|A_t - F_t|}{MAE_{in-sample\,1-step-ahead\,Naive}}$$

Hyndman & Koehler (2006), Hyndman (2006)

- Can always be calculated for intermittent demand (unless all values are zero)

BUT

- Suffers from the same problem as the MAE, as this measure is based on Absolute Errors.
- If data is very sparse, it may show a worse result than Naïve, even though Naïve is inappropriate.

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Mean Scaled Error

$$MScE = \frac{1}{n}\sum_{i=1}^{n}\frac{(A_t - F_t)}{MAE_{in-sample\,1-step-ahead\ Naive}}$$

- Can also always be calculated for intermittent demand (unless all values are zero) .

- Can be used as a complementary measure to MASE.
- MASE should not be used on its own for intermittent demand data.

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Periods in Stock (PIS) Measure

- Assumes a “fictitious” stock from which the over- or under-stock is measured
- It is the sum of the cumulative biases:

$$PIS_n = -\sum_{i=1}^{n}\sum_{j=1}^{i}(A_j - F_j)$$

Wallstrom & Segerstedt (2010)

- Scale dependent but can be adjusted - scaled Absolute Periods in Stock, Kourentzes (2014)
- Can be seen as a proxy for inventory measures and does not attempt to measure accuracy directly

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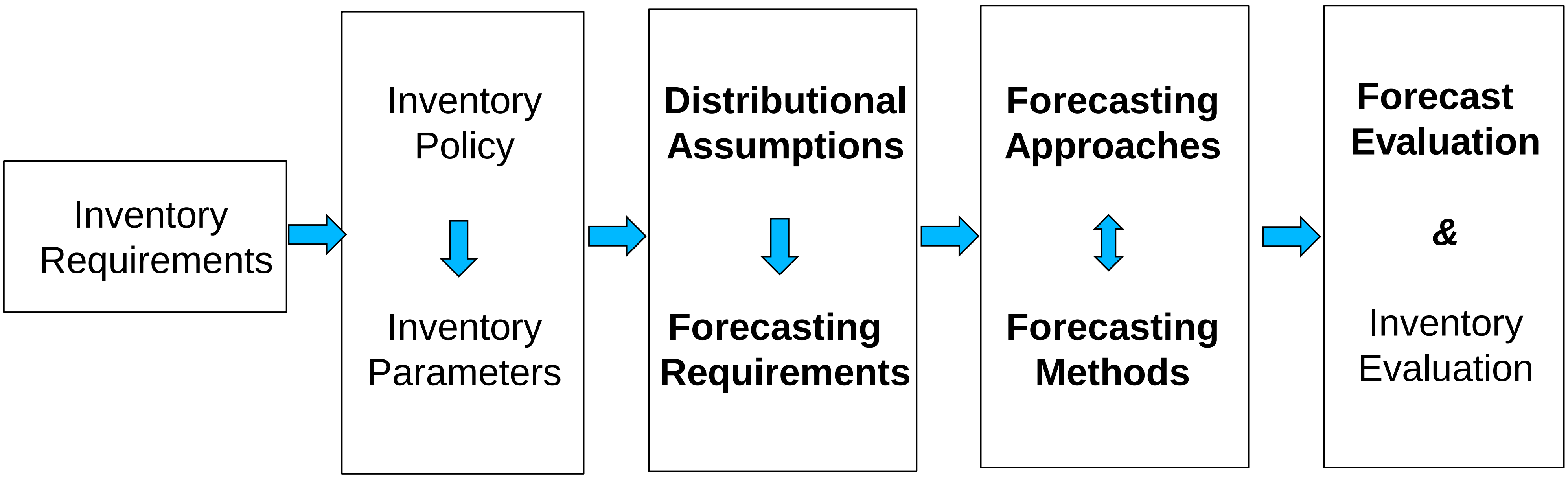
Back to the Framework

Need to understand forecasting in broader framework:



Research: greater emphasis on linkages needed.

Software: need to narrow innovation-adoption gap.

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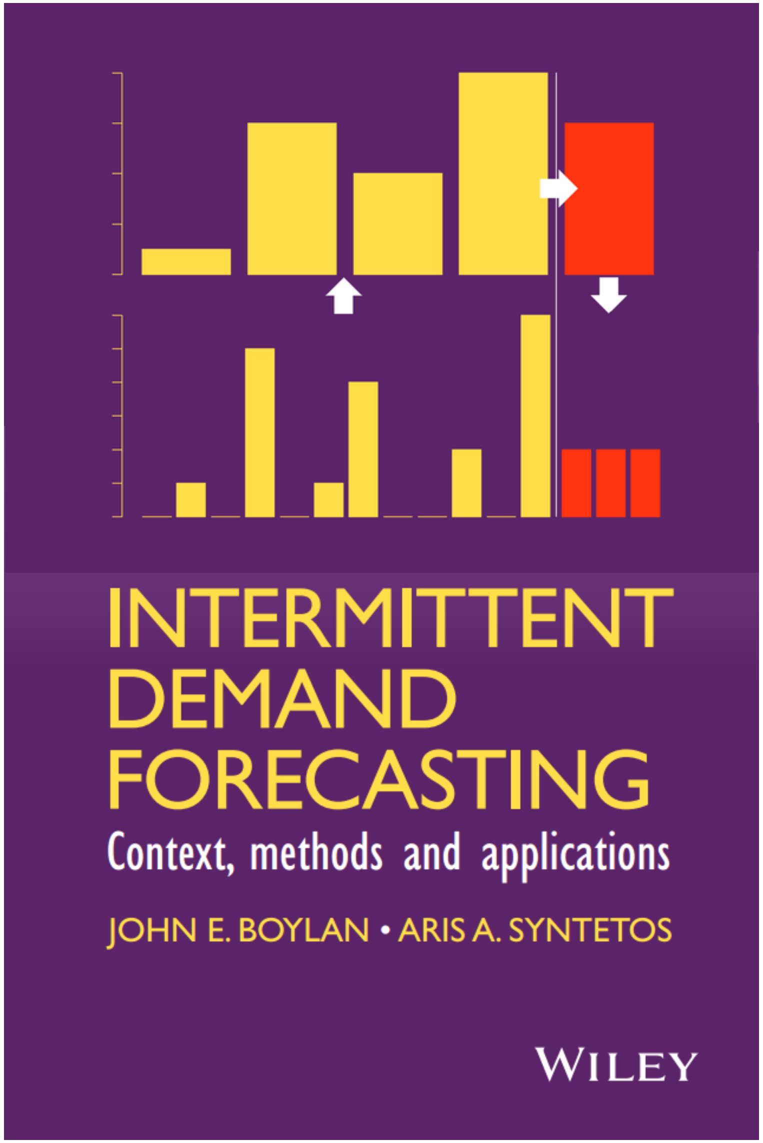
A book is needed on this subject!



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Thank you for your attention!

Q&A?

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